Chapter 4 - Insurance Benefits

Section 4.4 - Valuation of Life Insurance Benefits

(Subsection 4.4.1) Assume a life insurance policy pays \$1 immediately upon the death of a policy holder who takes out the policy at age x.

The present value of this payment is:

where $\delta = \ln(1 + i)$. This present value Z is a random variable, because the future life time of this person, T_x, is itself a random variable.

The Expected Present Value (EPV) of this future payment is:

$$= \int_0^\infty e^{-\delta\, t}\, f_x(t)\, dt$$

$$= \int_0^\infty e^{-\delta\, t}\, {}_tp_x\, \mu_{x+t}\, dt$$

The second moment of this random present value is :

$$E\left[\, Z^2 \,\right] = E\left[\, \{\, e^{-\delta\, T_x}\, \}^2 \,\right] = E\left[\, e^{-2\delta\, T_x} \,\right]$$

$$\equiv {}^2\overline{A}_x$$

Here is superscript 2 on the left indicates that this computation of $\overline{A}_x$ uses twice the force of interest (which is NOT the same thing as twice the interest rate). Consequently, the variance of the random present value Z is:

Of course, if the policy pays a benefit of s dollars, then the expected present value is

$$EPV = E[s\,Z\,] = s\,\overline{A}_x,$$

and the random benefit's variance is

$$Var[s\,Z\,] = s^2\left({}^2\overline{A}_x - \{\overline{A}_x\}^2\right).$$

Example 4-1: Suppose $f_X(t) = \frac{2t}{\omega^2}$ for $0 < t < \omega$, and zero elsewhere. Find the EPV and Var[Z] in this setting.

Example 4-2: Assume T_{40} is a continuous random variable with constant force of mortality .03. If a whole life policy pays 100 immediately upon death and the force of interest is $\delta = .04$, find the EPV of this insurance claim.

(Subsection 4.4.2) Assume the life insurance policy pays \$1 at the end of the policy year in which the person dies.

The present value of the random benefit is now:

$$Z = \nu^{K_x+1}$$

where $K_x = \lfloor T_x \rfloor$ is the curtate future life time random variable.

The Expected Present Value is therefore,

$$= \nu \, P[K_x = 0] + \nu^2 \, P[K_x = 1] + \nu^3 \, P[K_x = 2] + \cdots$$

$$= \nu \, q_x + \nu^2 \, {}_{1|}q_x + \nu^3 \, {}_{2|}q_x + \cdots$$

Recall that when $k \geq 1$,

The second moment of this random present value is:

$$^2A_x = E[Z^2] = \sum_{k=0}^{\infty} \left(\nu^{k+1}\right)^2 {}_{k|}q_x$$

$$= \sum_{k=0}^{\infty} \left(\nu^2\right)^{k+1} {}_{k|}q_x$$

which is the same as the computation of A_x when the force of interest is doubled, i.e. the effective interest rate is j satisfying

The variance of the random present value is

$$Var[\,Z\,] = {}^2A_x - (A_x)^2.$$

Example 4-3: Suppose $q_{x+0} = .1$, $q_{x+1} = .4$, and $q_{x+2} = 1.0$ with $i = .25$ then find A_x and $Var[\,Z\,]$.

(Subsection 4.4.3) Assume the life insurance policy pays \$1 at the end of the $(\frac{1}{m})^{th}$ fraction of the policy year in which the person dies. Typically, $m = 1, 2, 4$ or 12.

With $K_x^{(m)} = \frac{(\text{\# of sub-periods suvived})}{m}$, the EPV of the benefit is:

$$A_x^{(m)} = E\left[\nu^{K_x^{(m)}+\frac{1}{m}}\right] = \left(\nu^{\frac{1}{m}}\right)_{\frac{1}{m}}q_x + \left(\nu^{\frac{2}{m}}\right)_{\frac{1}{m}}\Big|_{\frac{1}{m}}q_x + \left(\nu^{\frac{3}{m}}\right)_{\frac{2}{m}}\Big|_{\frac{1}{m}}q_x + \cdots$$

The variance of the random present value is

$$Var[Z] = {}^2A_x^{(m)} - (A_x^{(m)})^2,$$

where ${}^2A_x^{(m)}$ is computed like $A_x^{(m)}$ with the force of interest doubled, i.e. with ν replaced with ν^2.

Example 4-4: A three year old red fox buys a whole life policy that pays 1 at the end of the half-year in which death occurs. Let $i = .05$ and assume UDD within each year. Use the life table to find the EPV of this insurance claim.

x	l_x	d_x
3	2194	1931
4	263	263
5	0	

(Subsection 4.4.4) Recursive Computation

In discrete cases with benefit payments of \$1 at the end of the policy year of death the values of A_x are computed based on a life table. These tables have a maximum life length ω beyond which no person is assumed to live. That is, there is a value ω such that

$$q_{\omega-1} = P[\,\text{person dies before age } \omega \mid \text{person lives to } \omega - 1\,] = 1.$$

If a policy was taken out by someone age $\omega - 1$, then

$$A_{\omega-1} \;=\; E\left[\,\nu^{K_{\omega-1}+1}\,\right] \;=\; \nu. \qquad \text{(because } K_{\omega-1} \equiv 0\text{)}$$

If a policy was taken out by someone age $\omega - 2$, then

Similarly, a policy taken out at age x has expected present value

$$A_x = \nu q_x + \nu^2 p_x q_{x+1} + \nu^3 p_x p_{x+1} q_{x+2} + \cdots$$

$$= \nu \Big[q_x + p_x \{ \nu q_{x+1} + \nu^2 p_{x+1} q_{x+2} + \nu^3 p_{x+1} p_{x+2} q_{x+3} + \cdots \} \Big]$$

showing that

So A_x can easily be computed as long as A_{x+1} is known. Since we know the final value, $A_{\omega-1} = \nu$, we can use backward recursion to compute A_x values for all the ages x in the life table.

Example 4-5: Use $i = .05$ and complete the A_x column.

x	q_x	A_x
0	.01246	
1	.02245	
2	.08619	
3	.37745	
4	1.00	

(Subsection 4.4.5) Term Insurance

Under a term insurance policy payment is only made if the insured person dies before a fixed term (n years) ends. After the n years the policy pays no benefits (has no value).

(a) Continuous Immediate Benefit Case: Here the benefit of $1 is paid immediately upon death of the policy holder. So the present value of the benefit is:

$$Z = \begin{cases} e^{-\delta T_x} & \text{if } T_x \leq n \\ 0 & \text{if } T_x > n \end{cases}$$

The expected present value of the benefit is

The second moment of the random present value is

$$^{2}\overline{A}^{1}_{x:\overline{n}|} = \int_0^n e^{-2\delta t} {}_t p_x \mu_{x+t} dt.$$

which results from doubling the force of interest. Its variance is then

$$Var[Z] = {}^{2}\overline{A}^{1}_{x:\overline{n}|} - \left(\overline{A}^{1}_{x:\overline{n}|}\right)^2.$$

(b) Discrete Annual ($m^{th}ly$) Benefit Case:

When the benefit is paid at the end of the year in which death occurs, the expected present value of the benefit is

and when it is paid at the end of the $(\frac{1}{m})^{th}$ portion of the year in which death occurs it is

$$A^{(m)1}_{x:\overline{n}|} = \sum_{k=0}^{nm-1} \left(\nu^{\frac{k+1}{m}}\right) {}_{\frac{k}{m}}\Big|_{\frac{1}{m}} q_x.$$

Example 4-6 : Suppose that beyond age x there is a constant force of mortality μ_x and a constant force of interest δ. Find $\bar{A}^{1}_{x:\overline{n}|}$.

(Subsection 4.4.6) Pure Endowment

A pure endowment benefit pays \$1 at the end of the policy term of n years if the insured person of age x survives the policy term of n years.

The present value of the benefit is the random variable

$$Z = \begin{cases} \nu^n \equiv e^{-\delta n} & \text{if } T_x \geq n \\ 0 & \text{if } T_x < n \end{cases}$$

The expected present value of the benefit is

There is no difference between continuous and discrete random life lengths in this setting. Pure endowment is a tool for describing more complex insurance instruments and is not sold as a separate policy.

The symbols

are used interchangeably. We see this as a discounted survival function.

Example 4-7 : Suppose T_x is de Moivre $(0, \omega - x)$ and the interest rate is constant, find $\overline{A}^{\,1}_{x:\overline{n}|}$.

(Subsection 4.4.7) Endowment Insurance

Endowment insurance pays a benefit of \$1 if the insured person of age x dies before the end of the policy term of n years or it pays \$1 at the end of the term if that person survives the term of n years.

The present value of the benefit is the random variable

$$Z = \begin{cases} \nu^{T_x} \equiv e^{-\delta T_x} & \text{if } T_x < n \\ \nu^n \equiv e^{-\delta n} & \text{if } T_x \geq n \end{cases}$$

$$= \nu^{min(T_x, n)}.$$

The expected present value of a continuous benefit paid immediately at death is therefore

Its variance is:

$$Var[Z] = {}^2\overline{A}_{x:\overline{n}|} - \left(\overline{A}_{x:\overline{n}|}\right)^2.$$

where ${}^2\overline{A}_{x:\overline{n}|}$ is computed in the same manner as $\overline{A}_{x:\overline{n}|}$ only with a doubled force of interest, i.e. δ is replaced by 2δ and ν is replaced by ν^2.

The annual and $m^{th}ly$ discrete cases are completely analogous, producing EPV's of:

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}} \qquad \text{annual}$$

and

$$A^{(m)}_{x:\overline{n}|} = A^{(m)}{}^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}. \qquad m^{th}ly$$

Note, for example that in the annual case,

$$Z = \begin{cases} \nu^{K_x+1} & \text{if } T_x < n \\ \nu^{n} & \text{if } T_x \geq n \end{cases}$$

$$= \nu^{\min(K_x+1,n)}$$

and so

Example 4-8 : You are given $\overline{A}^{1}_{x:\overline{n}|} = .4275$, $\delta = .055$ and $\mu_{x+t} = .045$ for all t. Find $\overline{A}_{x:\overline{n}|}$.

(Bonus Subsection 4.4.7.5) Special Continuous Models

(a) Constant Force of Mortality and Constant Force of Interest:

Consider a policy that pays \$1 immediately at the death of the insured. Assume also that there is a constant force of interest δ and a constant force of mortality μ_x that describes survival beyond age x. Now we recall that

$$ {}_tp_x = e^{-\mu_x t} \qquad \text{and thus} \qquad f_x(t) = \mu_x e^{-\mu_x t} \text{ for } t > 0 $$

For whole life it follows that

$$ \overline{A}_x = \int_0^\infty e^{-\delta t} \mu_x e^{-\mu_x t} dt \quad = \quad \mu_x \int_0^\infty e^{-(\delta+\mu_x) t} dt $$

$$ = \frac{-\mu_x}{(\delta + \mu_x)} e^{-(\delta+\mu_x) t} \Big|_0^\infty $$

which produces the formula

It quickly follows that

$$E[\,Z^2\,] = {}^2\overline{A}_x = \frac{\mu_x}{(2\delta + \mu_x)} \qquad \text{and}$$

$$Var[Z] = \frac{\mu_x}{(2\delta + \mu_x)} - \left(\frac{\mu_x}{(\delta + \mu_x)}\right)^2$$

Example 4-9 Same Setting: For term insurance, find $\overline{A}^{1}_{x:\overline{n}|}$ and $Var[\,Z\,]$.

Under these assumptions, the pure endowment term is:

$${}_nE_x = e^{-\delta n}\, {}_np_x \qquad \text{or}$$

Of course, endowment insurance of \$1 paid immediately at death or the end of the term (whichever comes first) has an EPV of

$$\bar{A}_{x:\overline{n}|} = \bar{A}^{1}_{x:\overline{n}|} + {}_nE_x.$$

(b) Uniform Distribution of Deaths and Constant Force of Interest:
Consider a policy that pays \$1 immediately at the death of the insured. Assume also that there is a constant force of interest δ and a uniform (de Moivre) distribution of deaths beyond age x. Now we recall that

$$ {}_tp_x = \frac{\omega - x - t}{\omega - x} \quad \text{and} \quad f_x(t) = \frac{1}{(\omega - x)} \text{ both for } 0 < t < \omega - x. $$

For whole life it follows that

$$ \overline{A}_x = E\left[e^{-\delta\, T_x}\right] = \int_0^{\infty} e^{-\delta\, t} f_x(t)dt $$

$$ = \int_0^{\omega - x} \frac{1}{(\omega - x)} e^{-\delta t} dt = \frac{-1}{\delta(\omega - x)} \left(e^{-\delta t}\right)\Big|_0^{\omega - x} \qquad \text{or} $$

Moreover, under these assumptions, the term insurance EPV for $n < (\omega - x)$ is

It is the same integral as above only with n as the upper bound.

Under these assumptions, the pure endowment term is:

$${}_nE_x = e^{-\delta n}\,{}_np_x \qquad\qquad \text{or}$$

(Subsection 4.4.8) Deferred Insurance Benefits

Sometimes a policy does not begin to offer death benefits until the end of a deferral period of $u > 0$ years.

Consider a term life insurance policy that does not begin insurance coverage until time $x + u$ and ends insurance coverage at $x + u + n$. The present value of a \$1 benefit paid immediately at death is:

$$Z = \begin{cases} 0 & \text{if } T_x < u \text{ or } T_x > u + n \\ e^{-\delta T_x} & \text{if } u < T_x \leq u + n \end{cases}$$

and the expected present value is:

Changing the variable of integration from t to $s = t - u$ in this integral produces,

$$
\begin{aligned}
{}_{u|}\bar{A}^{\,1}_{x:\overline{n}|} &= \int_0^n e^{-\delta(s+u)}\,{}_{s+u}p_x\,\mu_{x+s+u}\,ds \\
&= e^{-\delta u}\int_0^n e^{-\delta s}\,{}_{u}p_x\,{}_{s}p_{x+u}\,\mu_{x+s+u}\,ds \\
&= e^{-\delta u}\,{}_{u}p_x\int_0^n e^{-\delta s}\,{}_{s}p_{x+u}\,\mu_{x+s+u}\,ds, \qquad \text{that is,}
\end{aligned}
$$

The nature of the integral boundaries in the original description of this EPV yields

as an additional way to express the EPV of a deferred benefit in terms of EPV's of those that are not deferred.

Deferred policies are useful in describing or decomposing other policies. For example, a 10 year term policy has EPV

$$\overline{A}^{\,1}_{x:\overline{10}|} = \overline{A}^{\,1}_{x:\overline{6}|} + {}_{6}\Big|\overline{A}^{\,1}_{x:\overline{4}|}.$$

That is, it is equivalent to a 6 year term policy plus a 6-year deferred 4-year term policy. A whole life policy is equivalent to a n-year term policy plus a n-year deferred policy. So its EPV will satisfy

$$A_x = A^{\,1}_{x:\overline{n}|} + {}_{n}\Big|A_x \quad \text{or} \quad \overline{A}_x = \overline{A}^{\,1}_{x:\overline{n}|} + {}_{n}\Big|\overline{A}_x.$$

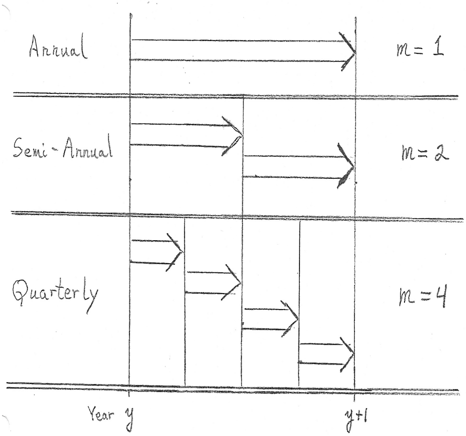
This implies, for example,

Therefore we can compute EPV's of term insurance policies from the EPV's of whole life policies.

Example 4-10: A 3-year deferred whole life policy pays 1 at the moment of death. You are also given that $\mu_t = .01$ for $0 \leq t \leq 2$ and $= .02$ for $t > 2$. We also have $\delta_t = .05$ for $0 \leq t \leq 2$ and $= .06$ for $t > 2$. Find the actuarial present value of this insurance.

Section 4.5 - Relationships among $\overline{A}_x$, A_x and $A_x^{(m)}$

As the number of potential benefit payment points per year, m, increases death benefits are paid sooner (or possibly at the same time point). Since the payment amount is the same, discounting over a shorter period produces a larger expected present value.



Therefore,

$$A_x < A_x^{(2)} < A_x^{(4)}$$

and, in general, $A_x^{(m)}$ is an increasing function of m.

It follows that when $m > 1$,

But the differences between these values are relatively small and can be approximated.

(a) Approximations assuming UDD (uniform distribution of deaths) within each year

Let $K_x^{(m)} = \frac{(\text{ \# of sub-periods survived})}{m}$ and examine

$$A_x^{(m)} = E\left[\nu^{K_x^{(m)}+\frac{1}{m}}\right]$$

$$= \sum_{k=0}^{\infty} E\left[\nu^{K_x^{(m)}+\frac{1}{m}} \middle| \begin{array}{c}\text{death in}\\ [x+k, x+k+1)\end{array}\right] P\left[\begin{array}{c}\text{death in}\\ [x+k, x+k+1)\end{array}\right]$$

Before continuing, we examine the conditional expected value on the right.

$$E\left[\nu^{K_x^{(m)}+\frac{1}{m}}\middle|\begin{array}{c}\text{death in}\\ [x+k,x+k+1)\end{array}\right]$$

$$=\left(\frac{1}{m}\right)\nu^{k+\frac{1}{m}}+\left(\frac{1}{m}\right)\nu^{k+\frac{2}{m}}+\cdots+\left(\frac{1}{m}\right)\nu^{k+\frac{m}{m}}$$

$$=\left(\frac{1}{m}\right)\nu^{k}\left(\nu^{\frac{1}{m}}+\nu^{\frac{2}{m}}+\cdots+\nu^{\frac{m}{m}}\right)$$

$$=\nu^{k+1}\left(\frac{\frac{(1-\nu)}{\nu}}{m(\nu^{-\frac{1}{m}}-1)}\right)=\nu^{k+1}\left(\frac{i}{i^{(m)}}\right)$$

where the last step follows because $(1-\nu)/\nu=i$ and $i^{(m)}=m\big((1+i)^{\frac{1}{m}}-1\big)$.

Substituting this in the term at the bottom of the previous page shows that under the UDD assumption

$$A_x^{(m)} = \sum_{k=0}^{\infty} {}_k p_x \, q_{x+k} \, \nu^{k+1} \left(\frac{i}{i^{(m)}} \right)$$

$$= \left(\frac{i}{i^{(m)}} \right) \sum_{k=0}^{\infty} \nu^{k+1} \, {}_k p_x \, q_{x+k} \qquad \text{so}$$

Taking the limit as $m \to \infty$ shows that under this UDD assumption

So motivated by the UDD assumption, we have the following approximations;

$$A_x^{(m)} \approx \left(\frac{i}{i^{(m)}}\right) A_x$$

$$\overline{A}_x \approx \left(\frac{i}{\delta}\right) A_x \qquad \text{and}$$

$$\overline{A}_{x:\overline{n}|} \approx \left(\frac{i}{\delta}\right) A^1_{x:\overline{n}|} + {}_nE_x.$$

(b) A claims acceleration approximation

This method is motivated by a UDD assumption, because under a UDD assumption a death is equally likely to occur in any one of the m time segments of the year. In this case the average claim payment time during a given year is

$$\frac{1}{m}\left(\frac{1}{m}\right)+\frac{1}{m}\left(\frac{2}{m}\right)+\cdots+\frac{1}{m}\left(\frac{m}{m}\right) = \frac{1}{m^2}\sum_{j=1}^{m} j = \frac{m+1}{2m}.$$

The approximation to $A_x^{(m)}$ is made by making any payments during the year at this average time within each year, producing

$$\begin{aligned}
A_x^{(m)} &\approx \nu^{\frac{m+1}{2m}}\, q_x + \nu^{1+\frac{m+1}{2m}}\, {}_{1|}q_x + \nu^{2+\frac{m+1}{2m}}\, {}_{2|}q_x + \cdots \\
&= \nu^{\frac{m+1}{2m}-1} \sum_{k=0}^{\infty} \nu^{k+1}\, {}_{k|}q_x \\
&= (1+i)^{\frac{m-1}{2m}} \sum_{k=0}^{\infty} \nu^{k+1}\, {}_{k|}q_x
\end{aligned}$$

Typically, we expect claims to increase over time. So when using the average time within the year, we expect to move more claims back in time than forward, thus accelerating claims (making them earlier than anticipated). The above computation motivates

Letting $m \to \infty$ produces

Likewise,

$$\overline{A}_{x:\overline{n}|} \approx (1+i)^{\frac{1}{2}} A^{1}_{x:\overline{n}|} + {}_nE_x.$$

Example 4-11: Consider the following: (1) $\overline{A}_x$ with CFM μ^* and CFOI δ, (2) $\overline{A}_x'$ with CFM $\mu^* + c$ and CFOI δ, and (3) $\overline{A}_x''$ with CFM μ^* and CFOI $\delta + c$. Assume $c > 0$. Find the ordering among these 3 EPV's.

Section 4.6 - Variable Benefits

When life insurance benefits are solely a function $H(T_x)$ of the future life length of the policyholder, then

and the variance of the present value is

$$Var(PV) = E\left[\nu^{2T_x} H^2(T_x)\right] - \left(EPV_H\right)^2.$$

Note that the second moment does not just require doubling the force of interest, the benefit amount must also be squared.

When the benefit function is linear, e.g. $H(t) = a + b\,t$ with a and b constants, then

$$EPV_H = a\,E\left[\nu^{T_x}\right] + b\,E\left[T_x\,\nu^{T_x}\right]$$

In the continuous case with the benefit being paid immediately upon death, the notation is

in a whole life insurance setting and

$$(\bar{\mathrm{I}}\bar{\mathrm{A}})^{1}_{x:\overline{n}|} \equiv \int_0^n t\,e^{-\delta\,t}\,{}_tp_x\,\mu_{x+t}dt$$

in a term insurance setting.

In a discrete setting with payments at the end of the year of death, the present value random variable is

$$Z = \nu^{K_x+1}(K_x + 1)$$

where $K_x = \lfloor T_x \rfloor$.

Thus the EPV of whole life insurance, for example, is

$$(\mathrm{IA})_{\mathrm{x}} = \sum_{\mathrm{k}=0}^{\infty} \nu^{\mathrm{k}+1}(\mathrm{k}+1)\,{}_{\mathrm{k}|}\mathrm{q}_{\mathrm{x}}$$

and for term life insurance it is

with ${}_{k|}q_x = {}_k p_x\, q_{x+k} = \frac{l_{x+k} - l_{x+k+1}}{l_x}$ coming from a life table.

There are some problem settings in which the life insurance benefits increase exponentially over time, i.e. they are multiplied by $(1+j)^{T_x}$. The present value of the benefit is then

$$Z = \nu^{T_x}(1+j)^{T_x} = \left(\frac{1+j}{1+i}\right)^{T_x} \equiv \nu_*^{T_x}$$

where

$$\nu_* = \frac{1}{1+i^*} = \left(\frac{1+j}{1+i}\right), \qquad \text{that is}$$

So the EPV's

$$\overline{A}_{x:i^*} \qquad \overline{A}^{1}_{x:\overline{n}|i^*} \qquad A_{x:i^*} \qquad \text{and} \qquad A^{1}_{x:\overline{n}|i^*}$$

for the continuous and discrete cases are computed as before, only with a new interest rate i^*.

Example 4-12: Consider a whole life policy payable immediately at death, with constant force of mortality of μ_x, constant force of interest δ, and variable payment of $e^{\alpha t}$ where μ_x, δ and α are all positive constants with $\alpha < \mu_x + \delta$. Find the EPV.

Example 4-13: Consider a whole life policy payable immediately at death, with constant force of mortality of $\mu_x = .03$ for all $t > 0$. Assume $\delta_t = .01$ for $0 \leq t < 65$ and $= .02$ for $t \geq 65$. In addition the benefit payment is $b_t = 200$ for $0 \leq t < 65$ and $= 100$ for $t \geq 65$. Find the EPV.

Supplement to Chapter 4

Pure Endowment Revisited

The pure endowment term

assumes compound interest using a constant force of interest. Here

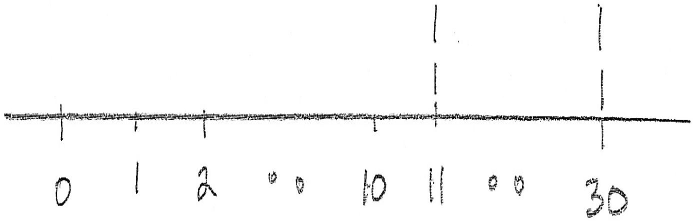
$$\nu^n = \left(\frac{1}{1+i}\right)^n = e^{-\delta\, n}$$

with $\delta = \ln(1+i)$.

Note there are no assumptions made here about survival. It only assumes use of the survival function ${}_tp_x$. The term ${}_nE_x$ represents discounting over a survived term of n years.

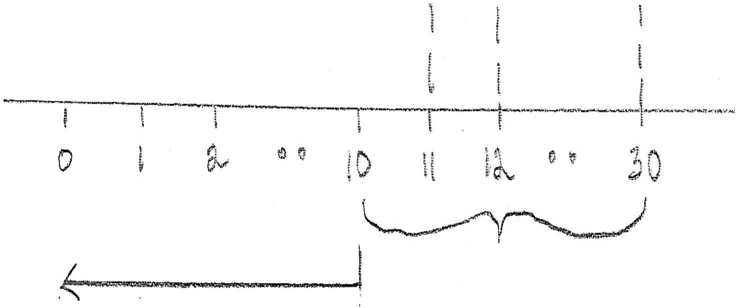
Moving the Vision Point When Computing an EPV

Sometimes potential insurance payments (benefits) don't begin right away. Pictured below is a 20-year term life insurance policy paying \$1 at the end of the year of death, but it is deferred 10 years, i.e. there are no benefits paid if death occurs during the first 10 years for the policyholder who buys the policy at time $t = 0$ when this person is age x.



Earlier we described the EPV of this deferred life insurance as:

But we now approach this computation in a different manner. Suppose we view the EPV of this policy from the vision point of $t = 10$.



If the policy had been purchased at $t = 10$ the EPV would be

$$A^{\;1}_{x+10:\overline{20}|}.$$

But, of course, it was purchased at $t = 0$. We must discount the EPV back to $t = 0$ via

Here the discounting process must account for more than just the interest rate. In order for any benefits to be paid, the policyholder must first survive from age x to age $x + 10$. Thus we must discount over a survived period of time, using the survival discount factor of ${}_{10}E_x$. Therefore, in general

This process of discounting over a survived period, enables us to compute EPV's by breaking the time line (or potential benefit payments) into disjoint insurance benefit segments and then discounting each segment to time $t = 0$, taking the necessary survival into account.

We refer to

In an expected life insurance benefit computation, if we were to move the vision point into the future from, for example, $t = 0$ to $t = u$ where $u > 0$, we would divide by the survival discount factor, in this case ${}_uE_{x+0}$, provided there were no potential benefit payments in the interval $[0, u]$.

Example 4-14: Consider a 8-year deferred 40-year term life insurance policy that pays 100 during the first 20 years and 200 during the second 20 years of insurance coverage with all payments at the end of the year of death. Find expressions for the purchase price (premium) of this policy at $t = 0$ and also the price if it was purchased at $t = 5$.

Putting the Pieces Together

Knowing the formulas for the exponential and de Moivre distributions and knowing how to discount while taking survival into account enables us to quickly solve complex problems by segmenting them into simpler settings in which both can be applied.

Example 4-15: A policyholder age $x = 40$ begins an endowment policy that pays 200 at the moment of death or at age 65, whichever comes first. Assume a de Moivre (0,50) distribution for future life length and $\delta = .05$. Find the EPV and Var[Z].

Example 4-16: A whole life policy pays 100 immediately at death. We are given $\delta_t = .04$ for $0 \leq t \leq 10$ and = .05 for $t > 10$ plus $\mu_{x+t} = .06$ for $0 \leq t \leq 10$ and = .07 for $t > 10$. Calculate the premium for this insurance.

Example 4-17 (Example 4-10 revisited): A 3-year deferred whole life policy pays 1 at the moment of death. You are also given that $\mu_t = .01$ for $0 \leq t \leq 2$ and $= .02$ for $t > 2$. We also have $\delta_t = .05$ for $0 \leq t \leq 2$ and $= .06$ for $t > 2$. Find the actuarial present value of this insurance.

Example 4-18: A whole life policy pays 1 immediately at death for the first 10 years and .5 after that. Given a constant force of mortality of μ^* and a constant force of interest of μ^*, plus EPV $\equiv$ E[Z] = .3324, find Var[Z].