Chapter 1 - Life Contingent Financial Instruments

The purpose of this course is to explore the mathematical principles that underly life contingent insurance products such as

- Life Insurance
- Pensions
- Lifetime Annuities

Section 1.2 - Some Important Terms

Premium - a payment from the policyholder to the insurance company These are often periodic payments.

Benefit - a payment from the insurance company to the policyholder or that person's beneficiaries (person or persons designated by the policyholder to receive the payment) This payment may be in the form of a lump sum or the policy could alternatively designate periodic payments.

Section 1.3 - Life Insurance

Life insurance is an attempt to indemnify (compensate for loss) the hardship created by the death of the policyholder. The policyholder pays a premium and this person's beneficiaries receive a lump sum payment upon the policyholder's death or upon survival of the policyholder to a predetermined maturity date.

Term Insurance - a lump sum benefit is paid upon the death of the policy holder provided it occurs before the end of a specific term.

(a) Decreasing Term - the sum insured (benefit) and (typically) the premium decrease over time.

(b) Level Term - the sum insured (benefit) and the premium stay the same over time.

(c) Renewable Term - the policyholder has the option to renew the policy at the end of the term without undergoing a health check.

Whole Life Insurance - a lump sum benefit is paid upon the death of the policy holder with premiums payable until death or until the policyholder reaches some maximum age, for example age 65.

Endowment Insurance - a lump sum benefit is paid upon the death of the policy holder or at the end of a specified term, whichever comes first.

Universal Life Insurance - Some of the premium is used to fund life insurance (either whole life or term depending on structure) while the remaining part of the premium is applied to an investment fund that grows the policy's cash value.

A "Participating" Insurance Policy (with profit) - for this policy, profits earned on invested premiums are shared with policyholders in the form of reduced premiums or reversionary bonuses (increases in the benefits promised).

Equity Linked Insurance

(a) Variable Annuity - premiums are deposited in an investment account. The benefit at maturity is the accumulated value of the premiums, but there is a minimum death benefit if the policyholder dies before maturity.

(b) Equity Indexed Annuity - premiums earn a guaranteed minimum return. At maturity the policyholder receives a portion of the return on a stock index if that exceeds the guaranteed minimum.

Underwriting

The health status of a policy applicant is important in determining the person's insurability and/or an appropriate premium amount. The process of collecting and evaluating the health information of prospective policyholders is called underwriting. Based on health information, applicants for insurance are classified in one of the following categories:

(a) preferred - low mortality risk due to lifestyle habits and family history

(b) normal (standard) - a few, but typical risk factors are present.

(c) rated - some significant risk factors are present. These must be taken into account when an appropriate premium is set (often an individual decision).

(d) uninsurable - strong risk factors make an insurance contract unwise at any price

Section 1.4 - Life Annuities

An annuity contract is an agreement designed to produce a series of payments.

Whole Life Annuity - payments continue from the start of the contract until the death of the annuitant (the policyholder) - whole life annuities are often purchased by persons entering retirement as a way to provide steady retirement income

Term Life Annuity - payments continue from the start of the contract until the death of the annuitant, but stops if the annuitant survives beyond a fixed time beyond the contract's start (called the term of the contract)

Some Classifications of Annuities:

Single Premium Deferred Annuity (SPDA) - Policyholder pays a single premium and benefit payments begin at some future specified date. If the policyholder dies before benefit payments begin, there is a death benefit. There may also be a guarantee period, a minimum benefit payment period with payments going to the estate of the annuitant if the annuitant does not survive this period.

Single Premium Immediate Annuity (SPIA) - This is like the SPDA, except the benefit payments begin when the contract begins.

Regular Premium Deferred Annuity (RPDA) - This is like the SPDA, except that instead of making just one premium payment, premiums are paid periodically throughout the deferral period.

Life Annuities often have the following structures:

(a) Joint Life Annuity - based on two lives, the annuity benefit payments continue as long as both individuals survive.

(b) Last Survivor Annuity - based on two lives, the annuity benefit payments continue as long as at least one of the individuals survive

(c) Reversionary Annuity - based on two lives (one designated the annuitant and the other the insured), the annuity only begins paying benefits in the event that the annuitant out lives the insured life. These payments then continue as long as the annuitant survives.

Section 1.5 - Pension Plans

Defined Contribution Plan (DC) - Both the employer and the employee contribute while the employee is working (usually a fixed percent of salary). These contributions grow in an investment fund over time and the accumulated funds are then available to provide income for the employee upon retirement. The funds are often used to purchase a SPIA. The 401K accounts work in this manner.

Defined Benefit Plan (DB) - Offers retirement income based on a defined formula which incorporates the number of years of service to this employer (a fixed percent of salary per year of employment) and the salary level of the individual (using the average salary during the last three [five] years of employment). The employer (and typically the employee) pay into this fund during the employed years. These contributions must be sufficient to provided the stated benefits promised to the employee which may also depend on the age of the employee.

Section 1.6 - Insurance Companies

A mutual insurance company is one that is owned by its policyholders. It has no shareholders. Profits are distributed to policyholders through dividends or bonuses.

A proprietary insurance company has shareholders. Profits of the company are distributed to shareholders and with-profit policyholders through a predetermined formula.

Insurance is sold not bought through the use of:

(a) Agents or financial advisors

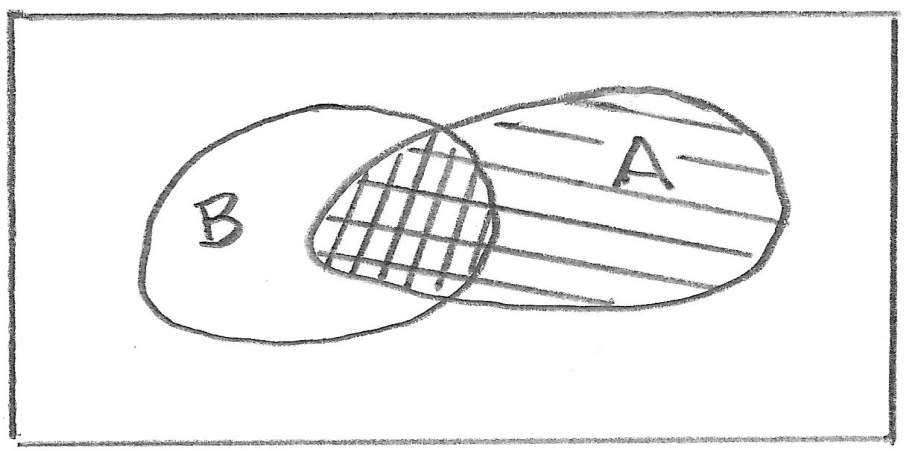
(b) Direct Marketing - television advertising, call-in centers, internet websites.

Appendix 1 - Probability

Conditional Probability

The conditional probability of event B given event A occurs, is the fraction of the probability in A that is also in B.

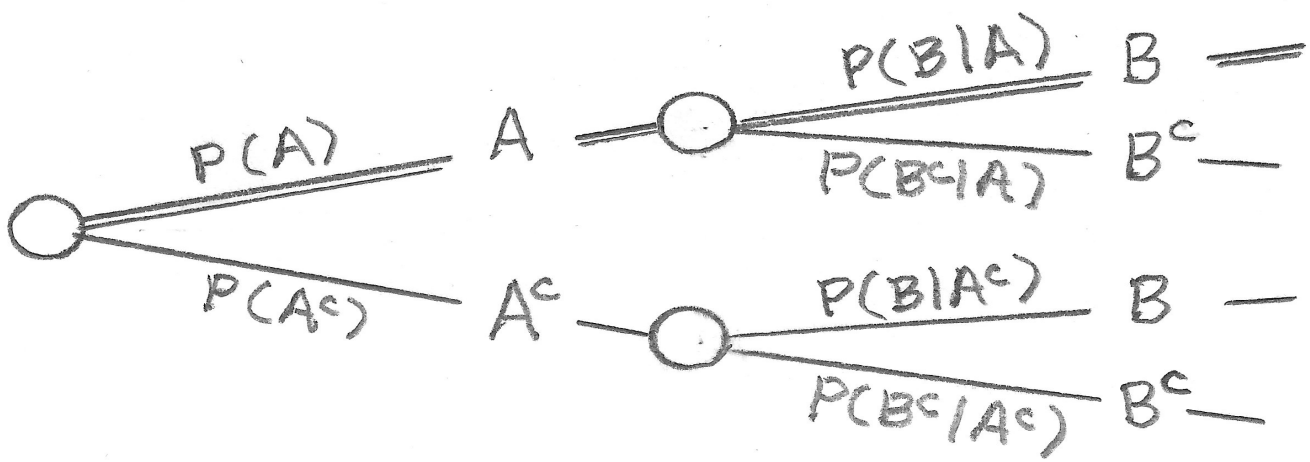
$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$



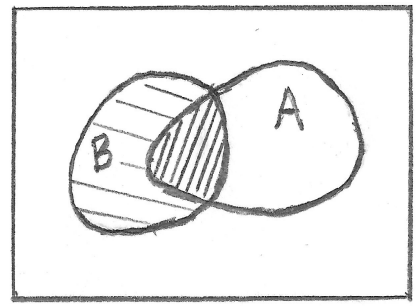
Multiplicative Law of Probability

The probability that both events A and B occur, is the probability of event A times the conditional probability of event B given that event A occurs.

$$P(A \cap B) = P(A)\,P(B \mid A)$$

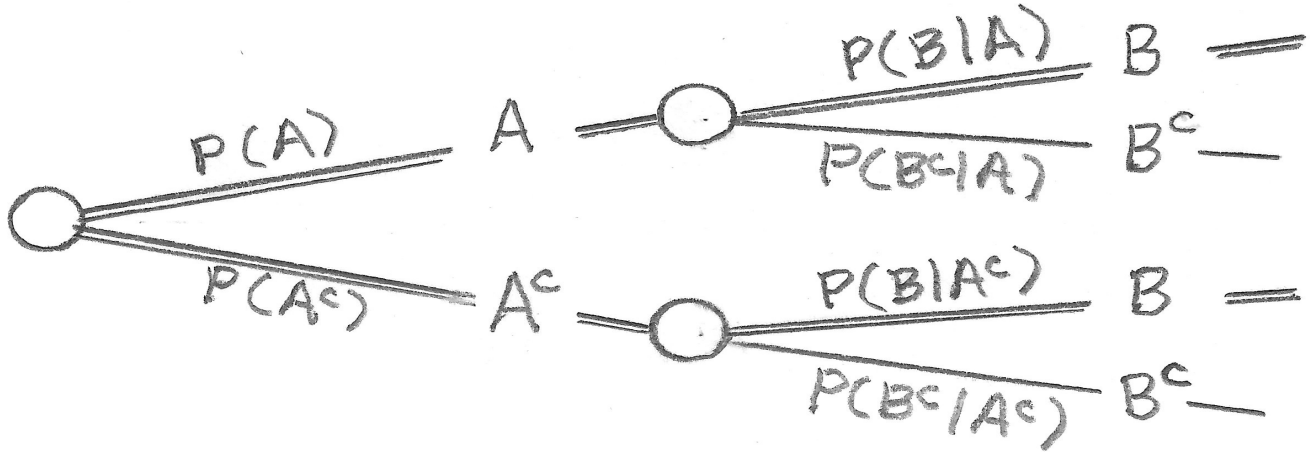


Law of Total Probability



$$P(B) = P(A \cap B) + P(A^c \cap B)$$

$$P(B) = P(A)\,P(B|A) + P(A^c)\,P(B|A^c)$$

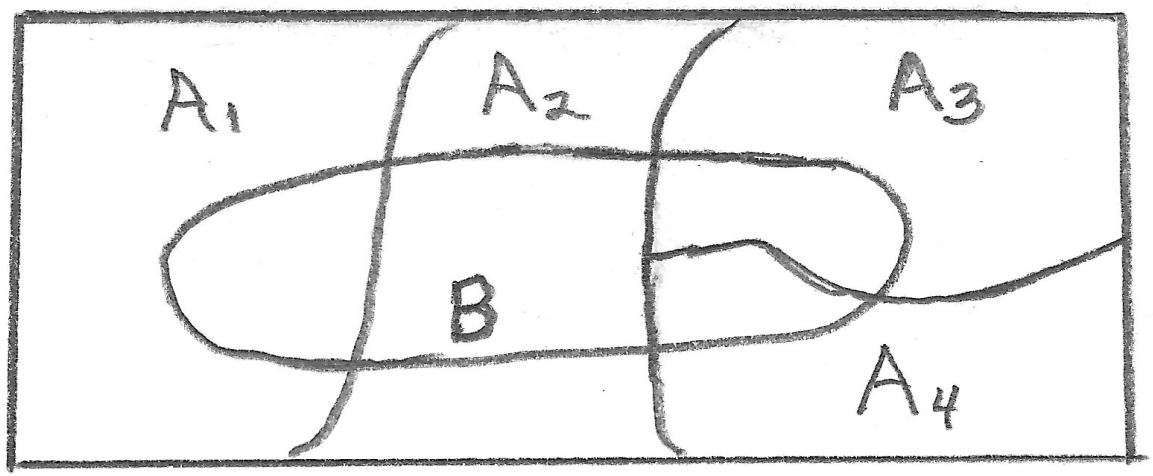


More generally, if

$$A_i \cap A_j = \phi \qquad \text{and} \qquad \cup_i A_i = S$$

then

$$P(B) = \sum_i P(A_i)\, P(B \mid A_i).$$



Appendix 2 - Random Variables

Expected Values

$$E\Big[\, h(\, Y\,)\,\Big] = \begin{cases} \sum\, h(y)\, p(y) & \text{in discrete case} \\ \\ \int\, h(y)\, f(y)\, dy & \text{in continuous case} \end{cases}$$

It is the average value of the function $h(\cdot)$ when evaluated at a random value Y.

Mean

$$E\Big[\, Y\,\Big] = \int\, y\, f(y)\, dy \quad \text{or} \quad \sum\, y\, p(y).$$

It is the average (balance point) of the distribution of the random variable Y.

Variance

$$Var\left[\, Y \,\right] = \left[\, \left\{\, Y - E[\, Y \,] \,\right\}^2 \,\right] = E\left[\, Y^2 \,\right] - \left\{\, E[\, Y \,] \,\right\}^2.$$

It is the average squared distance between the value of the random variable Y and its mean.

Standard Deviation

$$StDev\left[\, Y \,\right] = \sqrt{Var\left[\, Y \,\right]}.$$

It is a measure of variability of the random variable Y. It has the same unit of measurement as the values of Y.

Application to a Linear Function (Here a and b are constants.)

$$E\Big[\, a + b\,Y \,\Big] = a + b\,E\Big[\, Y \,\Big]$$

$$Var\Big[\, a + b\,Y \,\Big] = b^2\,Var\Big[\, Y \,\Big]$$

$$StDev\Big[\, a + b\,Y \,\Big] = |\,b\,|\,StDev\Big[\, Y \,\Big]$$