**Factorial Designs**

**Part A. Fixed Effects**

QA.1. A hotel is interested in studying the effects of washing machines and detergents on whiteness of bed sheets. The hotel has 4 washing machines and 3 brands of detergent. They randomly assign n=4 sheets for each combination of machine and detergent (each sheet is only observed for one combination of machine and detergent). After washing, the sheets are measured for whiteness (high scores are better). The model fit is:



pA.1.a. Give least squares estimates of the following parameters:









pA.1.b. Complete the ANOVA table.

pA.1.c. Is there a significant interaction between Machine and Detergent on whiteness scores? **Yes** / **No**

pA.1.d. Is there a significant main effect for Machines? **Yes** / **No**

pA.1.e. Is there a significant main effect for Detergents? **Yes** / **No**

QA.2. For the 2-way ANOVA with fixed effects and interaction, considering the following two models:



pA.2.a. Write out the Sums of Squares and Degrees of Freedom for each model:

Model 1: SSA = dfA =

 SSB = dfB

 SSAB = dfAB =

Model 2: SSTrts = dfTrts =

pA.2.b. (Algebraically) Show that SSA + SSB + SSAB = SSTrts and that dfA + dfB + dfAB = dfTrts

Hint: The expansion for SSAB is very much like the expansion for SSA.

QA.3. A 2-Factor (fixed effects ANOVA) is fit, with factor A at 5 levels, and factor B at 3 levels. There were 5 replicates for each combination of factor levels. Compute Bonferroni’s and Tukey’s minimum significant differences for comparing all pairs of means, each (Factors A and B, respectively) with an experiment-wise error rate of 0.05. Note: This is the same as the margin of error for the point estimates (half-width of simultaneous CI’s). Assume that interaction is not present, and give your results as functions of only MSE.

Factor: A B

Bonferroni

Tukey

QA.4. An unbalanced 2-Way ANOVA was fit (based on nT = 433 people), relating music empathizing scores (Y) to the factors: **E**xperience (Professional, Amateur, Non-Musician) and **G**ender (Male, Female). Due to unbalanced sample sizes for the 6 cells (sample sizes ranged from 38 to 160), a regression model was fit with the following dummy coded variables:



The following output gives partial ANOVA Results for 4 models: (1:E,G,EG 2:E,G 3:G,EG 4:E,EG)



Use these models to test the following hypotheses (always round down on error degrees of freedom, if necessary):

pA.4.a. H0: No Experience by Gender Interaction

pA.4.a.i. Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ p.5.a.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pA.4.b. H0: No Experience Main Effect (Controlling for Gender and EG)

pA.4.b.i. Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ p.5.b.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pA.4.c. H0: No Gender Main Effect (Controlling for Experience and EG)

pA.4.c.i. Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ p.5.c.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

QA.5. An experiment was conducted to determine the effects of food appearance on the plate (balanced/unbalanced and color/monochrome) on consumers hedonic liking of food. A sample of 68 restaurant customers was obtained, and assigned such that 17 received each combination of balance and color. The following table gives the means (SDs) for each treatment. The response (Y) was hedonic liking of the food dish (on a scale of -100 to +100).

Model: 

pA.5.a. The following table gives the means (SDs) for each treatment:



Complete the following ANOVA table:



pA.5.b. Test H0: No Interaction between Balance and Color

pA.5.b.i. Test Stat: \_\_\_\_\_\_\_ pA.5.b.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_ pA.5.b.iii. P-value **> or <** .05?

pA.5.c. Test H0: No Balance effect

pA.5.c.i. Test Stat: \_\_\_\_\_\_\_ pA.5.c.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_ pA.5.c.iii. P-value **> or <** .05?

pA.5.d. Test H0: No Color effect

pA.5.d.i. Test Stat: \_\_\_\_\_\_\_\_\_ pA.5.d.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_ pA.5.d.iii. P-value **> or <** .05?

QA.6. An experiment was conducted to compare *a* = 3 theories for the apparent modulus of elasticity (*Y*) of *b* = 3 apple varieties. The 3 theories were: Hooke’s, Hertz’s, and Boussineq’s; the 3 apple varieties were: Golden Delicious, Red Delicious, and Granny Smith. The researchers determined the elasticity for *n* = 15 based on each combination of theory and variety. For the purposes of this experiment, each factor is fixed.





Complete the following Analysis of Variance Table, and test for interaction effects and main effects.



QA.7 Consider a 2-factor, fixed effects, interaction model with *a* = *b* = *n* = 2. 

pA.7.a. Given the following data, obtain MSErr.



SSErr = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ MSErr = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pA.7.b. Use the matrix form of the model to obtain: the least squares estimate of the parameter vector **** , its estimated variance-covariance matrix, standard errors, and t-tests for all of the parameters.





QA.7. A 2-Way fixed effects model is fit, with factor A at 4 levels, and factor B at 3 levels. There are 3 replicates for each combinations of factors A and B. The error sum of squares is SSErr = 720.

pA.7.a. Suppose that the interaction is not significant. What will be Tukey’s and Bonferroni’s minimum significant differences be for comparing the means for factor A (averaged across levels of factor B).

Tukey’s HSD: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Bonferroni’s MSD: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 pA.7.b. Suppose that the interaction is significant. What will be Tukey’s and Bonferroni’s minimum significant differences be for comparing the means for factor A (within a particular level of factor B).

Tukey’s HSD: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Bonferroni’s MSD: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pA.7.c. How large would the interaction sum of squares, SSAB need to be to reject H0: No interaction between factors A&B?

QA.8. A forensic researcher is interested in the ridge densities between males and females and Caucasians and African-Americans. Samples of r=100 from each sub-population are obtained and for each subject, the number of ridges per 25mm2 is measured. Means (standard deviations) are given in the following table.



Complete the following ANOVA table, providing tests for interaction, as well as main effects for gender and race.



QA.9. Researchers measured water accumulation in multi-layer clothing. They considered 3 types of outer layers (wool, polyester, down) and the same 3 types of inner layers (wool, polyester, down). They had 2 replicates at each combination of outer and inner layers.



pA.9.a. Complete the following ANOVA table.



pA.9.b. The P-value for testing H0: ()11 = … = ()33 = 0 is **> 0.05 < 0.05 (Circle one)**

pA.9.c. The following table gives the cell means for all combinations of outer (rows) and inner (columns). Use Tukey’s method to compare all pairs of treatments (combinations of outer and inner layers), and draw lines connecting all treatments that are not significantly different. Note: W/P ⇒ Outer = Wool, Inner = Polyester



Tukey’s HSD = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 D/P D/W P/W W/W P/P W/P W/D P/D D/D

QA.10. An experiment was conducted to determine the effects of viewing a magical film versus a non-magical film in children. Samples of 32 6-year olds, and 32 8-year-olds were selected, and randomly assigned such that 16 of each age- group viewed the magical film and 16 of each age-group viewed the non-magical. The following table gives the means (SDs) for each treatment. The response (Y) was a score on an imagination scale (rating of a child acting out an object or animal).

Model: 

The following table gives the means (SDs) for each treatment:



Complete the following ANOVA table:



QA.11. An unbalanced two-way ANOVA was conducted to compare ethics scores on an exam (Y) among members of the U.S. Coast Guard. The factors were Gender (X1=1 if Male, -1 if Female) and Rank (X2=1 if Officers, -1 if Enlisted). The sample sizes were: M/O = 72, M/E = 180, F/O = 15, F/E =32. Four regressions models were fit:



pA.11.a. Test whether there is an interaction between Gender and Rank. H0: \_\_\_\_\_\_\_\_\_\_\_\_ HA: \_\_\_\_\_\_\_\_\_\_\_

Test Statistic \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ P-value **> 0.05** or **< 0.05**

pA.11.c. Test whether there is main effect for Gender. H0: \_\_\_\_\_\_\_\_\_\_\_\_ HA: \_\_\_\_\_\_\_\_\_\_\_

Test Statistic \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ P-value **> 0.05** or **< 0.05**

pA.11.c. Based on Model 2, give the predicted scores for all combinations of Gender and Rank.

****

QA.12. A study was conducted, measuring the effects of 3 electronic **Readers** and 4 **Illumination** levels on time for people to read a given text (100s of seconds). There were a total of 60 subjects, 5 assigned to each combination of Reader/Illumination level. For this analysis, consider both Reader and Illumination level as fixed effects.

pA.12.a Complete the following ANOVA table, and test for significant Reader/Illumination Interaction effects, as well as main effects for Reader and Illumination levels.



pA.12.b Use Tukey’s Method to make all pairwise comparisons among Readers. 

pA.12.c Use Bonferroni’s method to make all pairwise comparisons among Illumination levels. 

QA.13. A study compared the effects of *a* = 3 brands of squash rackets (Factor A: med, high, and higher price) and *b* = 2 levels of strings (Factor B: factory or new) the speed of a ball’s bounce off the racket (Y, in m/sec) after having been dropped from a standard height. There were *n* = 10 replicates per treatment (combination of racket brand and string type). Assume these are the only levels of interest to the researchers (that is, both are fixed factors).

pA.13.a. Write out the statistical model assuming errors are iid Normal.

pA.13.b. The following table gives the sample means. Compute the following least squares estimates of model parameters and 2:







pA.13.c. Complete the following Analysis of Variance Table, including stating the null hypothesis for each row in the table.



QA.14. A study was conducted to test for effects on willingness to pay during online auctions. There were 2 factors, each with 2 levels (both fixed): Urgency (Present (i=1) /Absent (i=2)) and Contrast (3 of 6 items “Featured” (High, j=1)/all 6 items “Featured” (Low, j=2)). There were 6 watches, and the response was the amount the participant was willing to pay for the watch. Although the researchers started with 80 subjects, 9 were eliminated due to incomplete information, so N = 71. The model fit is:



pA.14.a. Give the form of the (full rank) **X** matrix and **** vector for this model. Note that although **X** has 71 rows, there are only 4 “blocks” of distinct levels, each with a particular numbers of subjects.

pA.14.b. Obtain **X’X** as functions of the cell sample sizes (Just give the values on or above the main diagonal).

pA.14.c. The authors fit the following 4 models, with approximate Error sums of squares (divided by 1000 for ease of calculation):



SSErr1 = 884.0 SSErr2 = 937.0 SSErr3 = 941.5 SSErr4 = 978.3

Test: 

Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_\_ Do you conclude the Urgency effect “depends” on Contrast? Y / N

pA.14.d. The sample means for the 4 treatments are: . Treating the cell sample sizes as nij = 18 as an approximation, use Bonferroni’s method to compare all pairs of treatment means.

**Part B. Mixed and Random Effects**

QB.1.. The following partial ANOVA table gives the results of a balanced 2-Way ANOVA with interaction. Fill in the following values (for the mixed model, assume the unrestricted model):

 



QB.2.. For the 2-Way ANOVA with random effects,



Obtain the following quantities:



QB.3. In a large factory, with many **O**perators, **P**arts, and **D**evices, an experiment is conducted to measure the variation in measured strengths of parts. Samples of 5 **O**perators, 10 **P**arts, and 3 **D**evices were obtained; with each combination of Operators, Parts, and Instruments being replicated 2 times. The following model is fit (with all random effects independent).



pB.3.a. Complete the following ANOVA Table:



pB.3.b.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.3.b.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_

pB.3.c.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.3.c.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_

pB.3.d.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.3.d.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_



QB.4. For the 2-way ANOVA with one fixed factor, 1 random factor and a random interaction, considering the following model:



pB.4.a. Derive *E*(*Yij•*), *V*(*Yij•*), Cov(*Yij•, Yij’•*)

pB.4.b. Consider the following results:



Set-up a 95% Confidence Interval for (i - i’) based on these results (note that the variance components are unknown). Hint: Derive the mean and variance of  and make use of the estimated variance (standard error). Be specific (symbolically) and on degrees of freedom being used.

QB.5. The following partial ANOVA table gives the results of a balanced 2-Way ANOVA with interaction. Fill in the following values (for the mixed model, assume the unrestricted model, that is (ab)ij ~ NID(0,ab2)):

 



QB.6. For the 2-Way ANOVA with random effects and interactions,



Obtain the following quantities:



QB.7. In a large factory, with many **O**perators, **P**arts, and **D**evices, an experiment is conducted to measure the variation in measured strengths of parts. Samples of 6 **O**perators, 8 **P**arts, and 4 **D**evices were obtained; with each combination of Operators, Parts, and Instruments being replicated 3 times. The following model is fit (with all random effects independent).



pB.7.a. Complete the following ANOVA Table:



pB.7.b.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.7.b.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_

pB.7.c.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.7.c.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_

pB.7.d.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.7.d.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_



QB.8. A study was conducted to determine the effects of 2 map factors on subjects’ abilities to complete web-map tasks. A group of 96 subjects was obtained, **and each subject was measured in each of the 4 map conditions**. Y was the time to complete the task (sec).

* Factor A: Map centering on zooming in/out (Original Center/Re-center)
* Factor B: Directional Pan (Grouped together/ Distributed on edge of map)
* Factor C: Subject (96 individuals)

pB.8.a. Write out the appropriate statistical model, making clear what effects are fixed/random.

pB.8.b. Based on the following table, obtain SSA, SSB, and SSAB.



SSA = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ SSB = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ SSAB = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.8.c. Given the authors’ results, complete the following ANOVA table and give estimates of 





QB.9. A mixed model is fit with 2 factors, A fixed with a=3 levels, and B random with b=levels. There are r=4 replicates per treatment (combination of levels of A and B).

pB.9.a. Write the statistical model (including main effects and interactions).

pB.9.b. Derive (showing all work): 

QB.10. For the 2-way ANOVA with one fixed factor, 1 random factor and a random interaction, considering the following model:



pB.10.a. Derive *E*(*Yij•*), *V*(*Yij•*), Cov(*Yij•, Yij’•*)

pB.10.b. Consider the following results:



Set-up a 95% Confidence Interval for (i - i’) based on these results (note that the variance components are unknown). Hint: Derive the mean and variance of  and make use of the estimated variance (standard error). Be specific (symbolically) and on degrees of freedom being used.

QB.11. For the 2-Way Random Effects model, with a = 2, b = 4, and r = 3; DERIVE the following Variances:





QB.11. A study is conducted to measure intra- and inter-observer reliability in tasting experiments. A random sample of *a* = 10 **V**arieties of wine were selected at a wine store. Further, a random sample of *b* = 5 trained **J**udges were obtained from a society of wine aficionados. Each judge rated each wine *r* = 3 times. The judge gave the wine an evaluation for a particular attribute on a visual analogue scale from 0-100. (Note: The order of the 30 tastes for each judge were randomized and blinded, and the judges were given cab rides home). The model fit is:



pB.11.a. Complete the following ANOVA Table and give the expected mean square for each row. Note that for the F-Stats, you are testing: 



pB.11.b. Give unbiased (ANOVA) estimates of each of the four variance components:



pB.11.c. For this type of reliability study, there are two types of correlations: Inter-Judge (Among), and Intra-Judge (Within). The Inter-Judge and Intra-Judge Covariances, as well as Observation measurement Variance are used to obtain the two correlations. Give these terms (based on actual variance components) and their point estimates:



pB.11.d. The Inter-Judge and Intra-Judge Correlations are defined below, give “simple” point estimates of each:



QB.12. In a large factory, with many **O**perators, **P**arts, and **D**evices, an experiment is conducted to measure the variation in measured strengths of parts. Samples of 4 **O**perators, 10 **P**arts, and 3 **D**evices were obtained; with each combination of Operators, Parts, and Instruments being replicated 5 times. The following model is fit (with all random effects independent).



pB.12.a. Complete the following ANOVA Table:



pB.12.b.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.12.b.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_

pB.12.c.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.12.c.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_

pB.12.d.i Obtain an unbiased estimate of  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

pB.12.d.ii. Test H0:  vs HA:  Test Statistic: \_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_



QB.13. A 2-Way Random Effects model is fit, where a sample of *a* = 8 products were measured by a sample of *b* = 6 machinists, with *n* = 3 replicates per machinist per product. The model fit is as follows (independent random effects):



You are given the following sums of squares: 

Give the test statistic and rejection region for the following 3 tests. Note for test 1, your rejection region will be symbolic, give the specific numerator and denominator degrees of freedom. Also give unbiased (ANOVA) estimates of each variance component.



1: Test Stat: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Estimate: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2: Test Stat: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Estimate: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3: Test Stat: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Rejection Region: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Estimate: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

QB14. Mixed model with factor A fixed with a = 2 levels and factor B random with 3 levels, random interaction, and n = 2 replicates per combination. Give the model in terms of **Y = X + Zu + **Give all elements in full form.

QB.15. An experiment is conducted to estimate the effects of 3 brands of shoes on one mile run times for a sample of 4 runners. Each runner’s time is measured 5 times in each brand of shoe. The experimenters treat the shoes as fixed and the runners as random.

pB.15.a. Write out the statistical model, allowing for main effects and interaction of shoes and runners (be specific on parameters and ranges of subscripts):

pB.15.b. Complete the following table. Whenever possible, use actual numbers, not symbols. For the sum of squares, write out the relevant summation.

Source df Sum of Squares E{MS}

QB.16. A wine tasting is conducted, with a sample of *a* judges, each rating a sample of *b* wines. Note that each judge only rates each wine once.



pB.16.a. 

pB.16.b. Derive 

pB.16.c. The study had 9 judges and 10 varieties of wine. Complete the following ANOVA table testing:





pB.16.d. Obtain unbiased estimates of A2 and 2.

QB.17. An experiment is conducted to compare 4 navigation techniques (Factor A, Fixed), 2 input methods (Factor B, Fixed), in 36 subjects (Factor C, Random). Each subject is measured in each combination of levels of A and B once. Consider the model, where y is the task completion time:



Complete the following ANOVA table, testing all main effects and 2-way interactions.





QB.18. A study was conducted, regarding 2 Random factors: A: Wine Judge (*a* = 9) and B: Wine Brand (*b* = 10). Each judge rated each brand once, blind to the brand label. The model fit is:



pB.18.a. For this model, 

pB.18.a.i. 

pB.18.a.ii. **Derive**  **Be very specific on all parts of derivation.**

pB.18.b. Use the results from above to obtain 

pB.18.c. Write out *MSA* in terms of (a subset) of the components in *SS*ERR and obtain E{*MSA*}.

pB.18.d. Write out *MSB* in terms of (a subset) of the components in *SS*ERR and obtain E{*MSB*}.

pB.18.e. From the published study, we obtain the following sums of squares: 

Obtain unbiased estimates of the model’s three variances.

pB.18.f. Out of the (estimated) total variance , what percentages are accounted by the 3 sources:

Judges (A) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Brands (B) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Error (AB) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

QB.19. Based on the 2014 WNBA season, we have the point totals (Y) by game **Location** (Home/Away) for a **sample** of 10 **Players**. Each player played 17 home games and 17 away games. Consider the model:





pB.19.a. Complete the partial ANOVA table.

pB.19.b. Test whether there is an interaction between Player and Location (Home). H0: 2 = 0

pB.19.b.i. Test Stat: \_\_\_\_\_\_\_ p.5.b.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_ p.5.b.iii. P-value **> or <** .05?

pB.19.c. Test whether there is Location (Home vs Away) Main Effect. H0: 

pB.19.c.i. Test Stat: \_\_\_\_\_\_\_ p.5.c.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_ p.5.c.iii. P-value **> or <** .05?

pB.19.d. Test whether there is Player Main Effect. H0: 2 = 0

pB.19.d.i. Test Stat: \_\_\_\_\_\_\_ p.5.d.ii. Reject H0 if Test Stat is in the range \_\_\_\_\_\_\_\_\_ p.5.d.iii. P-value **> or <** .05?

pB.19.e. Give unbiased estimates of each of the variance components:

