# Worksheet - Introduction to Matrix Computation in R - US City Temps 

| Mean Temperature (F) |  |  |  |  |
| :--- | ---: | ---: | ---: | :---: |
| A | 1970 s | 1980 s | 1990 s | 2000s |
| LA | $\mathbf{6 2 . 9 5}$ | $\mathbf{6 3 . 4 1}$ | $\mathbf{6 3 . 6 3}$ | $\mathbf{6 2 . 9 7}$ |
| Miami | $\mathbf{7 6 . 0 1}$ | $\mathbf{7 6 . 2 2}$ | $\mathbf{7 7 . 3 5}$ | $\mathbf{7 7 . 1 8}$ |
| Orlando | $\mathbf{7 2 . 4 4}$ | $\mathbf{7 2 . 5 0}$ | $\mathbf{7 3 . 1 1}$ | $\mathbf{7 2 . 7 4}$ |

All instructions assume matrices are conformable to the operations ( $R$ will let you know if not!)

1. To create a matrix in $R$, enter the elements (default is by columns) separated by commas in concatenated form, followed by the numbers or rows and columns (there are many shorthand ways to do this).

A <- matrix(c(a11,a21, a31,a12,a22, a32,a31,a32, a33,a41, a42, a43), 3, 4)
2. To create an nxn Identity matrix, use $\operatorname{diag}(\mathbf{n})$
3. To obtain the transpose of the matrix $A$, use $\mathbf{t}(\mathbf{A})$
4. To multiply the matrix A by the scalar k, use $\mathbf{k *} \mathbf{A}$
5. To add two matrices, use $\mathbf{A}+\mathbf{B}$
6. To subtract one matrix from another, use $\mathbf{A}-\mathbf{B}$
7. To multiply 2 matrices (columns $(\mathrm{A})=\operatorname{rows}(\mathrm{B})): \mathbf{A} \% * \boldsymbol{\%} \mathbf{B}$
8. To create an mxn J matrix, $\mathbf{J}<-\boldsymbol{m a t r i x}(\mathbf{r e p}(\mathbf{1}, \mathbf{m} * \mathbf{n}), \mathbf{m}, \mathbf{n})$
9. To obtain a submatrix of $A$, containing rows $i_{1}, \ldots, i_{r}$ and columns $j_{1}, \ldots, j_{c}$ use
$\mathbf{A s}<-\mathbf{A}\left[\mathbf{c}\left(\mathbf{i}_{1}, \ldots, \mathbf{i}_{\mathbf{r}}\right), \mathbf{c}\left(\mathbf{j}_{1}, \ldots, \mathbf{j}_{\mathbf{c}}\right)\right] \quad$ if consecutive rows and columns: $\mathbf{A s}<-\mathbf{A}\left[\mathbf{i}_{1}: \mathbf{i}_{\mathbf{r}}, \mathbf{j}_{1}: \mathbf{j}_{\mathbf{c}}\right]$
10. To obtain a submatrix containing a subset of rows: $\mathbf{A r}<-\mathbf{A}\left[\mathbf{c}\left(\mathbf{i}, \ldots, \mathbf{i}_{\mathbf{r}}\right)\right.$, ]
11. To obtain a submatrix containing a subset of rows: $\mathbf{A c}<-\mathbf{A}[, \mathbf{c}(\mathbf{j} 1, \ldots, \mathbf{j} \mathbf{c})]$

## Using these commands:

A. Create the matrix $\mathbf{A}$
B. Create a $3 \times 1$ vector of 1 's ( $\mathbf{j} 3$ ), a $4 \times 1$ vector of 1 's ( $\mathbf{j} 4$ ), a $3 \times 3$ identity matrix ( $\mathbf{I 3}$ ), a $3 \times 3$ matrix of 1 's (J33), a $4 x 4$ identity matrix (I4), a $4 \times 4$ matrix of 1 's (J44)

## Using only the matrices and vectors above:

C. Obtain the $3 \times 1$ column vector of city means
D. Obtain the 1 x 4 column vector of decade means
E. Create the matrix of mean temperatures in Celsius $C=(5 / 9)(F-32)$
F. Obtain the variance-covariance matrix of the decade temperatures $\mathbf{S} \_\mathbf{c o l}=(1 /(3-1)) \mathbf{A}^{\prime}(\mathbf{I} 3-\mathbf{J 3 3}) \mathbf{A}$
G. Obtain the variance-covariance matrix of the city temperatures $\mathbf{S} \_\mathbf{r o w}=(1 /(4-1)) \mathbf{A}(\mathbf{I} 4-\mathbf{J} 44) \mathbf{A}^{\prime}$
H. Obtain the submatrix containing only the Florida cities and only the decades 1970s and 2000s.
I. Generate the matrix A using equation (4.3) on page 21 of Harville using the following algorithm.

- Create an mxn matrix of 0 's where $m=3, n=4$ Anew <- matrix(rep( $\mathbf{0}, \mathbf{m} * \mathbf{n}), \mathbf{m}, \mathbf{n}$ )
- for (i1 in $1: m$ ) \{
for (i2 in 1:n) \{
e_i1 <- matrix(I33[, i1],ncol=1); u_i2 <- matrix(I44[i2 ,],ncol=4)
Anew <- Anew + A[i1,i2]* e_i1 \%*\% u_i2
\}
\}
- Show that $\mathbf{I 3 3}[\mathbf{2}] \% * \,% \mathbf{A} \% * \% \mathbf{I 4 4}[, \mathbf{3}]$ gives the temperature for the correct city/decade.

