# Solutions to Logistic, Poisson, Nonlinear Regression Problems



**QA.1.** $H_0$: No Interaction between dose and variety $\beta_3 = 0$

$H_A$: Interaction Exists $(\beta_3 \neq 0)$

T.S. $X^2_{obs} = ((-2\ln L_2) - (-2\ln L_3)) = 885.6 - 883.3 = 2.3$

RR: $X^2_{obs} \geq \chi^2_{.05,1} = 3.841$ Do not reject $H_0$

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$H_0$: No variety effect, controlling for Dose $\beta_2 = 0$ (Model 2)

$H_A$: Variety effect exists $\beta_2 \neq 0$ (Model 2)

T.S. $X^2_{obs} = ((-2\ln L_1) - (-2\ln L_2)) = 912.7 - 885.6 = 27.1$

RR: $X^2_{obs} \geq \chi^2_{.05,1} = 3.841$ Reject $H_0$

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| | | VARIETY = A | VARIETY = B |
|---|---|---|---|
| Dose = 1, ~~VARIETY = A~~ | observed | $= \frac{5}{100} = 0.05$ | $\frac{10}{100} = 0.10$ |
| | Fitted | $= \frac{e^{-1.80+0.36(1)-0.87(1)}}{1+e^{-}} = \frac{e^{-2.31}}{1+e^{-2.31}} = .0903$ | $\frac{e^{-1.44}}{1+e^{-1.44}} = .1915$ |
| Dose = 16 | observed: | $\frac{95}{100} = .95$ | $\frac{100}{100} = 1.00$ |
| | Fitted | $\frac{e^{-1.80+0.36(16)-0.87(1)}}{1+e^{-}} = \frac{e^{3.09}}{1+e^{3.09}} = .9564$ | $\frac{e^{3.96}}{1+e^{3.96}} = .9813$ |

LPN2

QA2

$H_0: \beta_A = \beta_M = \beta_{AM} = 0$

$T.S.\ X^2_{obs} = ((-2\ln L_0) - (-2\ln L_1)) = 1349.16 - 1250.34 = 98.82$

$RR: X^2_{obs} \geq \chi^2_{.05,3} = 7.815$

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$H_0: \beta_{AM} = 0 \quad H_A: \beta_{AM} \neq 0 \quad TS: X^2_{obs} = 4.790 \quad P = .029$

Yes

$\text{Model } 0: \quad \hat{\pi} = \frac{e^{-.269}}{1+e^{-.269}} = \frac{.7641}{1.7641} = .433$ Both Groups

$\text{Model } 1: \ 25/0: \quad \hat{\pi} = \frac{e^{-1.257+.016(25)}}{1+e^{-1.257+.016(25)}} = \frac{e^{-.857}}{1+e^{-.857}} = .298$

~~$55/1: \quad \hat{\pi} = e^{.857+.013(1)+(.039)(1)55}$~~

$55/1: \quad \hat{\pi} = \frac{e^{-1.257+.016(55)+.013(1)+.039(1)(55)}}{1+e^{-1.257+.016(55)+.013(1)+.039(1)(55)}}$

$= \frac{e^{1.781}}{1+e^{1.781}} = \frac{5.936}{6.936} = 0.856$

QA3. Based on ACET Model $H_0: \beta_A = 0$ $H_A: \beta_A \neq 0$

LR Test Stat: $X^2_{LR} = (-2\ln L_0) - (-2\ln L_1) = 532.378 - 501.663 = 30.715$

$RR: X^2_{LR} \geq \chi^2_{.05,1} = 3.841$

Wald Test Stat: $X^2_w = 28.085$ $RR: X^2_w \geq 3.841$

$H_0: \beta_G = \beta_{AG} = 0$ $H_A: \beta_G$ and/or $\beta_{AG} \neq 0$

T.S. $X^2_{LR} = 501.663 - 501.318 = 0.345$

$RR: X^2_{LR} \geq \chi^2_{.05,2} = 5.991$

$$\hat{\pi}_{ACET} = \frac{e^{-.656 - 1.235(1)}}{1 + e^{\sim}} = \frac{e^{-1.891}}{1 + e^{-1.891}} = \frac{.151}{1.151} = .131$$

$$\hat{\pi}_{NoAcet} = \frac{e^{-.656 - 1.235(0)}}{1 + e^{\sim}} = \frac{e^{-.656}}{1 + e^{-.656}} = \frac{.519}{1.519} = .342$$

QA.4 P.4.a. $TS: X^2_{obs} = (-2\ln L_0) - (-2\ln L_1) = 185.50 - 180.50 = 5.00$

$RR: X^2_{obs} \geq \chi^2_{.05,1} = 3.841$

P.4.b. $TS: X^2_{obs} = (-2\ln L_3) - (-2\ln L_4) = 177.36 - 175.51 = 1.85$

$RR: X^2_{obs} \geq \chi^2_{.05,1} = 3.841$

P. ~~4.c.~~

continued

LPN4

P.4.~~b~~.i. $HI=0/SC=0:\ \hat{\pi}_{00} = \dfrac{e^{-1.431}}{1+e^{-1.431}} = \dfrac{.2391}{1.239} = 0.193$

$HI=1/SC=0:\ \hat{\pi}_{10} = \dfrac{e^{-1.431+.806}}{1+e^{\overline{\phantom{x}}}} = \dfrac{e^{-.625}}{1+e^{-.625}} = \dfrac{.535}{1.535} = 0.349$

$HI=0/SC=1:\ \hat{\pi}_{01} = \dfrac{e^{-1.431+.640}}{1+e^{\overline{\phantom{x}}}} = \dfrac{e^{-.791}}{1+e^{-.791}} = \dfrac{.453}{1.453} = 0.312$

$HI=1/SC=1:\ \hat{\pi}_{11} = \dfrac{e^{-1.431+.806+.640}}{1+e^{\overline{\phantom{x}}}} = \dfrac{e^{.015}}{1+e^{.015}} = \dfrac{1.015}{2.015} = 0.504$

P.4.c.ii 95% CI for $\beta_{HI}$: $.806 \pm \overbrace{1.96(.363)}^{.711} \equiv (.095, 1.517)$

95% CI for $e^{\beta_{HI}}$: $(e^{.095}, e^{1.517}) \equiv (1.100, 4.559)$

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QA.5

P.5.a.

| Variable | EST | SE | 1.96 Z*SE | CI LB | CI UB | OR | OR LB | OR UB |
|---|---|---|---|---|---|---|---|---|
| PC | -.563 | .248 | .486 | -1.049 | -.077 | .569 | .350 | .926 |
| RC | -.125 | .059 | .116 | -.241 | -.009 | .882 | .786 | .991 |
| SN | .421 | .217 | .425 | -.004 | .856 | 1.523 | .996 | 2.354 |
| I | .540 | .329 | .645 | -.105 | 1.185 | 1.716 | .900 | 3.271 |
| PS | .172 | .106 | .208 | -.036 | .380 | 1.188 | .965 | 1.462 |

P.5.b. i) $\hat{\pi}_{1,6,0,1,1} = \dfrac{\exp\{-.563+6(-.125)+0+.540+.172\}}{1+\exp\{\cdot\}}$

$= \dfrac{e^{-.601}}{1+e^{-.601}} = \dfrac{.548}{1.548} = .354$

ii) $\hat{\pi}_{0,2,1,3,3} = \dfrac{\exp\{2(-.125)+.421+3(.540)+3(.172)\}}{1+\exp\{\cdot\}} = \dfrac{e^{2.307}}{1+e^{2.307}} = \dfrac{10.044}{11.044}$

$= .909$

# QA.6.

**P.6.a.** TS: $X^2_{obs} = (-2\ln L_0) - (-2\ln L_1) = 95.95 - 52.33 = 43.62$

RR: $X^2_{obs} \geq \chi^2_{.05,4} = 9.49$  $P < .05$

**P.6.b.**

| Range | Var | EST | 1.96*SE | CI LB | CI UB | OR | OR LB | OR UB |
|---|---|---|---|---|---|---|---|---|
| 3.5-5.5 | pH | 1.886 | 1.060 | 0.826 | 2.946 | 6.593 | 2.284 | 19.030 |
| 0-70 | Nisin | -.066 | .037 | -.103 | -.029 | 0.936 | 0.902 | 0.971 |
| 25-50 | Temp | .110 | .094 | .016 | .204 | 1.116 | 1.016 | 1.226 |
| 11-19 | Brix | -.312 | .280 | -.592 | -.032 | 0.732 | 0.553 | 0.969 |
| | Constant | -7.246 | 6.309 | -13.555 | -.937 | — | — | — |

Highest: pH@5.5, NISIN@0, TEMP@50, Brix@11

Lowest pH@3.5, NISIN@70, TEMP@25, Brix@19

$$\hat{\pi}_{HIGH} = \frac{\exp\{1.886(5.5) - .066(0) + .110(50) - .312(11) - 7.246\}}{1 + \exp\{\cdot\}} = \frac{e^{5.195}}{1+e^{5.195}}$$

$$= \frac{180.37}{181.37} = .994$$

$$\hat{\pi}_{LOW} = \frac{\exp\{1.886(3.5) - .066(70) + .110(25) - .312(19) - 7.246\}}{1 + \exp\{\cdot\}} = \frac{e^{-8.443}}{1+e^{-8.443}}$$

$$= \frac{.0002}{1.0002} = .0002$$

QB.1. $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ $H_A$: Not all $\beta_i = 0$

T.S. $X^2_{obs} = (-2\ln L_0) - (-2\ln L_1) = 8400 - 7800 = 600$

RR: $X^2_{obs} \geq \chi^2_{.05,4} = 9.49$

$$\ln E\{Y\} = \ln A + \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \hat{\beta}_4 X_4$$

$$= \ln 70 + 5 - 2(70) + 3(15) - 1(8) + 5(0)$$

$$= 4.25 + 5 - 140 + 45 - 8 + 0 = -93.75$$

$\hat{\mu} = e^{-93.75} \simeq 0$ (Poor choice of coefficients)

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QB.2. TS: $z_{obs} = \frac{.215}{.0828} = 2.560$ RR: $z_{obs} \geq z_{.05} = 1.645$

$X = 0$: $\exp\{2.341 + .215(0)\} = \exp\{2.341\} = 10.39$

$X = 1$: $\exp\{2.341 + .215(1)\} = \exp\{2.556\} = 12.88$

$X = 2$: $\exp\{2.341 + .215(2)\} = \exp\{2.771\} = 15.97$

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QB.3. P.3.a. WALD $TS = \left(\frac{.069}{.014}\right)^2 = 24.29$ RR: $TS \geq \chi^2_{.05,1} = 3.841$

LR $TS = 2.88 - (-20.48) = 23.36$

P.3.b. Wald $TS = \left(\frac{-.001}{.002}\right)^2 = 0.25$ LR TS: $(-20.48) - (-21.67) = 1.19$ RR: $TS \geq 3.841$

P.3.c.

Model 1: $\exp\{.987\} = 2.683$

Model 2: $\exp\{-1.582 + .069(40)\} = \exp\{1.178\} = 3.248$

Model3: $\exp\{-2.857 + .136(40) - .001(40)^2\} = \exp\{.983\} = 2.672$

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QB.4. P.4a. TS: $X^2_{obs} = 256.31 - 84.362 = 171.948$

RR: $X^2_{obs} \geq \chi^2_{.05,2} = 5.991$

P.4.b. $\hat{\mu}_1 = \exp\{4.2517 - .0340(33) + .0213(30) - .0358(43.4) + 1.1065\}$
$= \exp\{3.32148\} = 27.70$ Residual$_1$ = $44 - 27.70 = 16.30$

$\hat{\mu}_2 = \exp\{2.6785 - .0115(33) + .0159(30) - .0188(43.4) + 3.7072 - .0609(33)(1)\}$
$= \exp\{3.65758\} = 38.77$ Residual$_2$ $= 44 - 38.77 = 5.23$

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QB.5. P.5.a. TS: $X^2_{obs} = (-2\ln L_1) - (-2\ln L_3) = 486.64 - 443.36 = 43.28$

RR: $X^2_{obs} \geq \chi^2_{.05,2} = 5.991$

P.5.b. Wald TS: $\left(-\frac{.5932}{.0913}\right)^2 = 42.21$ LRTS: $486.64 - 443.86 = 42.78$

RR: $TS \geq \chi^2_{.05,1} = 3.841$

P.5.6.

$$\hat{\mu}_{1993} = \exp\{\ln 425 - 2.0340 - .0058(26) - .5932(0)\} = \exp\{3.867\} = 47.81$$

$$\hat{\mu}_{1995} = \exp\{\ln 423 - 2.0340 - .0058(28) - .5932(1)\} = \exp\{3.258\} = 25.99$$

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QB.6. P.6.a. $TS: X^2_{obs} = 5368.02 - 3948.26 = 1419.76$

$$RR: X^2_{obs} \geq \chi^2_{.05,2} = 5.991$$

P.6.b. Wald TS: $\left(\frac{.0577}{.0014}\right)^2 = 1698.6$ LRTS: $5368.02 - 3955.47 = 1412.55$

$$RR: \chi^2_{obs} \geq \chi^2_{.05,1} = 3.841$$

P.6.c. $\hat{Y}_{Oregon} = \exp\{3.596 + 0.1777(1.574) + .0648(3.83) - .0023(1.574)(3.83)\}$

$$= \exp\{4.11\} = 60.95$$

$$\hat{Y}_{Florida} = \exp\{3.596 + .1777(1.193) + .0648(18.80) - .0023(1.193)(18.80)\}$$

$$= \exp\{4.97\} = 144.58$$

## QC1.

As $X_2 \uparrow \infty$, $E\{Y\} \to 0$

As $X_2 \uparrow$ $E\{Y\}$ Decreases

As $X_1 \uparrow$ $E\{Y\}$ Increases

$X_1 = 100, X_2 = 0 \Rightarrow \hat{Y} = \dfrac{7.6(100)}{1 + .019(100) + 0.17(0)} = \dfrac{760}{2.9} = 262.07$

$X_1 = 100, X_2 = 200 \Rightarrow \hat{Y} = \dfrac{760}{36.9} = 20.60$

$X_1 = 200, X_2 = 0 \Rightarrow \hat{Y} = \dfrac{1520}{4.8} = 316.67$

$X_1 = 200, X_2 = 200 \Rightarrow \hat{Y} = \dfrac{1520}{38.8} = 39.18$

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## QC2.

$Y' = \beta_0' + \beta_1' \ln X_1 + \beta_2' \ln X_2 \qquad Y = e^{Y'}$

$X_1 = 1, X_2 = 1 \Rightarrow \hat{Y}' = b_0' = 6.0 \qquad \hat{Y} = e^{6.0} = 403.43$

$X_1 = 1, X_2 = 10 \Rightarrow \hat{Y}' = b_0' + b_2' \ln 10 = 6.0 + 0.2(2.303) = 6.461$

$\Rightarrow \hat{Y} = e^{6.461} = 639.39$

$X_1 = 10, X_2 = 10 \Rightarrow \hat{Y}' = 6.0 + 0.7 \ln 10 = 7.612 \Rightarrow \hat{Y} = 2022.52$

$X_1 = 10, X_2 = 10 \Rightarrow \hat{Y}' = 6.0 + 0.7(2.303) + 0.2(2.303) = 8.073$

$\Rightarrow \hat{Y} = 3205.75$

**QC3.** $\hat{Y}_1 = 32.46\left[1 - e^{-1.51(1.11)}\right] = 32.46[1 - .187]$

$= 26.4 \qquad e_1 = Y_1 - \hat{Y}_1 = 33.6 - 26.4 = 7.2$

Max mean angle occurs @ $\beta_0$ (as $X \to \infty$)

$\hat{\beta}_0 \pm t_{.025, 16-2}\, se\{\hat{\beta}_0\} \equiv 32.46 \pm \underbrace{2.145(2.65)}_{5.68} \equiv (26.78, 38.14)$

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**QC4.** $X = 0 \Rightarrow E\{Y\} = \alpha + e^{-\beta}$

$X \to \infty \Rightarrow E\{Y\} = \alpha + 1$

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**QC5.** $X_i \to \infty \Rightarrow E\{Y_i\} = \beta_0 \qquad X_i = 0 \Rightarrow E\{Y_i\} = 0$

a) 95% CI for max velocity: $28.1 \pm \underbrace{t_{.025, 18-2}(0.73)}_{1.55} \equiv (26.55, 29.65)$ $\quad t_{.025,18-2} = 2.120$

b) Dose needed for 50% of max $\Rightarrow X_i = \beta_1$

$12.6 \pm \underbrace{2.120(0.76)}_{1.61} \equiv (10.99, 14.21)$

c) $\hat{Y}_0 = 0 \qquad \hat{Y}_{10} = \dfrac{281}{22.6} = 12.43 \qquad \hat{Y}_{20} = \dfrac{562}{32.6} = 17.24 \qquad \hat{Y}_{30} = 19.79$

12.43 $\qquad$ 4.81 $\qquad$ 2.55

LPN11

# QC6.

**P.6.a.** $E\{Y|X=0\} = \beta_0 + \beta_1 \quad E\{Y|X=\infty\} = \beta_0$

**P.6.b.** $\hat{Y}_0 = 16.6466 + 178.9799 e^{-.7201\sqrt{0}} = 195.63$

$\hat{Y}_{10} = 16.6466 + 178.9799(.1026) = 35.01$

$\hat{Y}_{20} = 16.6466 + 178.9789(.0400) = 23.79$

$\hat{Y}_{30} = 16.64\,66 + 178.9799(.0194) = 20.11$

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# QC.7.

**P.7.a.**

Brand1: $16.5 e^{-.0034(120)} = 10.97$

Brand2: $13.23 e^{-.0068(120)} = 5.85$

Brand3: $13.37 e^{-.0056(120)} = 6.83$

**P.7.b.** T.S. $F_{obs} = \dfrac{\frac{SSE_r - SSE_c}{df_r - df_c}}{\frac{SSE_c}{df_c}} = \dfrac{\frac{184.09 - 6.03}{43 - 39}}{\frac{6.03}{39}}$

$= \dfrac{\frac{178.06}{4}}{\frac{6.03}{39}} = \dfrac{44.515}{0.155} = 287.91$

RR: $F_{obs} \geq F_{.05,4,39} \doteq 2.606$

P.7.c.

$$F_{obs} = \frac{\frac{10.31-6.03}{41-39}}{\frac{6.03}{39}} = \frac{2.14}{0.155} = 13.81$$

$$RR: F_{obs} \geq F_{.05,2,39} \doteq 3.232$$

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Q.8. $\hat{Y} = \beta_0 \exp\{-e^{-\beta_1-\beta_2 X}\}$

P.8.a.

$$1994\ (X=0) \Rightarrow \hat{Y} = 13.37448 \exp\{-e^{-(-2.20756)}\}$$

$$= 13.37448(.000112394) = .0015032($$

$$2004\ (X=10) \Rightarrow \hat{Y} = 13.37448 \exp\{-e^{2.20756-4.0323}\}$$

$$= 13.37448 \exp\{-.16126\} = 13.37448(.8511)$$

$$= 11.3826$$

P.8.b. $n=12 \quad df = 12-3 = 9 \quad t_{.025,9} = 2.262$

$$13.37448 \pm \underbrace{2.262(0.2914)}_{.65915} \equiv (12.72, 14.03)$$

P.8.c. Crosses @ $\approx 6$ (2000)