Chapter 3 - Life Tables

Section 3.2 - Basic Life Tables

A life table displays the expected survival from some integer age to future integer ages.

For example, we begin at age x_0 with a hypothetical number of lives, l_{x_0}, (this number is called the radix of the table). Then the number of lives from this group which survive at least t years beyond this age is

Here both x_0 and t are viewed as integers. While l_{x_0} is typically an integer, generally, l_{x_0+t} is not. But in some cases it may be rounded to an integer.

Given the values l_{x_0+t} for $t = 0, 1, \cdots$, we can derive survival probabilities ${}_tp_x$ for any $x \geq x_0$, because

$$l_{x+t} = l_{x_0}({}_{x+t-x_0}p_{x_0})$$

$$= l_{x_0}({}_{x-x_0}p_{x_0})\,{}_tp_x$$

$$= l_x({}_tp_x).$$

So indeed we describe the survival probabilities via

when l_x values are given through the life table. The starting value, the radix l_{x_0}, is completely arbitrary and cancels out of computations such as (3.1).

Since l_x represents the expected number of survivors at year x, we can also compute mortality probabilities

$$_tq_x = 1 - \frac{l_{x+t}}{l_x}$$

and deferred mortality probabilities

In addition to l_x, another entry in a typical life table is the expected number of deaths during the year, ie

$$d_x = l_x - l_{x+1}. \qquad (3.4)$$

Clearly,

$$d_x = l_x\left(1 - \frac{l_{x+1}}{l_x}\right) = l_x(1 - p_x) \qquad \text{or}$$

which also describes q_x in terms of l_x and d_x.

Other quantities included in life tables

Average person years lived between ages x and $x+1$:

$$L_x = \frac{d_x}{2} + l_{x+1} = \frac{l_x + l_{x+1}}{2} \quad \text{for } x < \omega \quad \text{and} \quad L_\omega = l_\omega \, e_\omega$$

Total person years lived beyond age x:

Expectation of life at age x:

$$e_x = \frac{T_x}{l_x}.$$

Here we have assumed all deaths within a year occur halfway through the year. This is consistent with the UDD factional age assumption in the next section.

Table 1. Life table for the total population: United States, 2007

Spreadsheet version available from:ftp://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/59_09/Table01.xls.

Age	Probability of dying between ages x to $x+1$	Number surviving to age x	Number dying between ages x to $x+1$	Person-years lived between ages x to $x+1$	Total number of person-years lived above age x	Expectation of life at age x
	q_x	l_x	d_x	L_x	T_x	e_x
0–1	0.006761	100,000	676	99,406	7,793,398	77.9
1–2	0.000460	99,324	46	99,301	7,693,992	77.5
2–3	0.000286	99,278	28	99,264	7,594,691	76.5
3–4	0.000218	99,250	22	99,239	7,495,427	75.5
4–5	0.000176	99,228	17	99,219	7,396,188	74.5
5–6	0.000164	99,211	16	99,203	7,296,969	73.6
6–7	0.000151	99,194	15	99,187	7,197,766	72.6
7–8	0.000140	99,179	14	99,173	7,098,579	71.6
8–9	0.000124	99,166	12	99,159	6,999,407	70.6
9–10	0.000105	99,153	10	99,148	6,900,247	69.6
10–11	0.000091	99,143	9	99,138	6,801,099	68.6
11–12	0.000094	99,134	9	99,129	6,701,961	67.6
12–13	0.000132	99,125	13	99,118	6,602,831	66.6
13–14	0.000209	99,112	21	99,101	6,503,713	65.6
14–15	0.000314	99,091	31	99,075	6,404,612	64.6
15–16	0.000426	99,060	42	99,039	6,305,537	63.7
16–17	0.000529	99,018	52	98,991	6,206,498	62.7
17–18	0.000627	98,965	62	98,934	6,107,506	61.7
18–19	0.000715	98,903	71	98,868	6,008,572	60.8
19–20	0.000796	98,832	79	98,793	5,909,705	59.8
20–21	0.000881	98,754	87	98,710	5,810,911	58.8
21–22	0.000963	98,667	95	98,619	5,712,201	57.9
22–23	0.001017	98,572	100	98,522	5,613,582	56.9
23–24	0.001034	98,472	102	98,421	5,515,060	56.0
24–25	0.001023	98,370	101	98,320	5,416,639	55.1
25–26	0.001003	98,269	99	98,220	5,318,320	54.1
26–27	0.000990	98,171	97	98,122	5,220,100	53.2
27–28	0.000983	98,074	96	98,025	5,121,978	52.2
28–29	0.000988	97,977	97	97,929	5,023,952	51.3
29–30	0.001005	97,880	98	97,831	4,926,023	50.3
30–31	0.001030	97,782	101	97,732	4,828,192	49.4
31–32	0.001060	97,681	104	97,630	4,730,460	48.4
32–33	0.001099	97,578	107	97,524	4,632,831	47.5
33–34	0.001146	97,471	112	97,415	4,535,307	46.5
34–35	0.001201	97,359	117	97,300	4,437,892	45.6
35–36	0.001264	97,242	123	97,180	4,340,592	44.6
36–37	0.001340	97,119	130	97,054	4,243,412	43.7
37–38	0.001434	96,989	139	96,919	4,146,358	42.8
38–39	0.001548	96,850	150	96,775	4,049,438	41.8
39–40	0.001685	96,700	163	96,618	3,952,664	40.9
40–41	0.001836	96,537	177	96,448	3,856,045	39.9
41–42	0.002000	96,360	193	96,263	3,759,597	39.0
42–43	0.002188	96,167	210	96,062	3,663,334	38.1
43–44	0.002400	95,956	230	95,841	3,567,272	37.2
44–45	0.002629	95,726	252	95,600	3,471,431	36.3
45–46	0.002864	95,475	273	95,338	3,375,831	35.4
46–47	0.003107	95,201	296	95,053	3,280,493	34.5
47–48	0.003369	94,905	320	94,745	3,185,440	33.6
48–49	0.003661	94,586	346	94,412	3,090,694	32.7
49–50	0.003984	94,239	375	94,052	2,996,282	31.8
50–51	0.004337	93,864	407	93,660	2,902,230	30.9
51–52	0.004709	93,457	440	93,237	2,808,570	30.1
52–53	0.005091	93,017	474	92,780	2,715,333	29.2
53–54	0.005474	92,543	507	92,290	2,622,553	28.3
54–55	0.005863	92,037	540	91,767	2,530,263	27.5
55–56	0.006275	91,497	574	91,210	2,438,496	26.7
56–57	0.006726	90,923	612	90,617	2,347,286	25.8
57–58	0.007220	90,311	652	89,985	2,256,669	25.0
58–59	0.007773	89,659	697	89,311	2,166,684	24.2
59–60	0.008389	88,962	746	88,589	2,077,373	23.4
60–61	0.009081	88,216	801	87,816	1,988,784	22.5
61–62	0.009839	87,415	860	86,985	1,900,968	21.7
62–63	0.010657	86,555	922	86,094	1,813,983	21.0
63–64	0.011534	85,632	988	85,139	1,727,890	20.2
64–65	0.012491	84,645	1,057	84,116	1,642,751	19.4

Table 1. Life table for the total population: United States, 2007—Con.

Spreadsheet version available from:ftp://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/59_09/Table01.xls.

Age	Probability of dying between ages x to $x + 1$ q_x	Number surviving to age x l_x	Number dying between ages x to $x + 1$ d_x	Person-years lived between ages x to $x + 1$ L_x	Total number of person-years lived above age x T_x	Expectation of life at age x e_x
65–66	0.013600	83,587	1,137	83,019	1,558,635	18.6
66–67	0.014722	82,451	1,214	81,844	1,475,616	17.9
67–68	0.015959	81,237	1,296	80,589	1,393,772	17.2
68–69	0.017288	79,940	1,382	79,249	1,313,183	16.4
69–70	0.018755	78,558	1,473	77,822	1,233,934	15.7
70–71	0.020424	77,085	1,574	76,298	1,156,112	15.0
71–72	0.022385	75,511	1,690	74,666	1,079,814	14.3
72–73	0.024679	73,820	1,822	72,909	1,005,149	13.6
73–74	0.027320	71,999	1,967	71,015	932,239	12.9
74–75	0.030299	70,032	2,122	68,971	861,224	12.3
75–76	0.033636	67,910	2,284	66,768	792,254	11.7
76–77	0.037216	65,625	2,442	64,404	725,486	11.1
77–78	0.041160	63,183	2,601	61,883	661,082	10.5
78–79	0.045503	60,583	2,757	59,204	599,199	9.9
79–80	0.050281	57,826	2,908	56,372	539,995	9.3
80–81	0.055531	54,918	3,050	53,393	483,622	8.8
81–82	0.061293	51,869	3,179	50,279	430,229	8.3
82–83	0.067611	48,689	3,292	47,044	379,950	7.8
83–84	0.074528	45,398	3,383	43,706	332,906	7.3
84–85	0.082091	42,014	3,449	40,290	289,201	6.9
85–86	0.090346	38,565	3,484	36,823	248,911	6.5
86–87	0.099341	35,081	3,485	33,338	212,088	6.0
87–88	0.109125	31,596	3,448	29,872	178,749	5.7
88–89	0.119744	28,148	3,371	26,463	148,877	5.3
89–90	0.131244	24,778	3,252	23,152	122,415	4.9
90–91	0.143668	21,526	3,093	19,979	99,263	4.6
91–92	0.157056	18,433	2,895	16,986	79,284	4.3
92–93	0.171442	15,538	2,664	14,206	62,298	4.0
93–94	0.186853	12,874	2,406	11,671	48,092	3.7
94–95	0.203309	10,469	2,128	9,404	36,420	3.5
95–96	0.220822	8,340	1,842	7,419	27,016	3.2
96–97	0.239389	6,499	1,556	5,721	19,597	3.0
97–98	0.258999	4,943	1,280	4,303	13,876	2.8
98–99	0.279625	3,663	1,024	3,151	9,573	2.6
99–100	0.301225	2,638	795	2,241	6,422	2.4
100 and over	1.000000	1,844	1,844	4,181	4,181	2.3

Example 3-1: Using the enclosed life table find:

(a) the probability that a 21 year-old lives at least 20 more years:

(b) the probability that a 60 year-old dies within the next 3 years:

(c) the probability that a 50 year-old dies within 3 years of turning 58:

Example 3-2: Consider the mortality probabilities shown below and construct a life table with a radix of 100,000.

x	q_x	l_x	d_x	L_x	T_x	e_x
0	.15					
1	.25					
2	.55					
3	1.0					
4						

Section 3.3 - Fractional Age Assumptions

Uniform Distribution of Death (UDD) Fractional Age Assumption.

Assume that deaths are uniformly distributed between the beginnings of years.

- - - - - - - - - - - - - -

UDD 1

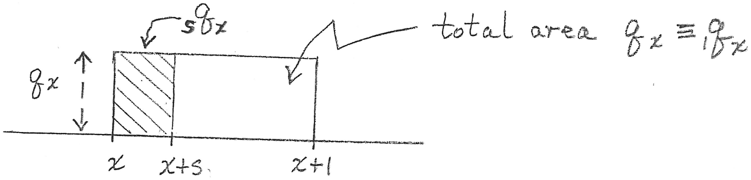
For every nonnegative integer x and any $0 \leq s < 1$, assume

- - - - - - - - - - - - - -

Here

$q_x \equiv {}_1q_x$ is the probability of death between x and $x + 1$

${}_sq_x$ is the probability of death between x and $x + s$.


sqx
qx
total area qx ≡ 1qx
x
x+s
x+1

UDD 2

Define R_x by

$$T_x \equiv \text{lifelength beyond age } x \text{ (measured continuously)}$$

where $K_x = \lfloor T_x \rfloor$ is the number of whole years lived (an integer), R_x denotes the fraction of the year lived between x and $x+1$,

(a) R_x is a uniform $(0, 1)$ random variable, and

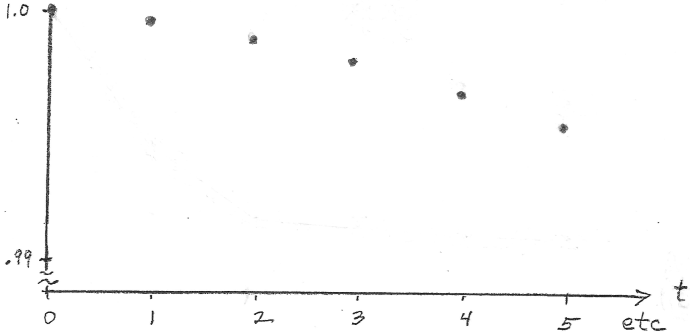
(b) K_x and R_x are independent random variables

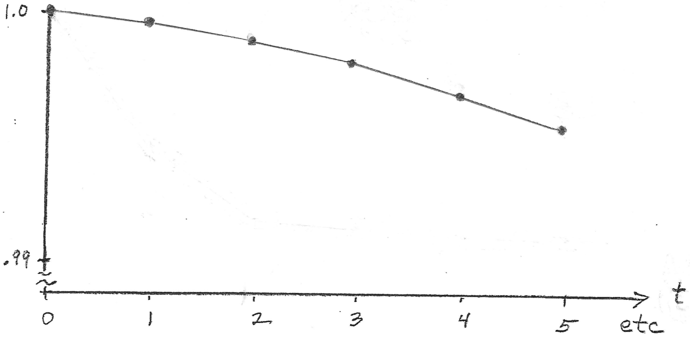
- - - - - - - - - - - - - -

Our textbook proves that UDD1 and UDD2 are alternative expressions of equivalent assumptions concerning the uniform distribution of deaths within each year.

Survival Function Interpretation of UDD

Suppose we plot the survival function $S_{x_0}(t)$ over the future lifelength years, that is, we plot $\left(t, \frac{l_{x_0+t}}{l_{x_0}}\right)$, for $t = 0, 1, \cdots$.

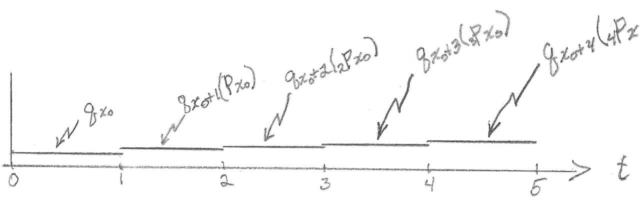



1.0
.99
t
0
1
2
3
4
5
etc

The slope of the line segment between integers t and $t+1$ is:

$$\frac{l_{x_0+t+1}}{l_{x_0}} - \frac{l_{x_0+t}}{l_{x_0}} = -\frac{d_{x_0+t}}{l_{x_0}} = -\left(\frac{d_{x_0+t}}{l_{x_0+t}}\right)\left(\frac{l_{x_0+t}}{l_{x_0}}\right) = -q_{x_0+t}({}_t p_{x_0}).$$

The distribution function $F_{x_0}(\cdot) = 1 - S_{x_0}(\cdot)$ has a constant derivative of $q_{x_0+t}({}_t p_{x_0})$ between integers t and $t+1$. So the density function of this continuous random variable T_{x_0} is a histogram:



Under the UDD fractional age assumption for any nonnegative integer x and any $0 \leq s \leq 1$, define

It follows that

holds whether $x \geq 0$ and or $t \geq 0$ are integers or not.

Example 3-3: Using the previous life table and UDD, find

(a) ${}_{8.2}q_{21.5}$

(b) Given $q_{35} = .001264$ and $q_{36} = .00134$, use only this information to find ${}_{.5}q_{35.8}$.

Note that if x is an integer and $0 \le s < t \le 1$, then because

we have

$$\frac{l_{x+t}}{l_{x+s}} = \frac{{}_tp_x}{{}_sp_x} = \frac{1 - t\,q_x}{1 - s\,q_x} \qquad \text{(these must have the same } x\text{).}$$

Note also that

$${}_{.5}q_{35.8} = {}_{.2}q_{35.8} + (1 - {}_{.2}q_{35.8})\,{}_{.3}q_{36} \qquad \text{and}$$

$${}_{.2}q_{35.8} = 1 - {}_{.2}p_{35.8} = 1 - \left(\frac{l_{36}}{l_{35.8}}\right)$$

$$= 1 - \left(\frac{p_{35}}{{}_{.8}p_{35}}\right) = 1 - \left(\frac{1 - q_{35}}{1 - .8\,q_{35}}\right)$$

So

$$ {}_{.2}q_{35.8} = 1 - \frac{1 - .001264}{1 - .8(.001264)} = .00025306 \qquad \text{and} $$

$$ {}_{.5}q_{35.8} = (.00025306) + (.99974694)(.3)(.001340) = .00065495 $$

In chapter 2 we saw that the future lifetime density satisfies

$$ f_x(t) = {}_t\, p_x \, \mu_{x+t} \quad . $$

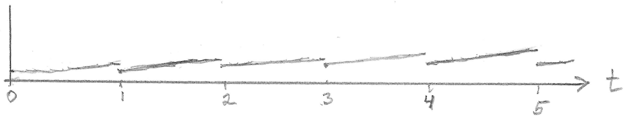
We will use this relationship to solve for the force of mortality under the UDD fractional age assumption. Again, let x be a nonnegative integer and $0 \leq s < 1$. Because of the histogram nature of the density of the future lifelength distribution,

$$ f_x(s) = q_x = {}_s p_x \, \mu_{x+s} $$

and thus

Thus we see that as s increases, the force of mortality also increases under the UDD fractional age assumption. Here

$$\lim_{s \searrow 0} \mu_{x+s} = q_x \qquad \text{and} \qquad \lim_{s \nearrow 1} \mu_{x+s} = \frac{q_x}{1 - q_x}.$$



Constant Force of Mortality (CFOM) Fractional Age Assumption.

A second assumption used to model fractional ages, is that the force of mortality between the beginnings of year x and $x + 1$ is constant (denote it by μ_x^*) . Let x be an arbitrary nonnegative integer and $0 \leq s < 1$. In chapter 2 we established that

$$= e^{-\mu_x^* \int_0^s dr} \quad = \quad e^{-s\,\mu_x^*}.$$

Letting s approach 1, shows that

$p_x = e^{-\mu_x^*}$ or that

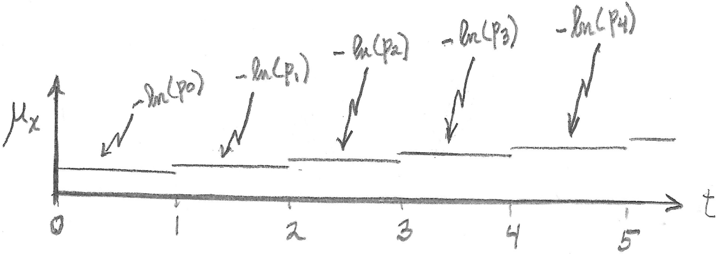
This shows the relationship between the constant force of mortality and the survival probability for that given year.

It follows that when x is an arbitrary nonnegative integer and $0 \leq s < 1$,

If, in addition, $t > 0$ satisfies $t + s \leq 1$ then

$$ {}_sp_{x+t} = (p_x)^s. $$

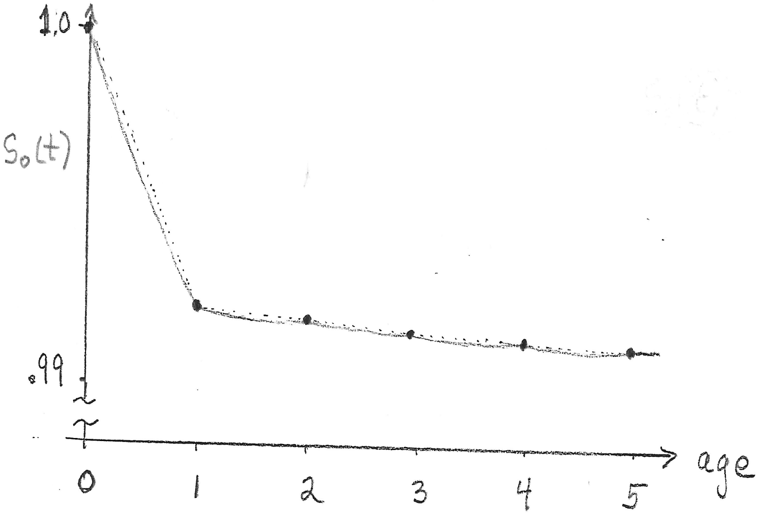
Under the CFOM fractional age assumption, the force of mortality function changes from year to year, but within a given year it is constant. Therefore the graph of the force of mortality function is a histogram, as pictured below.



Under the CFOM fractional age assumption, the survival function takes the form

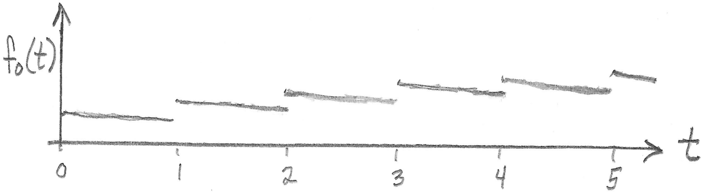
$$= \Big(\prod_{r=0}^{k^*} p_r\Big)(p_{k^*})^{t-k^*-1},$$

where $k^* = \lfloor t \rfloor$. It looks like


1.0
$S_0(t)$
.99
0
1
2
3
4
5
age

The future lifetime, T_0, has a distribution with density

for $k^* < t < k^* + 1$. In our example its graph decreases slightly between integers. It looks like



Under the CFM fractional age assumption,

$$\mu_x^* = -\ln(p_x) \qquad \text{or} \qquad p_x = e^{-\mu_x^*},$$

and it follows that

whenever μ_x^* is small. Thus when q_x is small, so is μ_x^* and they are roughly equal.

The survival function can be written as

$$S_0(x+s) = \Big(\prod_{j=0}^{x} p_j\Big)\Big(\frac{1}{p_x}\Big)\,{}_sp_x$$

when x is a nonnegative integer and $0 \le s \le 1$. The UDD fractional assumption uses

$${}_sp_x = 1 - s\,q_x$$

while the CFOM fractional assumption uses

$${}_sp_x = (p_x)^s.$$

As long as q_x is small,

$$(p_x)^s = e^{-s\,\mu_x^*} \approx (1 - s\,\mu_x^*)$$

that is, the two assumptions produce very similar results.

Hyperbolic Fractional Age Assumption.

Under the hyperbolic assumption, for $0 \leq s < 1$,

The force of mortality becomes

$$\mu_x = \frac{q_x}{1 - (1 - s)\, q_x}.$$

This is a decreasing function of s, which is the reason it is rarely used.

Table : Summary of formulas for fractional ages

Function	Uniform distribution of deaths	Constant force of mortality	Hyperbolic assumption
l_{x+s}	$l_x - sd_x$	$l_x p_x^s$	$\frac{l_{x+1}}{p_x + sq_x}$
${}_sq_x$	sq_x	$1 - p_x^s$	$\frac{sq_x}{1-(1-s)q_x}$
${}_sq_{x+t}$	$\frac{sq_x}{1-tq_x}$	$1 - p_x^s$	$\frac{sq_x}{1-[1-(s+t)]q_x}$
μ_{x+s}	$\frac{q_x}{1-sq_x}$	$-\ln p_x$	$\frac{q_x}{1-(1-s)q_x}$
m_x	$\frac{q_x}{1-0.5q_x}$	$-\ln p_x$	$-\frac{q_x^2}{p_x \ln p_x}$
$\mathring{e}_x$	$e_x + 0.5$		
$\mathring{e}_{x:\overline{n}\vert}$	$e_{x:\overline{n}\vert} + 0.5(1 - {}_np_x)$		

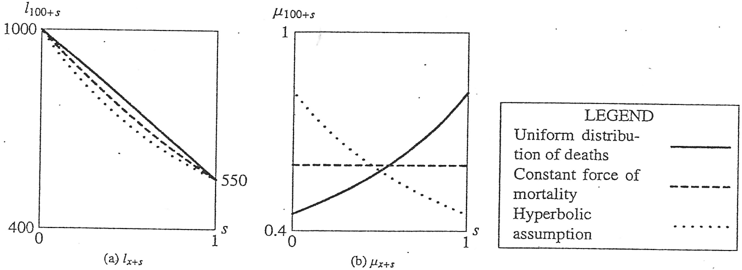


Figure : Comparison of the three assumptions for distribution of deaths between integral ages

Example 3-4: Use the US life tables to find the following under the UDD fraction age assumption:

(a) ${}_{6.5}p_{35.4}$

(b) ${}_{7.6}q_{25.8}$

(c) ${}_{7.5|6.2}q_{40.5}$

Example 3-5: Use the US life tables to find the following under the CFOM fraction age assumption:

(a) ${}_{6.5}p_{35.4}$

(b) ${}_{7.6}q_{25.8}$

(c) ${}_{3.5|7.6}q_{55.4}$

Section 3.4 - Measuring and Comparing Mortality

The mortality rate

$$q_x = P[\text{ a person who has lived to age x dies before reaching age x+1}]$$

provides a direct comparison of the risk of death at different ages.

Example: Comparison of the mortality rates of males and females in the U.S. (2002).

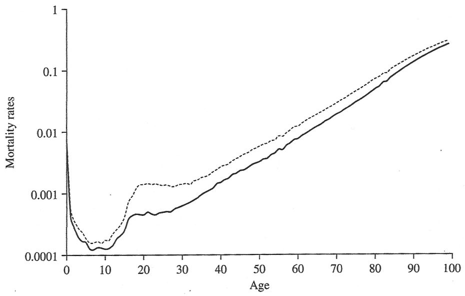


Figure 3.1 US 2002 mortality rates, male (dotted) and female (solid).

Note that

$$q_x = \frac{S_0(x) - S_0(x+1)}{S_0(x)} = \frac{F_0(x+1) - F_0(x)}{S_0(x)}$$

as long as $f_0(\cdot)$ does not change too much between x and $x+1$.

So the force of mortality is a continuous version of the same idea as the discrete version mortality rate.

Thus mortality comparisons are also made in terms of force of mortality.

Mortality rates differ among genders, between racial groups and between life insurance policy holders and those without insurance. Likewise mortality rates differ from one country to another country. These differences often result in separate life tables being constructed for each group.

Another influence on survival is provided by underwriting. Because of the examination of health records, only healthy persons are accepted as new policy holders. Consequently their survival experience is far better than the general population at least for a few years following taking out their policy.

Section 3.7 - Select and Ultimate Survival Models

We make the following modeling assumptions:

(1) Survival depends on

(a) the current age of the person

(b) the age x at which this individual joined the insured group -

(2) After a fixed number of years, d, the survival advantage of recent selection wears off and the survival probability is the same as it is for all insured individuals of that same age.

The survival probabilities

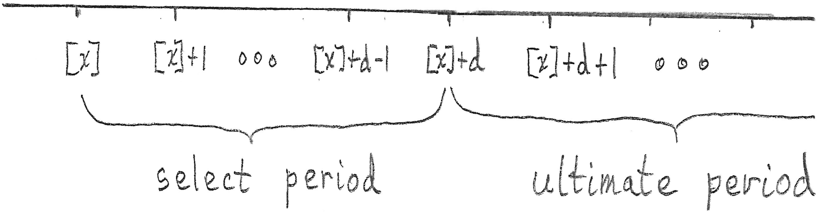
$$p_{[x]} \quad p_{[x]+1} \quad p_{[x]+2} \quad \cdots \quad p_{[x]+d-1}$$

describe the survival experience of the select at age $[x]$ group during the select period (the d years following selection).

After that the survival probabilities

$$p_{[x]+d} \equiv p_{x+d} \quad p_{[x]+d+1} \equiv p_{x+d+1} \quad \cdots$$

are the same as other individuals of the same age. This is described as the ultimate period of the model because the survival (mortality) probabilities are now equal to a ultimate (typical) set of probabilities for persons of this age.



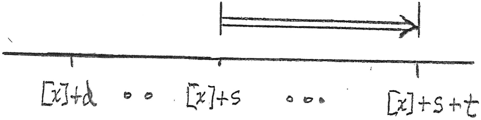
Earlier in this chapter we used a basic life table to find survival (mortality) probabilities via:

Rule:

Mortality $\quad {}_tq_x = 1 - \dfrac{l_{x+t}}{l_x}$

Clearly, when we are completely in the ultimate period of the select model, this rule still applies. That is, let $s \geq d$ and $t \geq 1$, then

$$ {}_t p_{[x]+s} $$



We use the counts in the ultimate life table to compute probabilities for survival (or mortality) in the usual manner (select age is irrelevant).

If, however, $0 \leq s < d$ and $t \geq 1$, then

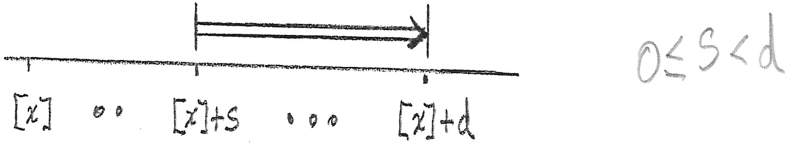
$$ {}_{t}p_{[x]+s} $$

must incorporate the survival experience of the select at age [x] group. For this group we have special survival (or mortality) probabilities

$$ p_{[x]} \quad p_{[x]+1} \quad p_{[x]+2} \quad \cdots \quad p_{[x]+d-1} $$

$$ (q_{[x]} \quad q_{[x]+1} \quad q_{[x]+2} \quad \cdots \quad q_{[x]+d-1}). $$

From these we can compute the probabilities of surviving until the start of the ultimate period via



These probabilities are then used to backfit (impute) expected survival counts $l_{[x]+s}$ for $0 \leq s < d$ via

So the expected survivals from the ultimate table at age $x + d$ is used to establish appropriate expected survivals during the select period.

The good news is that the rule on page 3-39 still applies. We just have to use the expected survival counts appropriate within the select or ultimate period, whichever is needed.

Example 3-6 Use the Select life table D.1 in the Appendix of the textbook (here $d = 2$) to find:

(a) ${}_5p_{[28]+3}$

(b) ${}_4q_{[29]}$

(c) ${}_6p_{[30]+1}$

(d) ${}_{3|2}q_{[55]+1}$

Table 3.7. *Select life table with a two-year select period.*

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$	x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
			100 000.00	20	50	98 552.51	98 450.67	98 326.19	52
			99 975.04	21	51	98 430.98	98 318.95	98 181.77	53
20	99 995.08	99 973.75	99 949.71	22	52	98 297.24	98 173.79	98 022.38	54
21	99 970.04	99 948.40	99 923.98	23	53	98 149.81	98 013.56	97 846.20	55
22	99 944.63	99 922.65	99 897.79	24	54	97 987.03	97 836.44	97 651.21	56
23	99 918.81	99 896.43	99 871.08	25	55	97 807.07	97 640.40	97 435.17	57
24	99 892.52	99 869.70	99 843.80	26	56	97 607.84	97 423.18	97 195.56	58
25	99 865.69	99 842.38	99 815.86	27	57	97 387.05	97 182.25	96 929.59	59
26	99 838.28	99 814.41	99 787.20	28	58	97 142.13	96 914.80	96 634.14	60
27	99 810.20	99 785.70	99 757.71	29	59	96 870.22	96 617.70	96 305.75	61
28	99 781.36	99 756.17	99 727.29	30	60	96 568.13	96 287.48	95 940.60	62
29	99 751.69	99 725.70	99 695.83	31	61	96 232.34	95 920.27	95 534.43	63
30	99 721.06	99 694.18	99 663.20	32	62	95 858.91	95 511.80	95 082.53	64
31	99 689.36	99 661.48	99 629.26	33	63	95 443.51	95 057.36	94 579.73	65
32	99 656.47	99 627.47	99 593.83	34	64	94 981.34	94 551.72	94 020.33	66
33	99 622.23	99 591.96	99 556.75	35	65	94 467.11	93 989.16	93 398.05	67
34	99 586.47	99 554.78	99 517.80	36	66	93 895.00	93 363.38	92 706.06	68
35	99 549.01	99 515.73	99 476.75	37	67	93 258.63	92 667.50	91 936.88	69
36	99 509.64	99 474.56	99 433.34	38	68	92 551.02	91 894.03	91 082.43	70
37	99 468.12	99 431.02	99 387.29	39	69	91 764.58	91 034.84	90 133.96	71
38	99 424.18	99 384.82	99 338.26	40	70	90 891.07	90 081.15	89 082.09	72
39	99 377.52	99 335.62	99 285.88	41	71	89 921.62	89 023.56	87 916.84	73
40	99 327.82	99 283.06	99 229.76	42	72	88 846.72	87 852.03	86 627.64	74
41	99 274.69	99 226.72	99 169.41	43	73	87 656.25	86 555.99	85 203.46	75
42	99 217.72	99 166.14	99 104.33	44	74	86 339.55	85 124.37	83 632.89	76
43	99 156.42	99 100.80	99 033.94	45	75	84 885.49	83 545.75	81 904.34	77
44	99 090.27	99 030.10	98 957.57	46	76	83 282.61	81 808.54	80 006.23	78
45	99 018.67	98 953.40	98 874.50	47	77	81 519.30	79 901.17	77 927.35	79
46	98 940.96	98 869.96	98 783.91	48	78	79 584.04	77 812.44	75 657.16	80
47	98 856.38	98 778.94	98 684.88	49	79	77 465.70	75 531.88	73 186.31	81
48	98 764.09	98 679.44	98 576.37	50	80	75 153.97	73 050.22	70 507.19	82
49	98 663.15	98 570.40	98 457.24	51					

Example 3-7: Using the US life table (2007) construct a select life table for ages 40-46 using select ages 40 and 41 with a 3-year select period and

$$q_{[40]} = .0015 \quad q_{[40]+1} = .0017 \quad q_{[40]+2} = .0021$$

$$q_{[41]} = .0016 \quad q_{[41]+1} = .0019 \quad q_{[41]+2} = .0023$$

Section 3.10 - Other Life Table Topics

(a) Life Table Analogues for Continuous Mortality Models

$$d_x = l_x - l_{x+1} = l_x(1 - p_x)$$

FOM $$\mu_x = \frac{-\frac{d}{dt}\left({}_{x+t}p_0\right)\big|_{t=0}}{{}_xp_0}$$

Expected future life length beyond x:

$$\overset{\circ}{e}_x = \int_0^\infty {}_tp_x dt$$ and

$$T_x = \int_0^\infty l_{x+t} dt$$

$$L_x = \int_0^1 l_{x+t} dt$$

(b) Percentiles of Future Life Length

The 100π percentile of future life length beyond age x is:

(Continuous Life Length Case) - the t_0 satisfying

$${}_{t_0}q_x = \pi$$

(Discrete Life Length Case) - the value $t_0 + s_0$ where t_0 is a nonnegative integer and $0 \leq s_0 < 1$ satisfy

In particular, the median future life length, $m(x)$, satisfies the above with $\pi = \frac{1}{2}$.

(c) Central Death Rate

m_x is defined as the number of deaths during the year divided by the average number alive during the year, i.e.

This differs slightly from q_x, which is the number of deaths during the year divided by the number alive at the beginning of the year.