**STA 6207 – Exam 2 – Fall 2015 PRINT Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**All Questions are based on the following 2 regression models, where SIMPLE REGRESSION refers to the case where *p*=1, and X is of full column rank (no linear dependencies among predictor variables).**

**Conduct all Tests at  = 0.05 significance level.**

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**Given: **

**Cochran’s Theorem:** Suppose **Y** is distributed as follows with nonsingular matrix **V:**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Critical Values for 2 and F-distributions**

# F-distributions indexed by numerator df across top of table

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **df** | **** | **F(.05,1)** | **F(.05,2)** | **F(.05,3)** | **F(.05,4)** | **F(.05,5)** | **F(.05,6)** | **F(.05,7)** | **F(.05,8)** | **F(.05,9)** |
| **1** | **3.841** | **161.446** | **199.499** | **215.707** | **224.583** | **230.160** | **233.988** | **236.767** | **238.884** | **240.543** |
| **2** | **5.991** | **18.513** | **19.000** | **19.164** | **19.247** | **19.296** | **19.329** | **19.353** | **19.371** | **19.385** |
| **3** | **7.815** | **10.128** | **9.552** | **9.277** | **9.117** | **9.013** | **8.941** | **8.887** | **8.845** | **8.812** |
| **4** | **9.488** | **7.709** | **6.944** | **6.591** | **6.388** | **6.256** | **6.163** | **6.094** | **6.041** | **5.999** |
| **5** | **11.070** | **6.608** | **5.786** | **5.409** | **5.192** | **5.050** | **4.950** | **4.876** | **4.818** | **4.772** |
| **6** | **12.592** | **5.987** | **5.143** | **4.757** | **4.534** | **4.387** | **4.284** | **4.207** | **4.147** | **4.099** |
| **7** | **14.067** | **5.591** | **4.737** | **4.347** | **4.120** | **3.972** | **3.866** | **3.787** | **3.726** | **3.677** |
| **8** | **15.507** | **5.318** | **4.459** | **4.066** | **3.838** | **3.688** | **3.581** | **3.500** | **3.438** | **3.388** |
| **9** | **16.919** | **5.117** | **4.256** | **3.863** | **3.633** | **3.482** | **3.374** | **3.293** | **3.230** | **3.179** |
| **10** | **18.307** | **4.965** | **4.103** | **3.708** | **3.478** | **3.326** | **3.217** | **3.135** | **3.072** | **3.020** |
| **11** | **19.675** | **4.844** | **3.982** | **3.587** | **3.357** | **3.204** | **3.095** | **3.012** | **2.948** | **2.896** |
| **12** | **21.026** | **4.747** | **3.885** | **3.490** | **3.259** | **3.106** | **2.996** | **2.913** | **2.849** | **2.796** |
| **13** | **22.362** | **4.667** | **3.806** | **3.411** | **3.179** | **3.025** | **2.915** | **2.832** | **2.767** | **2.714** |
| **14** | **23.685** | **4.600** | **3.739** | **3.344** | **3.112** | **2.958** | **2.848** | **2.764** | **2.699** | **2.646** |
| **15** | **24.996** | **4.543** | **3.682** | **3.287** | **3.056** | **2.901** | **2.790** | **2.707** | **2.641** | **2.588** |
| **16** | **26.296** | **4.494** | **3.634** | **3.239** | **3.007** | **2.852** | **2.741** | **2.657** | **2.591** | **2.538** |
| **17** | **27.587** | **4.451** | **3.592** | **3.197** | **2.965** | **2.810** | **2.699** | **2.614** | **2.548** | **2.494** |
| **18** | **28.869** | **4.414** | **3.555** | **3.160** | **2.928** | **2.773** | **2.661** | **2.577** | **2.510** | **2.456** |
| **19** | **30.144** | **4.381** | **3.522** | **3.127** | **2.895** | **2.740** | **2.628** | **2.544** | **2.477** | **2.423** |
| **20** | **31.410** | **4.351** | **3.493** | **3.098** | **2.866** | **2.711** | **2.599** | **2.514** | **2.447** | **2.393** |
| **21** | **32.671** | **4.325** | **3.467** | **3.072** | **2.840** | **2.685** | **2.573** | **2.488** | **2.420** | **2.366** |
| **22** | **33.924** | **4.301** | **3.443** | **3.049** | **2.817** | **2.661** | **2.549** | **2.464** | **2.397** | **2.342** |
| **23** | **35.172** | **4.279** | **3.422** | **3.028** | **2.796** | **2.640** | **2.528** | **2.442** | **2.375** | **2.320** |
| **24** | **36.415** | **4.260** | **3.403** | **3.009** | **2.776** | **2.621** | **2.508** | **2.423** | **2.355** | **2.300** |
| **25** | **37.652** | **4.242** | **3.385** | **2.991** | **2.759** | **2.603** | **2.490** | **2.405** | **2.337** | **2.282** |
| **26** | **38.885** | **4.225** | **3.369** | **2.975** | **2.743** | **2.587** | **2.474** | **2.388** | **2.321** | **2.265** |
| **27** | **40.113** | **4.210** | **3.354** | **2.960** | **2.728** | **2.572** | **2.459** | **2.373** | **2.305** | **2.250** |
| **28** | **41.337** | **4.196** | **3.340** | **2.947** | **2.714** | **2.558** | **2.445** | **2.359** | **2.291** | **2.236** |

Q.1. A study related Freight Volume (Y) in Shanghai to GDP (X1) and Fixed Investment (X2) over a period of n = 11 years. The authors fit the following 3 models:





p.1.a. Compute SSTotalCorrected

p.1.a. Compute SSRegression and SSResidual for each model.

p.1.b. Compute 

p.1.c. Test H0: 11 = 0 vs HA: 11 ≠ 0 (Note there are 2 ways of doing this).

p.1.d. What proportion of the variation in Y that is not explained by X1 is explained by X2?

Q.2. For the F-test for Lack-of-Fit, where:



Q.4. Consider the following Sums of Squares for Model 2:



p.4.a. Write out each as a Quadratic form of the vector **Y** and give the trace of its defining matrix (aka degrees of freedom). Note: trace(**AB**) = trace(**BA**) when matrices are compatible for multiplication.

p.4.a.i. SS(Model) = **Y’A1Y A1** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ trace(**A1**) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

p.4.a.ii. SS() = **Y’A2Y A2** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ trace(**A2**) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

p.4.a.iii. SS(Regression) = **Y’A3Y A3** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ trace(**A3**) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

p.4.b. What is the distribution of SS()/2? Show all work.

p.4.c. Show that SS() and SS(Regression) are independent. Show all work.

Q.5. Regression models are fit, relating price of Compact Hybrid Cars (Y, in $1000s) to Acceleration (X1, in km/hour/sec) and Miles per Gallon (max of Gas and Electric mph) for n=25 models from years 2009-2013.

Consider the following models with Residual Sums of Squares for each model (SSTotalCorr = 2196)



Note:



p.5.a. Compute Cp for Model 1.

p.5.b. Compute AIC for model 2

p.5.c. Compute SBC for models 3 and 4. Which model is preferred based on that criteria?