ANSWER KEY

STA 6167 – Exam 2 – Fall 2014 – **PRINT** Name Key

Note: Conduct all individual tests at α=0.05 significance level, and all multiple comparisons at an experiment-wise error rate of $\alpha_E = 0.05$.

Q.1. The broiler chicken study had 60 replicates at each of 2 levels of factor A (Base: Sorghum or Corn) and 2 levels of Factor B (Methionine: Present or Absent). One response reported was the weight of the wing drumette.

Model: $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad i=1,2\ j=1,2\ k=1,\ldots,60 \quad \sum_{i=1}^{2}\alpha_i = \sum_{j=1}^{2}\beta_j = \sum_{i=1}^{2}(\alpha\beta)_{ij} = \sum_{j=1}^{2}(\alpha\beta)_{ij} = 0 \quad \varepsilon_{ijk} \sim N(0,\sigma^2)$

p.1.a. The following table gives the means (SDs) for each treatment:

Base\Meth	Absent	Present	Mean
Sorghum	46.4 (8.0)	34.8 (6.0)	40.6
Corn	38.8 (6.0)	41.6 (10.0)	40.2
Mean	42.6	38.2	40.4

BASE: $120\left[(40.6-40.4)^2+(40.2-40.4)^2\right] = 9.6$

Meth: $120\left[(42.6-40.4)^2+(38.2-40.4)^2\right] = 1161.6$

Complete the following ANOVA table: FIXED Effects

Source	df	SS	MS	F	F(.05)
Base	2-1=1	9.6	9.6	9.6/59 = 0.16	~3.888
Methionine	2-1=1	1161.6	1161.6	1161.6/59 = 19.69	~3.888
B*M	1(1) =1	3110.4	3110.4	3110.4/59 = 52.72	~3.888
Error	2(2)(60-1)=236	13924	59	#N/A	#N/A
Total	2(2)(60)-1 = 239	18205.6	#N/A	#N/A	#N/A

p.1.b. Test H_0: No Interaction between Base and Methionine

p.1.b.i. Test Stat: 52.72 p.1.b.ii. Reject H_0 if Test Stat is in the range ≥ 3.888 p.1.b.iii. P-value > or < .05?

p.1.c. Test H_0: No Base effect

p.1.c.i. Test Stat: 0.16 p.1.c.ii. Reject H_0 if Test Stat is in the range ≥ 3.888 p.1.c.iii. P-value > or < .05?

p.1.d. Test H_0: No Methionine effect

p.1.d.i. Test Stat: 19.69 p.1.d.ii. Reject H_0 if Test Stat is in the range ≥ 3.888 p.1.d.iii. P-value > or < .05?

Q.2. An experiment was conducted to compare 4 brands of antiperspirant in terms of percentage sweat reduction. A sample of 24 subjects was obtained, and each subject was measured using each antiperspirant. Model:

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \quad i = 1,\ldots,4 \; j = 1,\ldots,24 \quad \sum_{i=1}^{4} \alpha_i = 0 \quad \beta_j \sim N\left(0, \sigma_b^2\right) \quad \varepsilon_{ij} \sim N\left(0, \sigma^2\right)$$

p.2.a. The 4 antiperspirant brand **mean** y-values are given below. Compute the overall mean.

$\bar{y}_1 = 15.6 \quad \bar{y}_2 = 25.0 \quad \bar{y}_3 = 26.5 \quad \bar{y}_4 = 26.5 \quad \bar{y} =$ 23.4

p.2.b. Complete the following partial ANOVA table: RBD

ANOVA					
Source	df	SS	MS	F	F(0.05)
Subject	24-1=23	14183.5	616.7	#N/A	#N/A
Brand	4-1=3	1976.75	658.9	3.87	~2.736
Error	23(3)=69	11740.25	170.1	#N/A	#N/A
Total	24(4)-1=95	27900.5	#N/A	#N/A	#N/A

p.2.c. Test H_0: No differences among Brand Effects $\quad H_A$: Differences exist among brands

p.2.c.i. Test Stat: 3.87 p.2.c.ii. Reject H_0 if Test Stat is in the range ≥ 2.736 p.2.c.iii. P-value > or < .05?

p.2.d. Use Tukey's Honest Significant Difference method to determine which (if any) brand means are significantly different.

$q(.05, 4, 69) \approx 3.724 \qquad \sqrt{\frac{MSE}{b}} = \sqrt{\frac{170.1}{24}} = 2.662$

$W = q\sqrt{\frac{MSE}{b}} = 3.724(2.662) = 9.91$

$A_1 \quad \underline{A_2 \quad A_3 \quad A_4}$

Tukey's W = 9.91

p.2.e. Compute the Relative efficiency of the Randomized Block Design (relative to Completely Randomized Design). How many subjects would be needed per treatment (in CRD) to have the same standard errors of sample means as RBD.

$$RE = \frac{(b-1)MSBL + b(t-1)MSE}{(bt-1)\,MSE} = \frac{14183.5 + 72(170.1)}{95(170.1)} = 1.64$$

$1.64(24) = 39.25$

Relative Efficiency = 1.64 # of subjects per treatment in CRD ≈ 40

Q.3. An experiment was conducted to compare 5 treatments (Seed Rate) in a latin square design. A field was partitioned into 5 rows and 5 columns, such that each treatment appeared in each row once, and each column once. The response is grain yield.

Latin Square

level	rowmean	colmean	trtmean
1	54.15	52.43	47.13
2	56.30	54.30	51.72
3	52.29	54.44	55.73
4	52.58	55.30	59.17
5	57.31	56.16	58.88

ANOVA					
Source	df	SS	MS	F	F(0.05)
Seed Rate	4	522.74	130.69	27.94	3.259
Field Row	4	99.13	24.78	#N/A	#N/A
Field Column	4	38.60	9.65	#N/A	#N/A
Error	4(3)=12	56.14	4.68	#N/A	#N/A
Total	5(5)-1=24	716.61	#N/A	#N/A	#N/A

p.3.a. Complete the ANOVA table.

p.3.b. Test H_0: No differences among Seed Rate Effects H_A: Differences exist among Seed rates

p.3.b.i. Test Stat: 27.94   p.3.b.ii. Reject H_0 if Test Stat is in the range >3.259   p.3.c.iii. P-value > or < .05?

p.3.c. Use Bonferroni's method to determine which (if any) Seed Rates are significantly different.

$$t(\underbrace{.025}_{\alpha/c},\ \underbrace{10}_{c},\ \underbrace{12}_{df}) = 3.428 \qquad \sqrt{\frac{2MSE}{t}} = \sqrt{\frac{2(4.68)}{5}} = 1.368$$

$$B = 3.428(1.368) = 4.69$$

$S_1 \quad S_2 \quad S_3 \quad S_5 \quad S_4$

Bonferroni's B = 4.69

p.3.d. Compute the Relative efficiency of the Latin Square Design (relative to Completely Randomized Design).

$$RE = \frac{MSR + MSC + (t-1)MSE}{(t+1)MSE} = \frac{24.78 + 9.65 + 4(4.68)}{6(4.68)} = 1.89$$

Relative Efficiency = 1.89

Q.4. Based on the 2014 WNBA season, we have the point totals (Y) by game **Location** (Home/Away) for a **sample** of 10 **Players**. Each player played 17 home games and 17 away games. Consider the model:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk} \quad i = 1,\ldots,a \quad j = 1,\ldots,b \quad k = 1,\ldots,r \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_j \sim N\left(0,\sigma_b^2\right) \quad \alpha\beta_{ij} \sim N\left(0,\sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$$

Mixed Model

ANOVA					
Source	df	SS	MS	F	F(0.05)
Player	10-1=9	3879.30	431.03	$\frac{431.03}{35.96} = 11.98$	3.179
Home	2-1=1	1.30	1.30	$\frac{1.30}{35.96} = .036$	5.117
P*H	9(1)=9	323.67	35.96	$\frac{35.96}{49.34} = 0.73$	$\simeq$ 1.927
Error	10(2)(17-1)=320	15787.29	49.34	#N/A	#N/A
Total	10(2)(17)-1 = 339	19991.56	#N/A	#N/A	#N/A

$$E\{MSE\} = \sigma^2 \qquad E\{MSAB\} = \sigma^2 + r\sigma_{ab}^2 \quad E\{MSB\} = \sigma^2 + r\sigma_{ab}^2 + ar\sigma_b^2 \quad E\{MSA\} = \sigma^2 + r\sigma_{ab}^2 + \frac{br\sum_{i=1}^{a}\alpha_i^2}{a-1}$$

p.4.a. Complete the partial ANOVA table.

p.4.b. Test whether there is an interaction between Player and Location (Home). H_0: $\sigma_{ab}^2 = 0$

p.4.b.i. Test Stat: 0.73 p.4.b.ii. Reject H_0 if Test Stat is in the range ≥ 1.927 p.4.b.iii. P-value > or < .05?

p.4.c. Test whether there is Location (Home vs Away) Main Effect. H_0: $\alpha_1 = \alpha_2 = 0$

p.4.c.i. Test Stat: .036 p.4.c.ii. Reject H_0 if Test Stat is in the range ≥ 5.117 p.4.c.iii. P-value > or < .05?

p.4.d. Test whether there is Player Main Effect. H_0: $\sigma_b^2 = 0$

p.4.d.i. Test Stat: 11.99 p.4.d.ii. Reject H_0 if Test Stat is in the range > 3.179 p.4.d.iii. P-value > or < .05?

p.4.e. Give unbiased estimates of each of the variance components:

$$\hat{\sigma}^2_{ab} = \frac{MSAB - MSE}{r} = \frac{35.96 - 49.34}{17} = -0.79$$

$$\hat{\sigma}^2_b = \frac{MSB - MSAB}{ar} = \frac{431.03 - 35.96}{2(17)}$$

$\hat{\sigma}^2_{ab}$ = $\simeq 0$ $\hat{\sigma}^2_b$ = 11.62 $\hat{\sigma}^2$ = $MSE = 49.34$

Q.5. A study was conducted to compare total distance covered by soccer players over a 16 minute game on fields of various sizes. The field sizes were 30x20meters, 40x30, and 50x40. A sample of 8 skilled soccer players were selected and are treated as blocks for this analysis. The total distance covered by the 8 players on the 3 field sizes are given in the following table. Use Friedman's test to test whether true mean distance covered differs among the 3 field sizes.

Player	30x20	40x30	50x40
1	1141 1	1558 3	1493 2
2	1573 1	1963 2	2036 3
3	1802 1	2140 2	2218 3
4	1745 1	2142 3	2078 2
5	1663 1	2116 3	2036 2
6	1288 1	1748 3	1696 2
7	1705 1	2105 2	2167 3
8	1340 1	1755 3	1748 2

$T_1 = 8$ $T_2 = 21$ $T_3 = 19$

$b = 8, k = 3$

$$F_R = \frac{12}{bk(k+1)} \sum_i T_i^2 - 3b(k+1)$$

$$= \frac{12}{8(3)(4)} [8^2 + 21^2 + 19^2] - 3(8)(4)$$

$$= \frac{866}{8} - 96 = 108.25 - 96 = 12.25$$

$$\chi^2_{.05,3-1} = 5.991$$

Friedman's Test Statistic 12.25 Rejection Region: ≥ 5.991 P-value < or > .05

Q.6. For the following problems, identify the factor(s), state whether they are fixed or random, give the analysis of variance table, with sources of variation and degrees of freedom, symbolic F-ratios and critical values (just give degrees of freedom) for relevant significance tests for treatment factors.

p.6.a. An ergonomic study was conducted to compare 6 car seat designs in terms of an overall comfort index (Y). A sample of 12 subjects was selected, and each subject rated each car seat one time.

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Source	df	F	Fcrit
Seat (Fixed)	6-1=5	MSSeat/MSE	F(.05, 5, 55)
Subject (Random)	12-1=11	—	
Error	5(11)=55	—	
Total	6(12)-1=71		

p.6.b. A food preference study was interested in the main effects and interactions among two factors on subjects' ratings of attractiveness of a plate of food. The factors under study were plate color (monochrome, color) and balance of food placement on the plate (symmetric (balanced), asymmetric (unbalanced)). A sample of 68 subjects were selected, and randomized such that 17 received each combination of color and balance. Each person only rated one plate.

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Source	df	F	Fcrit
Color (Fixed)	1	MSC/MSE	F(.05, 1, 64)
Balance (Fixed)	1	MSB/MSE	"
C*B "	1	MSCB/MSE	"
Error	2(2)(17-1)=64		
Total	67		

p.6.c. A study was conducted to compare 6 models of bread machines on quality of baked bread. There were 6 varieties of bread, and 6 chefs, and each variety was made by each machine once, and each chef used each machine once. The response was an overall quality rating based by a panel of judges (which was combined to a single rating).

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Source	df	F	Fcrit
Model (Fixed)	5	MSM/MSE	F(.05, 5, 20)
Varieties (F?)	5	MSV/MSE	—
Chefs (R)	5	MSC/MSE	—
Error	20		
Total	36-1=35		

p.6.d. A study was conducted to measure the reliability of collegiate gymnastics judges, and variation in gymnast skills. A sample of 8 judges was selected, and a sample of 4 gymnasts was selected. Each gymnast was filmed on 3 occasions, and each judge rated the 3 videos.

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Source	df	F	FCrit
Judge (R)	7	MSJ/MSJG	F(.05; 7, 21)
Gymnast (R)	3	MSG/MSJG	F(.05; 3, 21)
J*G (R)	21	MSJG/MSE	F(.05, 21, 64)
Error	64		
Total	95		

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