Chapter 11: Summary

1. Let $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_m$ be independently distributed with the X_i iid $\mathrm{N}(\mu_X, \sigma^2)$ and the Y_j iid $\mathrm{N}(\mu_Y, \sigma^2)$. Then $\bar{X} - \bar{Y} \sim \mathrm{N}(\mu_X - \mu_Y, \sigma^2(n^{-1} + m^{-1}))$.

2. The pooled estimator of σ^2 is s_p^2, where
$s_p^2 = [(n-1)s_X^2 + (m-1)s_Y^2]/(n+m-2)$, $(n-1)s_X^2 = \sum_{i=1}^n (X_i - \bar{X})^2$ and $(m-1)s_Y^2 = \sum_{j=1}^m (Y_j - \bar{Y})^2$.

3. Then $\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sigma(n^{-1}+m^{-1})^{1/2}} \sim \mathrm{N}(0,1)$.

4. A $100(1-\alpha)\%$ two-sided confidence interval for $\mu_X - \mu_Y$ for known σ is given by $\bar{X} - \bar{Y} \pm z_{\alpha/2}\sigma(n^{-1} + m^{-1})^{1/2}$.

5. A $100(1-\alpha)\%$ two-sided confidence interval for $\mu_X - \mu_Y$ for unknown σ is given by $\bar{X} - \bar{Y} \pm t_{n+m-2;\alpha/2} s_p (n^{-1} + m^{-1})^{1/2}$.

6. Let $T = (\bar{X} - \bar{Y})/[s_p(n^{-1} + m^{-1})^{1/2}]$. For testing
$H_0 : \mu_X = \mu_Y$ against $H_0 : \mu_X > \mu_Y$, reject when $T > t_{n+m-2;\alpha}$.
$H_0 : \mu_X = \mu_Y$ against $H_0 : \mu_X < \mu_Y$, reject when $T < -t_{n+m-2;\alpha}$.
$H_0 : \mu_X = \mu_y$ against $H_0 : \mu_X \neq \mu_Y$, reject when $|T| > t_{n+m-2;\alpha/2}$.

7. For paired samples, if (X_i, Y_i), $i = 1, \cdots, n$ are iid with means μ_X and μ_Y, variances σ_X^2 and σ_Y^2 and correlation coefficient ρ, then define $D_i = X_i - Y_i$, $i = 1, \cdots, n$, and $\bar{D} = \bar{X} - \bar{Y}$. Then $\mu_D = E(\bar{D}) = \mu_X - \mu_Y$ and $V(\bar{D}) = n^{-1}(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y)$.

8. Let $s_D^2 = (n-1)^{-1} \sum_{i=1}^n (D_i - \bar{D})^2$. Then a $100(1-\alpha)\%$ two-sided confidence interval for $\mu_X - \mu_Y$ is $\bar{D} \pm t_{n-1;\alpha/2} s_D / n^{1/2}$.