STA 6167 – Exam 2 – Spring 2015 – **PRINT** Name ANSWER KEY

For all significance tests, use α = 0.05 significance level.

Q.1. A split-plot experiment was conducted, comparing 4 coagulant treatments of camel chymosin (HBCC, LBCC, HCC, LCC) and age (4 levels) on Y = strand thickness of melted mozzarella cheese. The experiment was conducted on 3 cheese-making days (blocks), with coagulant treatment as the whole-plot factor, and age as the sub-plot factor.

p.1.a. Complete the ANOVA Table.

Source of Variation	df	SS	MS	F_obs	F(0.05)
WP Factor	4-1=3	959.7	319.9	571.25	4.757
Block	3-1=2	4.8	2.4	#N/A	#N/A
WP*Block	3(2)=6	3.36	0.56	#N/A	#N/A
SP Factor	4-1=3	68.7	22.9	38.49	3.009
WP*SP	3(3)=9	4.878	0.542	0.91	2.300
Error2	4(2)(3)=24	14.28	0.595	#N/A	#N/A
Total	4(3)(4)-1=47	1055.72	#N/A	#N/A	#N/A

p.1.b. Assuming the coagulant treatment/age interaction is not significant, compute Tukey's W for comparing all pairs of coagulant treatments.

$$W = q(0.05;\ a=4,\ df=6)\sqrt{\frac{MS_{WP*B}}{bc}}$$

$$= 4.896\sqrt{\frac{0.56}{3(4)}} = 4.896(0.216) = 1.058$$

p.1.c. Assuming the coagulant treatment/age interaction is not significant, compute Bonferroni's B for comparing all pairs of age.

$$C_c = \frac{4(3)}{2} = 6$$

$$B = t\left(\frac{.025}{6};\ 24\right)\sqrt{\frac{2\,MS_{E2}}{ab}} = 2.875\sqrt{\frac{2(.595)}{4(3)}}$$

$$= 2.875(0.315) = 0.905$$

Q.2. An experiment was conducted to Compare 4 Stretching protocols (1=Static Stretching, 2=Dynamic Stretching, 3=Combination Stretching, 4=No Stretch) among a sample of 10 dancers (blocks) on vertical jump height. The following table gives the vertical leaps for the dancers under the 4 stretching protocols, and some of the within dancer ranks of the protocols.

p.2.a. Complete the Rankings.

Dancer	Stretch1	Stretch2	Stretch3	Stretch4	Rank1	Rank2	Rank3	Rank4
1	32.76	35.79	34.69	30.71	2	4	3	1
2	32.67	37.23	37.24	32.06	2	3	4	1
3	23.04	25.71	25.74	25.71	1	2.5	4	2.5
4	45.63	46.68	47.03	45.38	2	3	4	1
5	29.29	32.61	34.32	30.06	1	3	4	2
6	28.9	34.53	31.17	27.96	2	4	3	1
7	42.23	41.73	43.41	40.14	3	2	4	1
8	50.07	54.27	54.04	51.14	1	4	3	2
9	49.63	56.42	53.98	49.52	2	4	3	1
10	45.88	49.33	48.87	44.91	2	4	3	1
				TOTAL	18	33.5	35	13.5

p.2.b. Compute the Rank Sums for each of the Stretch Protocols.

$T_1 =$ 18 $T_2 =$ 33.5 $T_3 =$ 35 $T_4 =$ 13.5

p.2.c. Conduct Friedman's Test to test whether the medians of distributions of Vertical Leaps differ among the Stretch Protocols. H_0: Distribution medians are all equal.

b = 10 blocks k = 4 stretching protocols (t r+1)

$$F_R = \left\{ \frac{12}{bk(k+1)} \sum_i T_i^2 \right\} - 3b(k+1)$$

$$= \frac{12}{10(4)(5)} \left[18^2 + 33.5^2 + 35^2 + 13.5^2 \right] - 3(10)(5)$$

$$= \left[\frac{12}{200}(2853.5) \right] - 150 = 171.51 - 150 = 21.21$$

Test Statistic 21.21 Rejection Region $F_R \geq \chi^2_{.05,3} = 7.815$ P-value: > 0.05 or < 0.05

Q.3. An experiment was conducted to study variation in assessments by raters across products in a 2-Way Random Effects Analysis of Variance. There were $a = 8$ (Random) Raters and $b = 5$ (Random) Products, with each Rater rating each Product $n = 4$ Times. The Raters were blind to which of the product varieties they were rating.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \alpha_i \sim N(0,\sigma_a^2) \quad \beta_j \sim N(0,\sigma_b^2) \quad (\alpha\beta)_{ij} \sim N(0,\sigma_{ab}^2) \quad \varepsilon_{ijk} \sim N(0,\sigma^2)$$

p.3.a. Complete the following ANOVA table.

Source	df 3 each	SS	MS 2 each	F_obs 3 each	F(.05) 2 each
Rater	8-1=7	1641	234.43	5.02	2.359
Product	5-1=4	892	223	4.77	2.714
R*P	7(4)=28	1308	46.71	6.36	F(.05,28,120)
Error	8(5)(4-1)=120	881	7.34	#N/A	#N/A
Total	159	4721	#N/A	#N/A	#N/A

p.3.b. The Expected Mean Squares for the various sources of Variation are:

$$E\{MSE\} = \sigma^2 \quad E\{MSRP\} = \sigma^2 + n\sigma_{ab}^2 \quad E\{MSR\} = \sigma^2 + n\sigma_{ab}^2 + bn\sigma_a^2 \quad E\{MSP\} = \sigma^2 + n\sigma_{ab}^2 + an\sigma_b^2$$

Give unbiased estimates of each of the variances:

(4) $\hat{\sigma}^2 = MSE = 7.34$

$\hat{\sigma}_{ab}^2 = \dfrac{MSRP - MSE}{n} = \dfrac{46.71 - 7.34}{4} = \dfrac{39.37}{4} = 9.84$

(6) $\hat{\sigma}_a^2 = \dfrac{MSR - MSRP}{bn} = \dfrac{234.43 - 46.71}{5(4)} = \dfrac{187.72}{20} = 9.39$

(6) $\hat{\sigma}_b^2 = \dfrac{MSP - MSRP}{an} = \dfrac{223 - 46.71}{8(4)} = \dfrac{176.29}{32} = 5.51$

$\hat{\sigma}^2 =$ 7.34 $\quad \hat{\sigma}_{ab}^2 =$ 9.84 $\quad \hat{\sigma}_a^2 =$ 9.39 $\quad \hat{\sigma}_b^2 =$ 5.51

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Q.4. A study was conducted to compare 3 treatments for depression (Sudarshan Kriya Yoga (SKY), Electroconvulsive Therapy (ET), and Imipramine (IMN)) in subjects suffering from melancholia. A sample of 45 subjects was obtained and randomized so that 15 received SKY, 15 received ET, and 15 received IMN. One response measured was the patients' scores on the 17-item Hamilton Depression Scale (HRSD). Patients were each measured at 5 time points.

p.4.a. Complete the ANOVA table. 3each 2 2each 3each 2each

Source	df	SS	MS	F_obs	F(0.05)
Treatment	3-1=2	184	92	0.56	~3.232
Subject(Trt)	3(15-1)=42	6945	165.36	#N/A	#N/A
Time	5-1=4	11119	2779.75	97.19	~2.425
Trt*Time	2(4)=8	577	72.13	2.52	~1.993
Error	42(4)=168	4805	28.60	#N/A	#N/A
Total	225-1=224	23630	#N/A	#N/A	#N/A

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p.4.b. Is there a significant treatment by time interaction? Yes or No

p.4.c. The standard error of the difference between two treatment means at the same time point, and estimated degrees of freedom are:

$$s\left\{\overline{Y}_{i\bullet k}-\overline{Y}_{i'\bullet k}\right\}=\sqrt{\frac{2\left(MS_{SUBJECTS(TRTS)}+(t-1)MS_{ERROR}\right)}{nt}}=2.73 \quad \text{with approximate df: } \hat{\nu}=\frac{\left[(t-1)MS_{ERROR}+MS_{SUBJECT(TRT)}\right]^2}{\left[\frac{\left[(t-1)MS_{ERROR}\right]^2}{a(n-1)(t-1)}+\frac{\left[MS_{SUBJECT(TRT)}\right]^2}{a(n-1)}\right]}=107.4$$

The means for the 3 treatments at the final time point are: ECT = 2.5, IMN = 6.3, SKY = 8.3. Use Bonferroni's method to compare all pairs of treatments at the final time point. Note that low scores mean lower depression.

$t\left(\frac{.025}{3}; 107.4\right) \approx 2.431$ 4

$B = 2.431(2.73) = 6.64$ 4

$\underline{2.5 \quad 6.3 \quad 8.3}$ 2

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Q.5. Samples of red King crabs were obtained from t = 5 locations, and their muscle tissue was measured for heavy metal concentrations (μg/g dry weight). Sample sizes, means, and standard deviations for the locations are given below (the overall mean is 32.97) for the model: $Y_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,\ldots,5;\ j = 1,\ldots,n_i \quad \varepsilon_{ij} \sim N(0,\sigma^2)$

Location	i	n_i	ybar_i	SD_i
SE Bering Sea 1	1	11	40.64	10.22
SE Bering Sea 2	2	10	26.01	8.4
Kodiak Island	3	10	24.87	5.37
SE Alaska	4	9	33.15	15.69
Alice Arm, BC	5	6	43.7	14.31

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$$11(40.64-32.97)^2+10(26.01-32.97)^2+10(24.87-32.97)^2+9(33.15-32.97)^2+6(43.70-32.97)^2=2478.72$$

$$10(10.22^2)+9(8.4^2)+9(5.37^2)+8(15.69^2)+5(14.31^2)=4932.35$$

p.5.a. Test H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

7 $SST = 2478.72 \quad df_T = 5-1=4 \quad MST = 619.68$

7 $SSE = 4932.35 \quad df_E = N-5 = 41 \quad MSE = 120.30$

2 $F_{obs} = \dfrac{619.68}{120.30} = 5.15 \qquad F(.05, 4, 41) \approx 2.606$ 2

Test Statistic: $F = 5.15$ Rejection Region: $F \geq 2.606$ Reject H_0? Yes or No 2

p.5.b. Based on Bonferroni's method for comparing all pairs of locations, which pair(s) of location will have the highest minimum significant difference, and the lowest. Compute those values:

$t\left(\frac{.025}{10}; 41\right) \approx 2.971$

Highest B ⇒ Lowest n_i's ⇒ Trts 4 ($n_i=9$), Trt 5 ($n_i=6$)

Highest: Pair(s) = 3 4, 5 $\quad B_{ij}$ = 3 $2.971\sqrt{MSE\left(\frac{1}{9}+\frac{1}{6}\right)}$ (MSE = 120.30)

Lowest B ⇒ Smallest n_i's (1 v 2, 1 v 3)

Lowest: Pair(s) = 3 1v2, 1v3 $\quad B_{ij}$ = 3 $2.971\sqrt{MSE\left(\frac{1}{11}+\frac{1}{10}\right)}$ (MSE = 120.30)

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Q.6. A bioavailability study for sulpride involved 3 treatments (sulpride alone (A), sulpride with sucralafate (B), and sulpride with antacid (C)). It was conducted in 3 time periods with 6 subjects in a replicated latin square. The ANOVA and the design are given below, for Y = Urinary excretion of sulpride in 24 hours. The means are:

$\overline{Y}_A = 31.0 \quad \overline{Y}_B = 18.6 \quad \overline{Y}_C = 20.9 \quad \overline{Y}_{\bullet\bullet} = 23.5$

p.6.a. Complete the ANOVA table. $N = 6(3) = 18$

3 each 2 2 each 2 2

Source	df	SS	MS	F_obs	F(0.05)
Trts	3-1=2	527	263.5	16.86	4.459
Periods	3-1=2	51	25.5	#N/A	#N/A
Subjects	6-1=5	380	76	#N/A	#N/A
Error	17-9=8	125	15.63	#N/A	#N/A
Total	18-1=17	1083	#N/A	#N/A	#N/A

Subject\Period	1	2	3
1	A	B	C
2	C	A	B
3	B	C	A
4	C	A	B
5	B	C	A
6	A	B	C

p.6.b. How many times does each treatment appear in each subject? 1 (2)

p.6.b. How many times does each treatment appear in each Period? 2 (2)

p.6.c. Use Tukey's HSD to compare all pairs of treatment means.

$q(.05, 3, 8) = 4.041$ (4)

$W = 4.041\sqrt{\frac{15.63}{6}} = 4.041(1.614) = 6.522$ (2)

(4)

W_{ij} = 6.522

Connect trts that are not significantly different B C A (2)

18.6 20.9 31.0

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Q.7. A study was conducted to compare "likability" of 7 food packages (Treatments). A sample of 103 consumers (Blocks) rated the packages in random order.

MSB 8 SSB 3 others 2

p.7.a. Complete the ANOVA table. 3 each

Source	df	SS	MS	F_obs	F(0.05)
Treatments	7-1=6	459	76.5	19.92	~2.182
Blocks	103-1=102	23136.7	226.83	#N/A	#N/A
Error	6(102)=612	2350.3	3.84	#N/A	#N/A
Total	721-1=720	25946	#N/A	#N/A	#N/A

2

$$F = \frac{MST}{MSE} = 19.92 = \frac{76.5}{MSE} \Rightarrow MSE = \frac{76.5}{19.92} = 3.84$$

$$SSE = 612(3.84) = 2350.3$$

2

p.7.b. Do you conclude there are significant differences in terms of likability among the 7 packages? Yes or No

p.7.c. Compute the Relative Efficiency of the Randomized Block Design (relative to the Completely Randomized Design).

$$RE(RBD, CRD) = \frac{(b-1)MSB + b(t-1)MSE}{(bt-1)MSE}$$

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$$= \frac{23136.7 + 2373.12}{2764.8} = \frac{25509.82}{2764.8} = 9.23$$

p.7.d. How many replicates per treatment and overall would be needed for a Completely Randomized Design to have equivalent precision for estimating the treatment means as this design has?

$9.23(103) \approx 950$ $\quad$ $950(7) \approx 6650$

3 $\quad$ 2

Per Package: 950 $\quad$ Overall: 6650

42

[redacted]

Q.8. For the following problems, identify the factor(s), state whether they are fixed or random and if they are nested, give the analysis of variance table, with sources of variation and degrees of freedom, and appropriate error terms (e.g. MSAB, MSE, $MS_{Subjects(Trts)}$).

p.8.a. An experiment on growth of Atlantic Salmon has 3 diets to be compared. The researchers have 12 fish tanks and randomly assign them so that 4 tanks are assigned to each diet. A total of 1200 fish are pre-weighed and randomly assigned to so that there are 100 in each tank. Y = weight change over a fixed period of time, one measurement per fish.

Factors (Fixed or Random): Diet (Fixed) Tank (Diet) (F or R)

2 1 2 1

ANOVA

Source	df	Error Term
Diet	3-1=2 2	Tank(Diet) if Random / Error if fixed 1
Tank(Diet)	3(4-1)=9 1	Error 1
Error	12(100-1)=1188 1	

p.8.b. A study considered weight gain in Rats. There were 2 Diets (Normal and Oat β-Glucan) and 2 exercise conditions (Exercise and No exercise). A sample of 32 rats is obtained, and randomly assigned so that 8 receive exactly one of the 4 Diet/Exercise conditions. Y = weight change over a fixed period of time, one measurement per rat.

Factors (Fixed or Random): Diet (F) Exercise (F) D*E (F)

1 1 1 1 1 1

ANOVA

Source	df	Error Term
Diet	2-1=1 1	MSE 1
Exercise	2-1=1 1	MSE 1
D*E	1(1)=1 1	MSE 1
Error	4(8-1)=28 1	

p.8.c. A study was conducted to compare 7 models of bread machines on quality of baked bread. There were 7 varieties of bread, and 7 chefs, and each variety was made by each machine once, and each chef used each machine once. The response was an overall quality rating based by a panel of judges (which was combined to a single rating).

Factors (Fixed or Random): Machine (F), Chefs (F or R), Varieties (F)

2 1 1

ANOVA

Source	df	Error Term
Varieties	7-1=6 1	MSE 1
Machine	6 1	MSE 1
Chefs	6 1	MSE 1
Error	6(5)=30 1	

24