STA 6167 - Exam 2 - Spring 2015 - PRINT Name ANSWER KEY

For all significance tests, use $\alpha = 0.05$ significance level.

Q.1. A split-plot experiment was conducted, comparing 4 coagulant treatments of camel chymosin (HBCC, LBCC, HCC, LCC) and age (4 levels) on Y = strand thickness of melted mozzarella cheese. The experiment was conducted on 3 cheese-making days (blocks), with coagulant treatment as the whole-plot factor, and age as the sub-plot factor.

p.T.a. Complete the ANOVA Table.

| Source of Variation | df | SS | MS | F_obs | F(0.05) |
|---------------------|--------------|---------|-------|--------|---------|
| WP Factor | 4-1=3 | 959.7 | 319.9 | 571.25 | 4.757 |
| Block | 3-1=2 | 4.8 | 2.4 | #N/A | #N/A |
| WP*Block | 3(2)=6 | 3.36 | 0.56 | #N/A | #N/A |
| SP Factor | 4-1=3 | 68.7 | 22.9 | 38.49 | 3.009 |
| WP*SP | 3(3)=9 | 4.878 | 0.542 | 0.91 | 2.300 |
| Error2 | 4(2)(3)=24 | 14.28 | 0.595 | #N/A | #N/A |
| Total | 4(3)(4)-1=47 | 1055.72 | #N/A | #N/A | #N/A |

p.1.b. Assuming the coagulant treatment/age interaction is not significant, compute Tukey's W for comparing all pairs of coagulant treatments.

$$W = 9(0.05; 9=4, df=6) \sqrt{\frac{MSWP*B}{bC}}$$

$$= 4.896 \sqrt{\frac{0.56}{3(4)}} = 4.896(0.216) = 1.058$$

p.1.c. Assuming the coagulant treatment/age interaction is not significant, compute Bonferroni's B for comparing all pairs of age.

$$B = \left(\frac{-0.25}{6}, 24\right) \sqrt{\frac{2 M S \epsilon 2}{a b}} = 2.875 \sqrt{\frac{2(-595)}{4(3)}}$$
$$= 2.875 \left(0.315\right) = 0.905$$

Q.2. An experiment was conducted to Compare 4 Stretching protocols (1=Static Stretching, 2=Dynamic Stretching, 3=Combination Stretching, 4=No Stretch) among a sample of 10 dancers (blocks) on vertical jump height. The following table gives the vertical leaps for the dancers under the 4 stretching protocols, and some of the within dancer ranks of the protocols.

p.2.a. Complete the Rankings.

| Dancer | Stretch1 | Stretch2 | Stretch3 | Stretch4 | Rank1 | Rank2 | Rank3 | Rank4 |
|--------|----------|----------|----------|----------|-------|-------|-------|-------|
| 1 | 32.76 | 35.79 | 34.69 | 30.71 | 2 | 4 | 3 | 1 |
| 2 | 32.67 | 37.23 | 37.24 | 32.06 | 2 | 3 | 4 | 1 |
| 3 | 23.04 | 25.71 | 25.74 | 25.71 | 1 | 2.5 | 4 | 2.5 |
| 4 | 45.63 | 46.68 | 47.03 | 45.38 | 2 | . 3 | 4 | 1 |
| 5 | 29.29 | 32.61 | 34.32 | 30.06 | 1 | 3 | 4 | 2 |
| 6 | 28.9 | 34.53 | 31.17 | 27.96 | 2 | 4 | 3 | 1 |
| . 7 | 42.23 | 41.73 | 43.41 | 40.14 | 3 | 2 | 4 | 1 |
| 8 | 50.07 | 54.27 | 54.04 | 51.14 | 1 | 4 | · 3 | 2 |
| 9 | 49.63 | 56.42 | 53.98 | 49.52 | 2 | 4 | 3 | l |
| 10 | 45.88 | 49.33 | 48.87 | 44.91 | 2 | Y | 3 | (|
| | | | | TOTA | L 18 | 33. | 5 35 | 13 5 |

p.2.b. Compute the Rank Sums for each of the Stretch Protocols.

$$T_1 = \frac{18}{1}$$
 $T_2 = \frac{33.5}{1}$ $T_3 = \frac{35}{1}$ $T_4 = \frac{13.5}{1}$

p.2.c. Conduct Friedman's Test to test whether the medians of distributions of Vertical Leaps differ among the Stretch

Protocols. H₀: Distribution medians are all equal.
$$b = 10 \text{ b/ocks}$$
 $k = 4 \text{ stretching pso of occols}$

$$F_{R} = \left\{ \frac{12}{bk(k+1)} \sum_{i} \frac{7i}{i} \right\} - 3b(k+1)$$

$$= \frac{12}{b(4)(5)} \left[18^{2} + 33.5^{2} + 35^{3} + 13.5^{2} \right] - 3(10)(5)$$

$$= \left[\frac{12}{200} \left(2853.5 \right) \right] - 150 = 171.51 - 150 = 21.21$$

Test Statistic 21:21 Rejection Region $\frac{7.815}{1.21}$ P-value: > 0.05 or (<0.05)

Q.3. An experiment was conducted to study variation in assessments by raters across products in a 2-Way Random Effects Analysis of Variance. There were a = 8 (Random) Raters and b = 5 (Random) Products, with each Rater rating each Product n = 4 Times. The Raters were blind to which of the product varieties they were rating.

$$Y_{ijk} = \mu + lpha_i + eta_j + \left(lphaeta
ight)_{ij} + arepsilon_{ijk} \quad lpha_i \sim N\!\left(0,\sigma_a^2
ight) \quad eta_j \sim N\!\left(0,\sigma_b^2
ight) \quad \left(lphaeta
ight)_{ij} \sim N\!\left(0,\sigma_{ab}^2
ight) \quad arepsilon_{ijk} \sim N\!\left(0,\sigma^2
ight)$$

p.3.a. Complete the following ANOVA table.

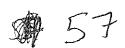
| Source | df 3 each | SS | MS Zeach | F_obs 2.440 | F(.05) 2 es |
|---------|---------------|------|----------|-------------|----------------|
| Rater | 8-1=7 | 1641 | 234,43 | 5.02 | 2.359 |
| Product | 5-1=4 | 892 | 223 | 4.77 | 2.714 |
| R*P | 7(4)=28 | 1308 | 46.71 | 6.36 | F(.05, 28, 16) |
| Error | 8(5)(4-1)=120 | 881 | 7.34 | #N/A | #N/A |
| Total | 159 | 4721 | #N/A | #N/A | #N/A |

p.3.b. The Expected Mean Squares for the various sources of Variation are:

$$E\left\{MSE\right\} = \sigma^2 - E\left\{MSRP\right\} = \sigma^2 + n\sigma_{ab}^2 - E\left\{MSR\right\} = \sigma^2 + n\sigma_{ab}^2 + bn\sigma_a^2 - E\left\{MSP\right\} = \sigma^2 + n\sigma_{ab}^2 + an\sigma_b^2$$

Give unbiased estimates of each of the variances:

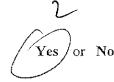
$$\hat{\sigma}^2 = \frac{7.34}{\hat{\sigma}_{ab}^2} = \frac{9.84}{\hat{\sigma}_{ab}^2} = \frac{9.84}{\hat{\sigma}_a^2} = \frac{9.35}{\hat{\sigma}_b^2} = \frac{5.51}{\hat{\sigma}_b^2}$$



Q.4. A study was conducted to compare 3 treatments for depression (Sudarshan Kriya Yoga (SKY), Electroconvulsive Therapy (ET), and Imipramine (IMN)) in subjects suffering from melancholia. A sample of 45 subjects was obtained and randomized so that 15 received SKY, 15 received ET, and 15 received IMN. One response measured was the patients' scores on the 17-item Hamilton Depression Scale (HRSD). Patients were each measured at 5 time points.

| p.4.a. Complete the ANG | OVA table. 3 eac | 2 | read | Beach | Verc |
|-------------------------|------------------|-------|---------|-------|---------|
| Source | df | SS | MS | F_obs | F(0.05) |
| Treatment | 3-1=2 | 184 | 92 | 0.56 | ~3.232 |
| Subject(Trt) | 3(15-1)=42 | 6945 | 165.36 | #N/A | #N/A |
| Time | 5-1=Y | 11119 | 2779.75 | 97.19 | ~2.9US |
| Trt*Time | 2(4)=8 | 577 | 72.13 | 2.52 | ~1.993 |
| Error | 42(4)=168 | 4805 | 28.60 | #N/A | #N/A |
| Total | 225-1-224 | 23630 | #N/A | #N/A | #N/A |

p.4.b. Is there a significant treatment by time interaction?



p.4.c. The standard error of the difference between two treatment means at the same time point, and estimated degrees of freedom are:

$$s\left\{\overline{Y}_{i-k} - \overline{Y}_{i'-k}\right\} = \sqrt{\frac{2\left(MS_{SUBJECTS(TRTS)} + (t-1)MS_{ERROR}\right)}{nt}} = 2.73 \text{ with approximate df: } \hat{\nu} = \frac{\left[(t-1)MS_{ERROR} + MS_{SUBJECT(TRT)}\right]^2}{\left[\frac{\left[(t-1)MS_{ERROR}\right]^2}{a(n-1)(t-1)} + \frac{\left[MS_{SUBJECT(TRT)}\right]^2}{a(n-1)}\right]} = 107.4$$

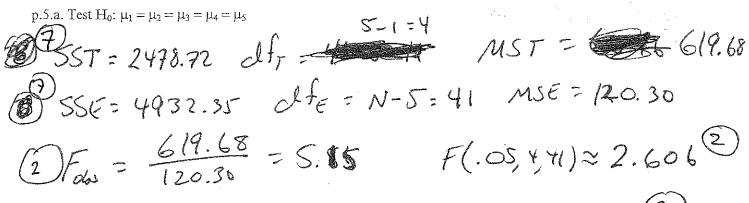
The means for the 3 treatments at the final time point are: ECT = 2.5, IMN = 6.3, SKY = 8.3. Use Bonferroni's method to compare all pairs of treatments at the final time point. Note that low scores mean lower depression.

$$t(\frac{-025}{3};107.4) \approx 2.431$$
 (4)
$$B = 2.431(2.73) = 6.64$$
 (4)
$$2.5 = 6.3 = 8.3$$
 (2)

Q.5. Samples of red King crabs were obtained from t = 5 locations, and their muscle tissue was measured for heavy metal concentrations (μ g/g dry weight). Sample sizes, means, and standard deviations for the locations are given below (the overall mean is 32.97) for the model: $Y_{ij} = \mu_i + \varepsilon_{ij}$ $i = 1,...,5; j = 1,...,n_i$ $\varepsilon_{ij} \sim N(0,\sigma^2)$

| Location | i | n_i | ybar_i | SD_i |
|-----------------|---|-----|--------|-------|
| SE Bering Sea 1 | 1 | 11 | 40.64 | 10.22 |
| SE Bering Sea 2 | 2 | 10 | 26.01 | 8.4 |
| Kodiak Island | 3 | 10 | 24.87 | 5.37 |
| SE Alaska | 4 | 9 | 33.15 | 15.69 |
| Alice Arm, BC | 5 | 6 | 43.7 | 14.31 |

 $11(40.64 - 32.97)^{2} + 10(26.01 - 32.97)^{2} + 10(24.87 - 32.97)^{2} + 9(33.15 - 32.97)^{2} + 6(43.70 - 32.97)^{2} = 2478.72$ $10(10.22^{2}) + 9(8.4^{2}) + 9(5.37^{2}) + 8(15.69^{2}) + 5(14.31^{2}) = 4932.35$



Test Statistic: F= 5.15 Rejection Region: F2 2.606 Reject H₀? Ves or No

p.5.b. Based on Bonferroni's method for comparing all pairs of locations, which pair(s) of location will have the highest minimum significant difference, and the lowest. Compute those values:

$$L(\frac{025}{10}; 4) \approx 2.971$$

 $Highest: Pair(s) = \frac{2.971}{1000}$
 $L(\frac{025}{10}; 4) \approx 2.971$
 $L(\frac{025}{100}; 4) \approx 2$

Lowest B=> Sadlet 1: (1,2, 1,3)

Lowest: Pair(s) =
$$\frac{1}{10.30}$$

$$B_{ij} = \frac{2.971}{10.30}$$

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Q.6. A bioavailability study for sulpride involved 3 treatments (sulpride alone (A), sulpride with sucralafate (B), and sulpride with antacid (C)). It was conducted in 3 time periods with 6 subjects in a replicated latin square. The ANOVA and the design are given below, for Y = Urinary excretion of sulpride in 24 hours. The means are:

$$\overline{Y}_A = 31.0$$
 $\overline{Y}_B = 18.6$ $\overline{Y}_C = 20.9$ $\overline{Y}_{\bullet \bullet} = 23.5$

p.6.a. Complete the ANOVA table. N = 6(3) = 8

| | 3 ear | h v | r esc | h 2 | 2 |
|----------|---------|------|-------|-------|---------|
| Source | df | SS | MS | F_obs | F(0.05) |
| Trts | 3-1-2 | 527 | 263.5 | 16.86 | 4.451 |
| Periods | 3-1=2 | 51 | 25.5 | #N/A | #N/A |
| Subjects | 6-1-5 | 380 | 76 | #N/A | #N/A |
| Error | 17-9=8 | 125 | 15.63 | #N/A | #N/A |
| Total | 18-1=17 | 1083 | #N/A | #N/A | #N/A |

| Subject\Period | 1 | 2 | 3 |
|----------------|---|---|----|
| 1 . | Α | В | Ċ |
| 2 | C | A | В |
| 3 | В | С | Α |
| 4 | С | Α | В |
| 5 | В | С | A. |
| 6 | Α | В | С |

p.6.b. How many times does each treatment appear in each subject?

1



p.6.b. How many times does each treatment appear in each Period?

2



p.6.c. Use Tukey's HSD to compare all pairs of treatment means.

$$9(.05,3,8) = 4.041$$
 (4)
 $W = 4.041\sqrt{\frac{15.63}{6}} = 4.041(1.614) = 6.522$
(4)

Q.7. A study was conducted to compare "likability" of 7 food packages (Treatments). A sample of 103 consumers (Blocks) rated the packages in random order. MSR (A) SSB (3)

SS

231

p.7.a. Complete the ANOVA table. 3 each

df

7-1=6

103-1-102 6(102)=612

721-1=720

Source

Blocks

Error

Total

Treatments

| | /4/5/15 | ofher. | (2) |
|-------|---------|--------|---------|
| | MS | F_obs | F(0.05) |
| 459 | 76.5 | 19.92 | -2.182 |
| 36,7 | 226.83 | #N/A | #N/A |
| 350.3 | 3.8Y | #N/A | #N/A |

#N/A

#N/A

2

$$F = \frac{MST}{MSE} = 19.92 = \frac{76.5}{MSE} = 7 MSE = \frac{76.5}{19.92} = 3.84$$

 $SSE = 612(3.84) = 2350.3$

#N/A

p.7.b. Do you conclude there are significant differences in terms of likability among the 7 packages (Yes by No

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p.7.c. Compute the Relative Efficiency of the Randomized Block Design (relative to the Completely Randomized Design).

$$RE(RBD, CRD) = \frac{(b-1)MSB + b(t-1)MSE}{(bt-1)MSE} = \frac{23136.7 + 2373.12}{2764.8} = \frac{25509.82}{2764.8} = 9.23$$

p.7.d. How many replicates per treatment and overall would be needed for a Completely Randomized Design to have equivalent precision for estimating the treatment means as this design has?

$$9.23(103) \approx 950$$

6650 Overall:

