Experimental Design Problems

Part A: 1-Way ANOVA (Completelely Randomized Design)

QA.1. An experiment was conducted as a Completely Randomized Design (1-Way ANOVA) to compare t = 4 methods of packaging steaks, in terms of the amount of bacteria measured after 9 days of storage. There were n_i = 3 replicates per treatment. The treatment means and sums of squares were:

$$\overline{Y}_{1\bullet} = 7.48 \quad \overline{Y}_{2\bullet} = 5.50 \quad \overline{Y}_{3\bullet} = 7.26 \quad \overline{Y}_{4\bullet} = 3.36 \quad \sum_{i=1}^{4} n_i \left(\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}\right)^2 = 32.87 \qquad \sum_{i=1}^{4} \left(n_i - 1\right) s_i^2 = 0.93$$

p.1.a. Conduct the F-test for testing H_0: $\mu_1=\mu_2=\mu_3=\mu_4$ $(\alpha_1=\alpha_2=\alpha_3=\alpha_4=0)$

Test Statistic: ______________________ Rejection Region: __________________ Reject H_0? **Yes** or **No**

p.1.b. Compute Tukey's Honest Significant Difference for simultaneously comparing all pairs of packages, with a family-wise error rate of 0.05. Identify significant differences among all pairs of means.

Trt4 Trt2 Trt3 Trt1

p.1.c. Compute Bonferroni's Minimum Significant Difference for simultaneously comparing all pairs of packages, with a family-wise error rate of 0.05

Trt4 Trt2 Trt3 Trt1

QA.2. A 1-Way ANOVA is conducted to compare the effects of 4 methods of preparing steel. Five replicates of each method are obtained, and the breaking strength is measured. Suppose that the between treatment sum of squares is 1200, and the within treatment sum of squares is 2400. Give the test statistic for testing whether the true mean breaking strengths differ among the 4 methods. Give the minimum significant difference for pairs of methods, based on Bonferroni's method with an experimentwise error rate of 0.05.

QA.3. For a 1-Way ANOVA, based on 3 treatments, and 30 subjects per treatment, give the Treatment and Error Degrees of Freedom:

Df_{Trt} = ______________________ df_{Err} = ______________________________

QA.4. A Completely Randomized Design is conducted to compare 5 varieties of fertilizer on plant yield. Each variety is randomly assigned to 7 plots of land (each plot only receives one variety).

DF(Treatments) ______________ DF(Error)_______________ DF(Total) _______________

QA.5. When using Bonferroni's method of adjustment for simultaneous Confidence Intervals, as the number of intervals increases, the width of the individual confidence intervals will decrease. ____________

QA.6. An experiment is run to compare t=4 meat packaging conditions. There were n_i=3 replicates per treatment in the Completely Randomized Design. The response was a measure of bacteria count (high values are bad). The treatment means and standard deviations are given below for the model: $Y_{ij} = \mu_i + \varepsilon_{ij}$.

p.6.a. Compute the Treatment and Error Sum of Squares:

Treatment	Mean	SD	SS(Treatments)	SS(Error)
1	7.48	0.44		
2	5.50	0.27		
3	7.26	0.19		
4	3.36	0.40		
Overall	5.90	Total		

p.6.b. Compute the F-Statistic for testing H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

p.6.c. Conclude packaging condition true means not all equal if test statistic falls in the range ___________

p.6.d. Based on your test, the P-value will be **Larger / Smaller** than 0.05

QA.7. An experiment is conducted as a Completely Randomized Design with t = 5 treatments and n_i = 5 replicates per treatment. The error sum of squares is SSE = 250. Compute Bonferroni's minimum significant difference for all pairwise comparisons with experiment-wise error rate of α_E = 0.05.

B_{ij} =

QA.8. A Completely Randomized Design is conducted with 3 treatments, and 8 replicates per treatment (independent samples). Once the measurements have been ranked from smallest to largest, adjusting for ties, you compute the rank sums to be: T_1=110, T_2 = 100, T_3 = 90. You conduct the Kruskal-Wallis test, α = 0.05:

p.8.a. Test Statistic:

p.8.b. Conclude treatment means (medians) are significantly different if Test Stat falls in range: ____________

QA.9. An experiment was conducted as a Completely Randomized Design (1-Way ANOVA) to compare t = 4 methods of packaging steaks, in terms of the amount of bacteria measured after 9 days of storage. There were n_i = 3 replicates per treatment. The treatment means and sums of squares were:

$$\overline{Y}_{1\bullet} = 7.48 \quad \overline{Y}_{2\bullet} = 5.50 \quad \overline{Y}_{3\bullet} = 7.26 \quad \overline{Y}_{4\bullet} = 3.36 \quad \sum_{i=1}^{4} n_i\left(\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}\right)^2 = 32.87 \qquad \sum_{i=1}^{4}(n_i - 1)s_i^2 = 0.93$$

p.9.a. Conduct the F-test for testing H_0: $\mu_1=\mu_2=\mu_3=\mu_4$ $(\alpha_1=\alpha_2=\alpha_3=\alpha_4=0)$

Test Statistic: ___________________________ Rejection Region: ____________________ Reject H_0? **Yes** or **No**

p.9.b. Compute Tukey's Honest Significant Difference for simultaneously comparing all pairs of packages, with a family-wise error rate of 0.05. Identify significant differences among all pairs of means.

Trt4 Trt2 Trt3 Trt1

p.9.c. Compute Bonferroni's Minimum Significant Difference for simultaneously comparing all pairs of packages, with a family-wise error rate of 0.05

Trt4 Trt2 Trt3 Trt1

QA.10. . Researchers studied nest humidity levels among 54 species of birds. The nests were classified as (1=Cup, 2=Scrape, 3=Covered). The following table gives the sample sizes, means, and standard deviations among the 3 nest types.

p.10.a. Test whether the population mean nest humidity levels differ among the 3 nest types (first obtain the relevant sums of squares and degrees of freedom). H_0: $\mu_1 = \mu_2 = \mu_3$

NestType	n	mean	SD
1	39	20.84	4.76
2	9	19.98	4.19
3	6	31.74	3.20
Overall	54	21.91	#N/A

ANOVA						
Source	df	SS	MS	F	F(.05)	Reject H0?
Nest Type						
Error						
Total						

p.10.b. Use Bonferroni's method to obtain the minimum significant difference between each pair of means.

Cup vs Scrape: __________________ Cup vs Covered: __________________ Scrape vs Covered: __________________

QA.11. A study compared infarct volumes of mice exposed to one of 3 treatments in a completely randomized design (1=vehicle control, 2=compound X, 3=compound Y). There were a few extreme outliers, so the Kruskal-Wallis test will be applied. The following table gives the sample sizes and rank sums for the 3 treatments. Conduct the Kruskal-Wallis test to determine whether the population medians differ among the 3 treatments.

Trt	N	RankSum
1	13	326
2	15	375
3	14	202

QA.12. A published report, based on a balanced 1-Way ANOVA reports means (SDs) for the three treatments as:

Trt 1: 70 (8) Trt 2: 75 (6) Trt 3: 80 (10)

Unfortunately, the authors fail to give the sample sizes.

p.12.a. Complete the following table, given arbitrary levels of the number of replicates per treatment:

r	SSTrt	SSErr	MSTrt	MSErr	F_obs	F(.05)
2						
6						
10						

p.12.b. The smallest r, so that these means are significantly different is:

i) $r \le 2$ ii) $2 < r \le 6$ iii) $6 < r \le 10$ iv) $r > 10$

QA.13. An experiment is conducted as a Completely Randomized Design to compare the durability of 5 green fabric dyes, with respect to washing. A sample of 30 plain white t-shirts was obtained, and randomized so that 6 received each dye (with each shirt receiving exactly one dye). A measure of the color brightness of the shirts after 10 wash/dry cycles is obtained (with higher scores representing brighter color). The error sum of squares is reported to be $SSE = 2000$. The mean scores for the 5 dyes are: $\overline{Y}_{1\bullet} = 30$ $\overline{Y}_{2\bullet} = 25$ $\overline{Y}_{3\bullet} = 40$ $\overline{Y}_{4\bullet} = 35$ $\overline{Y}_{5\bullet} = 20$

p.13.a. Compute Tukey's HSD, and determine which (if any) pairs of means are significantly different with an experiment-wise (overall) error rate of $\alpha_E = 0.05$.

Tukey's HSD: ________________

p.13.b. Compute the Bonferroni MSD, and determine which (if any) pairs of means are significantly different with an experiment-wise (overall) error rate of $\alpha_E = 0.05$.

Bonferroni's MSD: ________________

QA.14. A study compared efficiency levels (based on a complex algorithm) among three types of Trade Shows in Spain. The authors classified Trade Shows as being one of 3 sectors (Consumer Goods, Investment Goods, and Services). The Trade Shows were ranked based on their efficiencies (1=Lowest). Based on the sample sizes and the Rank Sums from the following table, conduct the Kruskal-Wallis Test (Note: Total is NOT a "treatment," it is just useful in computations).

Sector	n	RankSum
Consumer	21	466
Investment	16	312
Services	8	257
Total	45	1035

Test Statistic: ________________________________ Rejection Region: __________________________

QA.15. A study compared antioxidant activity of t = 8 brands of craft beer in a 1-Way ANOVA. One response reported was DPPH radical scavenging activity. Each brand was had n = 3 replicates measured.

Brand	Mean	SD
L	794.9	27.5
P	376	32.4
W	706.2	30.9
B9	586.5	17.7
N	864.6	42.4
R	670.6	19.9
T	1310.3	19
E	679.2	25

$$\sum_{i=1}^{t}\left(\overline{y}_{i\bullet}-\overline{y}_{\bullet\bullet}\right)^2 = 508911.8 \qquad \sum_{i=1}^{t} s_i^2 = 6253.88$$

p.15.a. Complete the following Analysis of Variance table used to test H_0: $\mu_1 = ... = \mu_8$

Source	df	SS	MS	F_obs	F(.05)
Brand					
Error					
Total					

p.15.b. Do we reject the null hypothesis, and conclude the population means differ among the brands? **Yes** or **No**

p.15.c. Compute Tukey's minimum significant difference and determine which brands are significantly different.

P B9 R E W L N T

QA.16. A study classified a sample of French Ski resorts into 3 classifications (large, medium, and small) based on their volume of business. The researchers obtained a measure of each resort's Luenberger Productivity Index (LPI) was obtained. The authors conducted a Kruskal-Wallis test to test whether population median LPI scores differ by resort size group. The numbers and rank sums for each resort size group are given below.

Size	n	RankSum
Large	16	428
Medium	31	932
Small	17	720

Test Statistic: ______________________ Rejection Region __________________ P-value is **> 0.05** or **< 0.05**

QA.17. An experiment was conducted to determine the effect of g = 3 different food portion/container sizes on food intake in a Completely Randomized Design. There were a total of N = 90 subjects who were randomized so that 30 received each condition (each subject was observed in one of the 3 conditions). The conditions were: 1= medium portion/small container, 2 = medium portion/large container, 3 = large proportion/large continer. The response was food intake (Y, in grams) that the subject consumed while watching a television show. The model and summary statistics are given below.

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} = \mu_i + \varepsilon_{ij} \quad n_1 = 30, \overline{y}_{1\bullet} = 30, s_1 = 30 \quad n_2 = 30, \overline{y}_{2\bullet} = 69, s_2 = 44 \quad n_3 = 30, \overline{y}_{3\bullet} = 60, s_3 = 45$$

p.17.a. Compute the Between Treatment Sum of Squares (SST) and Within Treatment Sum of Squares (SSE).

SST = ____________________________________ SSE = ____________________________________

p.17.b. Test H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$

Test Statistic: ________________________ Rejection Region ______________________ P-value > or < 0.05

p.17.c. Use Tukey's method to compare all pairs of treatments.

Tukey's W = ______________________ Trt1 Trt3 Trt2

QA.18. Consider the following 3 scenarios for a (Fixed Effects) Completely Randomized Design.

$$y_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,2,3;\ j = 1,...,n \quad \varepsilon_{ij} \sim N\left(0, \sigma^2\right)$$

$a)\ \mu_1 = 80, \mu_2 = 100, \mu_3 = 120, \sigma = 20, n = 5 \quad b)\ \mu_1 = 90, \mu_2 = 100, \mu_3 = 110, \sigma = 10, n = 3 \quad c)\ \mu_1 = 95, \mu_2 = 100, \mu_3 = 105, \sigma = 5, n = 7$

Rank the from smallest to largest in terms of $\dfrac{E\{MST\}}{E\{MSE\}}$

Smallest: _________________________ Middle: ______________________ Largest: _____________________

QA.19. A delivery company is considering buying one of 3 drones for deliveries. They fly each drone 12 times, measuring the distance from the landing point to the target. Due to the skewed distribution of the distances, they use the non-parametric Kruskal-Wallis procedure to test for differences among the drones' true medians. The rank sums are 200, 218, and 248 for the 3 drones. Test H_0: $M_1 = M_2 = M_3$.

Test Statistic: ________________________ Rejection Region: _____________________ P-value < or > .05

QA.20. Unless the number of treatments is 2, Tukey's HSD (W) will always be smaller that Bonferroni's MSD (B) for a given set of data. **True / False**

QA.21. An experiment was conducted to compare the effects of 4 fragrances on various office workers characteristics. There were 50 subjects per treatment (fragrance). One response measured was the workers' concentration levels. The experiment was conducted as a Completely Randomized Design.

$$y_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1, \ldots, 4;\ j = 1, \ldots, 50 \quad \varepsilon_{ij} \sim N\left(0, \sigma^2\right)$$

Trt (i)	n_i	ybar_i	s_i
Control	50	103.3	9.0
Citrus	50	105.2	8.8
Grapefruit	50	103.7	9.4
Rose	50	104.2	9.3

p.21.a. Compute the Between treatment sum of squares (SST) and its degrees of freedom (df_T)

SST = __________________________________ df_T = ________________________________

p.21.b. Compute the Within treatment sum of squares (SSE) and its degrees of freedom (df_E)

SSE = __________________________________ df_E = ________________________________

p.21.c.Test whether there is evidence of treatment effects. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \quad H_A:$ Not all μ_i are equal

Test Statistic: ________________________________ Rejection Region: ____________________________

QA.22. A study compared three methods of making espresso: Bar Machine (BM, i=1), Hyper Espresso Method (HIP, i=2), and I-Espressos System (IT, i=3). There were n=9 replicates per method (N=27). The following summary statistics were computed for the response Foam Index (%). $\overline{y}_{1\bullet} = 32.4 \quad \overline{y}_{2\bullet} = 61.3 \quad \overline{y}_{3\bullet} = 39.7 \quad MSE = 71.53$

p.22.a. Use Tukey's method to compare all pairs of methods. BM IT HIP

p.22.b. Compute the minimum significant difference for all pairs of means based on the Bonferroni method.

QA.23. An experiment is conducted to compare t = 3 diets for parrots. The diets are described as follow.

Diet 1: Corn Diet 2: Sunflower seeds Diet 3: Corn + Sunflower seeds

Give two orthogonal contrasts of interest among these 3 treatments (diets).

$l_1 =$ ______ $\mu_1 +$ ______ $\mu_2 +$ ______ μ_3 $l_2 =$ ______ $\mu_1 +$ ______ $\mu_2 +$ ______ μ_3

QA.24. A Kruskal-Wallis test is conducted to compare 4 treatments, with n = 3 replicates per treatment. The total of the 3 rank sums will be what?

QA.25. A study involved men's rating of attractiveness of women. A photograph of a woman was photoshopped so that the woman's t-shirt was one of 4 colors: White, Red, Blue, or Green. There were a total of N = 120 subjects, with subjects being randomly assigned to colors in a Completely Randomized (n = 30 subjects per Treatment). The summary statistics are given below.

p.25.a. Complete the following ANOVA table. Is there evidence to conclude that color effects attractiveness ratings? **Yes / No**

Color	n	Mean
White	30	5.12
Red	30	5.95
Blue	30	5.07
Green	30	4.93
Overall	120	5.27

ANOVA					
Source	df	SS	MS	F	F(.05)
Trts (Color)					
Error		1.50		#N/A	#N/A
Total			#N/A	#N/A	#N/A

p.25.b. Give a contrast comparing the Red Shirt mean with mean of the remaining 4 colors.

Contrast Coefficients: $l_{RvWBG} =$ ________ μ_R ________ μ_W ________ μ_B ________ μ_G

p.25.c. Give the estimated contrast, its standard error, and the t-test for testing $H_0: l = 0 \quad H_A: l \neq 0$

$\hat{l} =$ __________ $\hat{SE}\{\hat{l}\} =$ ____________ Test Stat: ____________ Rejection Region: ____________

QA.26. A 1-Way ANOVA is fit with t = 5 treatments and n_i = 4 replicates per treatment. The Mean Square Error is 300. Compute Tukey's HSD and Bonferroni's Minimum Significant Difference for comparisons among all pairs of treatment means.

Tukey HSD = ________________________ Bonferroni MSD = ________________________

Part B: Randomized Block Design

QB.1. A study is conducted to compare 4 varieties of cat food on weight gain in kittens. 4 Kittens are selected at random from each of 12 litters with 4 or more kittens. Of the 4 kittens selected from each litter, one is assigned to variety A, one to B, one to C, and one to D (at random). Weight change at 16 weeks is obtained for each kitten. Complete the following ANOVA table and use Bonferroni's method to compare all pairs of variety (population) mean weight change.

Source	df	SS	MS	F	Critical F
Variety		600			
Litter					
Error		990			
Total	47	3000			

Variety Means: A: 21 B: 28 C: 22 D: 27

H_0: No Variety Differences

H_A: Variety Differences Exist

Test Statistic________________________ Rejection Region _________________

Critical t-value for Bonferroni's Method: __________________

Standard error of Difference between 2 Variety Means:

B_{ij}

Comparison	Confidence Interval	Conclude
A vs B		
A vs C		
A vs D		
B vs C		
B vs D		
C vs D		

QB.2. An experiment is conducted to compare the effects of 4 types of fertilizer on the growth of a particular plant. A sample of 8 locations (blocks) in a large yard are selected and 4 plants are planted at each location. At each location, the 4 plants are randomly assigned such that one receives fertilizer A, one receives fertilizer B, one receives fertilizer C, and one receives fertilizer D. Complete the following Analysis of Variance Table.

Source	df	SS	MS	F	F(.05)
Fertilizer		395.8			
Location		329.3			
Error					
Total		745.3			

The means for the fertilizers are: A=27.1, B=29.0, C=33.7, D=35.9. Use Bonferroni's method to make

pairwise comparisons among all pairs of varieties with an experimentwise error rate of 0.05

QB.3. A Randomized Block Design is conducted to compare the bioavailabilities of 4 formulations of a test drug. A sample of 8 subjects is obtained, and each subject receives each formulation once (in random order with adequate time between administrations of drug).

DF(Treatments) ______________ DF(Block) ______________ DF(Error) ________ DF(Total) ___________

QB.4. A randomized block design is conducted to compare t=3 treatments in b=4 blocks. Your advisor gives you the following table of data form the experiment (she was nice enough to compute treatment, block, and overall means for you), where: $TSS = \sum\left(Y - \overline{Y}\right)^2$

Blk\Trt	1	2	3	BlkMean
1	20	22	24	22
2	10	13	16	13
3	28	25	34	29
4	10	12	14	12
TrtMean	17	18	22	19
TSS				
658				

p.4.a. Complete the following ANOVA table:

Source	df	SS	MS	F_obs	F(.05)	Reject H0: No Effect?
Treatments						
Blocks						
Error						
Total						

p.4.b. Compute the Relative Efficiency of having used a Randomized Block instead of a Completely Randomized Design

RE(RB,CR) = ________________________________

p.4.c.. Compute Tukey's minimum significant difference for comparing all pairs of container types:

Tukey's W = _______________________________

p.4.d. Give results graphically using lines to connect Trt Means that are not significantly different: T1 T2 T3

QB.5. Jack and Jill wish to compare the effects of 3 internet pop-up advertisements (ad1, ad2, ad3) on click throughs. Their response is the fraction of all website visitors who are exposed to the pop-up who click through (analyzed as click-throughs per 1000 exposures). They identify a large number of potential websites that are comparable with respect to:complexity and traffic.

p.5.a. Jack conducts a Completely Randomized Design, sampling 60 websites and randomly assigns them so that 20 receive ad1, 20 receive ad2, and 20 receive ad3. He obtains the following results:

$\bar{y}_1 = 25 \quad \bar{y}_2 = 35 \quad \bar{y}_3 = 45 \quad SS_{Trts} = 4000 \quad SS_{ERR} = 68400$

Give Jack's test for testing H_0: No advertisement effects:

p.5.a.i. Test Statistic:

p.5.a.ii. Reject H_0 if Jack's test statistic falls in the range ____

p.5.b. Jill conducts a Randomized Block Design, sampling 12 websites (blocks) and assigns each ad to each website (randomizing the order of the ads to the websites). She obtains the following results:

$\bar{y}_1 = 25 \quad \bar{y}_2 = 35 \quad \bar{y}_3 = 45 \quad SS_{Trts} = 2400 \quad SS_{Blocks} = 36000 \quad SS_{ERR} = 11000$

Give Jill's test for testing H_0: No advertisement effects:

p.5.b.i. Test Statistic:

p.5.b.ii. Reject H_0 if Jill's test statistic falls in the range ____

p.5.c. Obtain Jack's and Jill's minimum significant differences based on Bonferroni's method for comparing all pairs of advertisement effects

Jack's B_{ij} = _______________________________ Jill's B_{ij} = _________________________________

QB.6. A study was conducted to compare 3 speed reduction marking (SRM) conditions on drivers' acceleration in an automobile simulator. A sample of 15 drivers was selected, and each driver drove the simulator under the 3 SRM conditions (**N**o SRM, **L**ongitudinal SRM, **T**raverse SRM).

p.6.a The following tables give the treatment (and overall) means, and the partial ANOVA table. Complete the ANOVA table and test H_0: $\mu_N = \mu_L = \mu_T$.

Treatment	Mean
No SRM	0.1613
Longitudinal SRM	0.1260
Traverse SRM	-0.0320
Overall	0.0851

ANOVA						
Source	df	SS	MS	F	F(.05)	Reject H0?
Trts						
Drivers		0.9865				
Error						
Total		2.3309				

p.6.b Use Tukey's method to obtain simultaneous 95% confidence intervals for comparing all pairs of treatment means.

QB.7. An experiment was conducted to determine whether initiation times for cricket players are effected by ball color and illumination level. There were 6 treatments (combinations of ball color (Red/White) and Illumination level (571/1143/1714)). There were 5 subjects (blocks) who were observed under each condition. The mean initiation time for each player under each condition (treatment) is given in the following table. Use Friedman's test to determine whether there are any significant differences among the treatment medians.

Subject	Trt1	Trt2	Trt3	Trt4	Trt5	Trt6
1	125	121	131	124	110	120
2	178	183	156	175	169	168
3	178	167	159	157	167	166
4	147	126	147	146	150	136
5	144	153	162	171	157	163

Test Statistic: ______________________________ Rejection Region: ______________________________

QB.8. An experiment was conducted to compare 4 brands of antiperspirant in terms of percentage sweat reduction. A sample of 24 subjects was obtained, and each subject was measured using each antiperspirant. Model:

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \quad i = 1,...,4 \; j = 1,...,24 \qquad \sum_{i=1}^{4} \alpha_i = 0 \quad \beta_j \sim N\left(0, \sigma_b^2\right) \quad \varepsilon_{ij} \sim N\left(0, \sigma^2\right)$$

p.8.a. The 4 antiperspirant brand **mean** y-values are given below. Compute the overall mean.

$$\overline{y}_1 = 15.6 \quad \overline{y}_2 = 25.0 \quad \overline{y}_3 = 26.5 \quad \overline{y}_4 = 26.5 \quad \overline{y} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

p.8.b. Complete the following partial ANOVA table:

ANOVA					
Source	df	SS	MS	F	F(0.05)
Subject		14183.5		#N/A	#N/A
Brand		1976.75			
Error		11740.25		#N/A	#N/A
Total		27900.5	#N/A	#N/A	#N/A

p.8.c. Test H_0: No differences among Brand Effects H_A: Differences exist among brands

p.8.c.i. Test Stat: _______ p.2.c.ii. Reject H_0 if Test Stat is in the range _________ p.2.c.iii. P-value > **or** < .05?

p.8.d. Use Tukey's Honest Significant Difference method to determine which (if any) brand means are significantly different.

Tukey's W = ________________________________

p.8.e. Compute the Relative efficiency of the Randomized Block Design (relative to Completely Randomized Design). How many subjects would be needed per treatment (in CRD) to have the same standard errors of sample means as RBD.

Relative Efficiency = __________________________________ # of subjects per treatment in CRD _______________

QB.9. A study was conducted to compare total distance covered by soccer players over a 16 minute game on fields of various sizes. The field sizes were 30x20meters, 40x30, and 50x40. A sample of 8 skilled soccer players were selected and are treated as blocks for this analysis. The total distance covered by the 8 players on the 3 field sizes are given in the following table. Use Friedman's test to test whether true mean distance covered differs among the 3 field sizes.

Player	30x20	40x30	50x40
1	1141	1558	1493
2	1573	1963	2036
3	1802	2140	2218
4	1745	2142	2078
5	1663	2116	2036
6	1288	1748	1696
7	1705	2105	2167
8	1340	1755	1748

Friedman's Test Statistic _______________ Rejection Region: ______________________ P-value < or > .05

QB.10. A study compared t = 4 warm-up protocols in terms of vertical jump ability in dancers. There were b = 10 dancers, each dancer was measured under each warm-up protocol and the experiment is a Randomized Block Design with dancers as blocks.

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} = \mu_i + \beta_j + \varepsilon_{ij}$$

The treatments and their means are: Static Stretch: 38.0 Dynamic Stretch: 41.4 Static&Dynamic Stretch: 41.0 Control: 37.8

p.10.a. Complete the following ANOVA table.

Source	df	SS	MS	F_obs	F(.05)
Treatment (Warm-up)					
Block (Dancer)		850.6			
Error		42.0			
Total					

p.10.b. Do you reject $H_0 : \alpha_1 = ... = \alpha_4 = 0 \quad (\mu_1 = ... = \mu_4)$? **Yes** / **No**

p.10.c. Compute the Relative Efficiency of the RCB to the Completely Randomized Design. How many subjects would be needed per treatment to have the same standard error of a treatment (warm-up protocol) in a CRD?

Relative Efficiency _____________________ # of Subjects per treatment ___________________________

p.10.d. Compute Bonferroni's minimum significant difference and determine which treatments are significantly different.

Control Static Static&Dynamic Dynamic

QB.11. An experiment was conducted comparing various treatments (involving various hydrocolloids and amounts of wheat flower) with the goal of reducing oil content in a food product. The experiment was conducted in separate replicates (blocks). One response measured was Oil Content of the sample. The partial ANOVA table is given below.

Source	df	SS	MS	F	F(.05)
Treatments	12	261.146			
Blocks	2	0.523		#N/A	#N/A
Error		0.689		#N/A	#N/A
Total		262.358	#N/A	#N/A	#N/A

p.11.a. Complete the table. Is the P-value for testing H_0: No Treatment Effect > 0.05 or < 0.05

p.11.b. Give the number of Treatments and number of Blocks in the experiment. # Trts = _______ # Blks = ________

p.11.c. What is the estimated standard error of the difference between any 2 treatment means? $SE\left\{\overline{Y}_{i\bullet} - \overline{Y}_{j\bullet}\right\}$

p.11.d. Suppose we wish to use Scheffe's method to compare all pairs of treatment means. What would be the minimum significant difference?

QB.12. An experiment was conducted as a Randomized Block Design with 3 treatments (Weight Belts: None (Control), Air Belt, and Comp Vest Belt) in 12 blocks (Subjects). The response was the maximum acceptable work load. The mean squares for Treatments (Belts), Blocks (Subjects), and Error (Trt/Block Interaction) and the Belt means are given below.

Treatments: MST = 131.3 Blocks: MSB = 3628.9 Error: MSE = 38.2 $\overline{Y}_{1\bullet} = 34.45$ $\overline{Y}_{2\bullet} = 38.84$ $\overline{Y}_{3\bullet} = 40.93$

p.12.a. Use Tukey's method to compare the all pairs of belt means.

Tukey's HSD: ___________________________ No Belt Air Belt Comp Vest Belt

p.12.b. Compute the Relative Efficiency of the Randomized Block Design to the Completely Randomized Design.

RE = __________________________________

p.12.c. How many subjects would be needed per treatment in a Completely Randomized Design to have the same precision in terms of the difference between mean that was obtained in this experiment? How many total?

Subjects per Treatment _________________________ Total Subjects __________________________________

Part C: Latin Square Design

QC.1. An experiment was conducted to compare 5 treatments (Seed Rate) in a latin square design. A field was partitioned into 5 rows and 5 columns, such that each treatment appeared in each row once, and each column once. The response is grain yield.

level	rowmean	colmean	trtmean
1	54.15	52.43	47.13
2	56.30	54.30	51.72
3	52.29	54.44	55.73
4	52.58	55.30	59.17
5	57.31	56.16	58.88

ANOVA					
Source	df	SS	MS	F	F(0.05)
Seed Rate		522.74			
Field Row		99.13		#N/A	#N/A
Field Column		38.60		#N/A	#N/A
Error				#N/A	#N/A
Total		716.61	#N/A	#N/A	#N/A

p.1.a. Complete the ANOVA table.

p.1.b. Test H_0: No differences among Seed Rate Effects H_A: Differences exist among Seed rates

p.1.b.i. Test Stat: _______   p.3.b.ii. Reject H_0 if Test Stat is in the range _________   p.3.c.iii. P-value > **or** < .05?

p.1.c. Use Bonferroni's method to determine which (if any) Seed Rates are significantly different.

Bonferroni's B = _________________________________

p.1.d. Compute the Relative efficiency of the Latin Square Design (relative to Completely Randomized Design).

Relative Efficiency = ___________________________________

QC.2. A latin square design is conducted comparing sales of juice in 5 **container types** (Treatment factor). The experiment is conducted in 5 **stores** (row blocking factor), over 5 **weeks** (column blocking factor) in a manner such that each container is sold in each store once, and each week once. Results of sales are given below:

Container Means: C1: 80   C2: 100   C3: 90   C4: 60   C5: 85 SS_{Row} = 1000 SS_{Column} = 400 SS_{Error} = 240 p.2.a. Compute the Relative Efficiency of having used a Latin Square instead of a Completely Randomized Design

RE(LS,CR) = ________________________________

p.2.b. Compute Bonferroni's minimum significant difference for comparing all pairs of container types:

Bonferroni's B = ________________________________

p.2.c. Give results graphically using lines to connect Containers that are not significantly different:

C4 C1 C5 C3 C2

QC3. Researchers conducting a Latin Square Design with t=5 treatments, row blocks, and column blocks report a relative efficiency (relative to completely randomized design) of 3. How many replicates per treatment would they need if they conducted this experiment as a completely randomized design to have equivalently precise standard errors of sample means as they obtained from the latin square?

QC.4. A study was conducted as a latin square design to compare countries of origin (Treatments: 1=Australia, 2=Chile, 3=France, 4=New Zealand) on ratings of quality of wine. There were 120 raters (30 per Week (Row Factor)) and 4 Bottle Labels (Column Factor)). The response for each "cell" in the design was the sum of the quality ratings of the 30 subjects for that week/label /country.

p.4.a Complete the following ANOVA table (hint: there are 16 total measurements).

ANOVA						
Source	df	SS	MS	F	F(.05)	Reject H0?
Week (Row)		729.2				
Label (Column)		414.7				
Country (Trt)		1937.7				
Error		397.9				
Total		3479.4				

p.4.b The sample means for the 4 countries are: Australia=89.5, Chile=116.5, France=94.25, New Zealand=90.5. Use Tukey's method to compare all pairs of country means.

QC.5. A latin square design was used to test for treatment effects among 5 mixes of concrete in terms of tensile strenth. There were 5 molds, and 5 workers who made and poured the concrete molds. The design is shown below.

	Worker1	Worker2	Worker3	Worker4	Worker5
Mold1	Mix1	Mix2	Mix3	Mix4	Mix5
Mold2	Mix2	Mix3	Mix4	Mix5	Mix1
Mold3	Mix3	Mix4	Mix5	Mix1	Mix2
Mold4	Mix4	Mix5	Mix1	Mix2	Mix3
Mold5	Mix5	Mix1	Mix2	Mix3	Mix4

p.5.a. The mix means are: 70, 65, 50, 80, and 75 for Mixes 1-5, respectively. The error sum of squares is $SSE = 600$. Use Bonferroni's method to compare all pairs of Mix means.

Bonferoni's MSD: ____________________ Mix3 Mix2 Mix1 Mix5 Mix4

p.5.b. The sums of squares for molds and workers are $SSR = 1000$ and $SSC = 400$, respectively. Compute the Relative Efficiency of the Latin Square design, relative to the Completely Randomized Design.

RE(LS,CR) = ____

QC.6. A researcher is using a latin square design to compare 4 brands of car tires in terms of miles driven before reaching a given wear level. For one blocking factor they use tire position (Driver Front, Passenger Front, Driver Rear, Passenger Rear). They choose to use 12 cars as the other blocking factor. Note that each brand will be on each car once, and on each tire position 3 times. What will be the error degrees of freedom?

QC.7. A Latin Square Design is used for an experiment with 3 treatments. The column (blocking) factor has 3 levels and the row blocking factor has 12 levels (thus, multiple squares have been formed). Give the critical F-value for testing for treatment effects.

$F_{.95;df1,df2}$ = _________________________

QC.8. A marketing experiment was conducted as a latin square with t = 6 treatments (Shelf Space, with levels 2,4,6,8,10,12 feet) conducted in t = 6 Rows (Stores) over t = 6 Columns (Weeks). Each shelf space was run in each store one time and during each week one time. The response was number of packages of baking soda sold during the week in the store.

p.8.a. Complete the partial ANOVA table is given below. Is H_0: No Difference in Shelp Space Effects Rejected? **Yes / No**

ANOVA					
Source	df	SS	MS	F	F(.05)
Store		8287		#N/A	#N/A
Week		982		#N/A	#N/A
ShelfSpace		395			
Error		1488		#N/A	#N/A
Total		11151	#N/A	#N/A	#N/A

p.8.b. Compute the minimum significant difference for comparing Shelf Space Means, based on Bonferroni's method. The smallest and largest means were 40.5 for 2 feet, and 51.2 for 10 feet. Are they significantly different?

Minimum Significant Difference ______________________________ 2 and 10 feet significantly different? **Yes / No**

Part D: 2-Factor (and Higher) Crossed ANOVA

QD.1. Based on the 2014 WNBA season, we have the point totals (Y) by game **Location** (Home/Away) for a **sample** of 10 **Players**. Each player played 17 home games and 17 away games. Consider the model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk} \quad i = 1,\ldots,a \quad j = 1,\ldots,b \quad k = 1,\ldots,r \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_j \sim N\left(0,\sigma_b^2\right) \quad \alpha\beta_{ij} \sim N\left(0,\sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$$

ANOVA					
Source	df	SS	MS	F	F(0.05)
Player		3879.30			
Home		1.30			
P*H		323.67			
Error		15787.29		#N/A	#N/A
Total		19991.56	#N/A	#N/A	#N/A

$$E\left\{MSE\right\} = \sigma^2 \qquad E\left\{MSAB\right\} = \sigma^2 + r\sigma_{ab}^2 \quad E\left\{MSB\right\} = \sigma^2 + r\sigma_{ab}^2 + ar\sigma_b^2 \quad E\left\{MSA\right\} = \sigma^2 + r\sigma_{ab}^2 + \frac{br\sum_{i=1}^{a}\alpha_i^2}{a-1}$$

p.1.a. Complete the partial ANOVA table.

p.1.b. Test whether there is an interaction between Player and Location (Home). H_0: $\sigma_{ab}^2 = 0$

p.1.b.i. Test Stat: _______ p.4.b.ii. Reject H_0 if Test Stat is in the range _________ p.4.b.iii. P-value > **or** < .05?

p.1.c. Test whether there is Location (Home vs Away) Main Effect. H_0: $\alpha_1 = \alpha_2 = 0$

p.1.c.i. Test Stat: _______ p.4.c.ii. Reject H_0 if Test Stat is in the range _________ p.4.c.iii. P-value > **or** < .05?

p.1.d. Test whether there is Player Main Effect. H_0: $\sigma_b^2 = 0$

p.1.d.i. Test Stat: _______ p.4.d.ii. Reject H_0 if Test Stat is in the range _________ p.4.d.iii. P-value > **or** < .05?

p.1.e. Give unbiased estimates of each of the variance components:

$\hat{\sigma}_{ab}^2 =$ ______________________ $\hat{\sigma}_b^2 =$ ______________________ $\hat{\sigma}^2 =$ ______________________

QD.2 The broiler chicken study had 60 replicates at each of 2 levels of factor A (Base: Sorghum or Corn) and 2 levels of Factor B (Methionine: Present or Absent). One response reported was the weight of the wing drumette.

Model: $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad i=1,2 \; j=1,2 \; k=1,\ldots,60 \quad \sum_{i=1}^{2}\alpha_i = \sum_{j=1}^{2}\beta_j = \sum_{i=1}^{2}(\alpha\beta)_{ij} = \sum_{j=1}^{2}(\alpha\beta)_{ij} = 0 \quad \varepsilon_{ijk} \sim N(0,\sigma^2)$

p.2.a. The following table gives the means (SDs) for each treatment:

Base\Meth	Absent	Present	Mean
Sorghum	46.4 (8.0)	34.8 (6.0)	40.6
Corn	38.8 (6.0)	41.6 (10.0)	40.2
Mean	42.6	38.2	40.4

Complete the following ANOVA table:

Source	df	SS	MS	F	F(.05)
Base					
Methionine					
B*M		3110.4			
Error		13924		#N/A	#N/A
Total			#N/A	#N/A	#N/A

p.2.b. Test H_0: No Interaction between Base and Methionine p.2.b.i. Test Stat: _______ p.2.b.ii. Reject H_0 if Test Stat is in the range _________ p.2.b.iii. P-value > **or** < .05?

p.2.c. Test H_0: No Base effect

p.2.c.i. Test Stat: _______ p.2.c.ii. Reject H_0 if Test Stat is in the range _________ p.2.c.iii. P-value > **or** < .05?

p.2.d. Test H_0: No Methionine effect

p.2.d.i. Test Stat: _________ p.1.d.ii. Reject H_0 if Test Stat is in the range ________ p.2.d.iii. P-value > **or** < .05?

QD.3. A study is conducted to compare 5 methods of oiling bowling alleys (Factor A) on scores by professional bowlers. A random sample of 10 professional bowlers (Factor B) are observed twice on each of these 5 oiling methods (the scores are totals pins over 7 games/100). These are the only oiling methods of interest. The partial ANOVA table is given below for the model:

$$Y_{ijk} = \mu + \alpha_i + b_j + (ab)_{ij} + \varepsilon_{ijk} \quad \sum \alpha_i = 0 \quad b_j \sim N\left(0, \sigma_b^2\right) \quad (ab)_{ij} \sim N\left(0, \sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim N\left(0, \sigma^2\right)$$

Source	df	SS	MS	F	Critical F
Oiling Method	4	43.0	10.75		
Bowler	9	10.1	1.12		
OilxBowler	36	24.3	0.675		
Error	50	37.5	0.75		
Total	99	114.9			

Conduct the following tests:

p.3.a H_0: No bowler/oiling method interaction: $\sigma_{ab}^2 = 0$ H_A: bowler/oiling method interaction: $\sigma_{ab}^2 > 0$

Test Statistic _______________ Rejection Region __________ Do you conclude there is a significant interaction? _____

p.3.b H_0: No Oiling Method Differences: $\alpha_1=\alpha_2=\alpha_3=\alpha_4=\alpha_5 = 0$ H_A: Differences exist among oil methods (Not all $\alpha_i = 0$)

Test Statistic __________ Rejection Region _________ conclude there is a significant oiling method effect? _______

p.3.c. H_0: No bowler effect: $\sigma_b^2 = 0$ H_A: bowler effect exists: $\sigma_b^2 > 0$

Test Statistic _______________ Rejection Region __________ Conclude there is a significant interaction? _______

QD.4. An experiment is conducted to measure the effects of 4 weave types and 3 test speeds on the breaking strength of fibers. Four replicates are obtained at each combination of weave type and test speed. These are the only weave types and fibers of interest to the researchers.

p.4.a Complete the following ANOVA table, and conduct the tests for interactions and main effects.

Source	df	SS	MS	F	F(.05)
Weave Type		3224.82			
Test Speed		3186.53			
Interaction		20.98			
Error		389.28			
Total		6821.62			

p.4.b H_0: No Interaction between weave type and test speed **Reject H_0 / Fail to Reject H_0**

p.4.c H_0: No weave type effects **Reject H_0 / Fail to Reject H_0**

p.4.d. H_0: No test speed effects **Reject H_0 / Fail to Reject H_0**

QD.5. An experiment is conducted to determine the effects of 3 **ripening stages** (Factor A) and 2 **screw speeds** (Factor B) on Water Solubility Index in Bananas. There are 3 replicates at each combination of levels of Factors A and B. The Sample means are given in the following table, as well as row and column means, and the partial ANOVA table. Both factors are considered **fixed** in this design.

Means

Factor A\B	1	2	RowMean
1	23.4	24.1	23.75
2	24.3	25.8	25.05
3	25.4	26.2	25.80
ColMean	24.37	25.37	24.87

ANOVA

Source	df	SS	MS	F_obs	F(.05)
A		12.91			
B					
A*B		0.57			
Error		1.53			
Total					

p.5.a. Complete the ANOVA Table.

p.5.b. Test H_0: No Interaction between ripening stage (A) and screw speed (B).

p.5.b.i. Test Stat: ________ p.5.b.ii. Reject H_0 if Test Stat is in the range __________ p.5.b.iii. P-value > **or** < .05?

p.5.c. Compute Tukey's minimum significant difference (W) and Bonferroni's minimum significant difference (B) when we wish to compare all 3 ripening stages at a given level of screw speed. (That is, when screw speed=1, or when screw speed=2).

Tukey's W: ______________________________ Bonferroni's B ___________________________________

QD.6. An experiment was conducted to measure variability in gauge readings among **operators** (Factor A) and **parts** from a production process (Factor B). The 3 **operators** are the only ones at the company, so they are **fixed**. The 20 **parts** are a random sample from many parts produced, so they are **random**. Each operator makes r = 2 measurements per part.

p.6.a. Assuming the mixed model with fixed operator effects and random (and independent) parts and interaction effects, complete the following ANOVA table:

Source	df	SS	MS	F_obs	F(.05)	Reject H0: No Effect?
Operators		2.6				
Parts		1185.4			1.867	
O*P		27.1			1.603	
Error						
Total		1274.6				

p.6.b. Assuming no Operator/Part interaction, based on Tukey's method, how far apart would 2 operators means need to differ by to be considered significantly different, when we simultaneously compare all pairs of operators? Tukey's W = _________________________

QD.7 Based on the following Analysis of Variance table, based on a balanced 2-Way ANOVA, answer the following questions.

ANOVA					
Source	**df**	**SS**	**MS**	**F_obs**	**P-value**
A	**4**	**600**	**150**	**7.5**	**0.0001**
B	**3**	**270**	**90**	**4.5**	**0.0065**
A*B	**12**	**360**	**30**	**1.5**	**0.1495**
Error	**60**	**1200**	**20**		
Total	**79**	**2430**			

p.7.a. Number of levels of Factor A ___________________

p.7.b. Number of levels of Factor B ___________________

p.7.c. Number of replicates per treatment (combination of levels of A and B) ___________________

p.7.d. Estimate of standard deviation of measurements within same treatment ___________________

p.7.e. P-value for test of H_0: No interaction between the effects of levels of factors A and B ___________

p.7.f. P-value for test of H_0: No effects among levels of factor B ___________

p.7.g. Number of pairs of levels of factor A in a multiple comparison procedure ___________

QD.8 Consider the following table from a 2-Factor Fixed Effects Model

Table 4. Ash content (g kg-1 DM) of selected saltgrass accessions grown during 10 weeks in water culture at four salinity levels

Accession	Salinity level (dS m-1) 1.5	10	30	50
AL1	65 ± 2.2	78 ± 4.1	90 ± 2.9	98 ± 3.1
AL3	63 ± 2.8	79 ± 5.0	92 ± 4.3	94 ± 4.9
Arg1	84 ± 3.1	99 ± 4.4	102 ± 3.7	107 ± 3.1
Arg2	86 ± 3.6	94 ± 3.5	96 ± 3.2	102 ± 3.8
CA1	72 ± 2.4	88 ± 2.9	103 ± 3.5	105 ± 3.7
CA4	66 ± 2.5	84 ± 4.2	89 ± 4.1	89 ± 4.7
CA13	70 ± 1.9	90 ± 3.3	97 ± 3.6	96 ± 4.0
CA17	68 ± 2.6	85 ± 4.4	94 ± 4.0	94 ± 3.8
CH1	79 ± 3.4	94 ± 3.6	103 ± 4.4	106 ± 5.6
CH2	75 ± 3.1	95 ± 4.8	100 ± 5.9	99 ± 4.1
CT2	71 ± 2.9	84 ± 3.1	90 ± 3.7	92 ± 4.2
DE1	75 ± 1.8	88 ± 2.9	96 ± 3.5	99 ± 3.3
DE3	74 ± 2.5	82 ± 2.6	90 ± 3.0	91 ± 3.4
GA2	56 ± 1.4	77 ± 1.9	86 ± 2.2	88 ± 2.9
GA3	62 ± 1.6	79 ± 2.4	89 ± 2.8	90 ± 3.2
GA6	66 ± 1.9	76 ± 1.8	81 ± 2.7	82 ± 3.5

* a Values are means ± SE of six replicates

p.8.a. Give the degrees of freedom for the Analysis of Variance.

Source	df
Accession	
Salinity	
A*S	
Error	
Total	

p.8.b. Set up the calculation of the Error Sum of Squares (SE represents standard error of the mean)

QD.9. An experiment is conducted to compare 4 varieties of cheddar cheese (fixed effect) in terms of bitterness scores among a sample of 5 raters (random effect). Each rater tastes each variety twice (they are not told which variety they are tasting, and bitterness is rated on a visual analogue scale, ranging from 0 (low) to 10 (high).).

Model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad i=1,\ldots,4 \; j=1,\ldots,5 \; k=1,\ldots,2 \quad \sum_{i=1}^{4}\alpha_i = 0 \quad \beta_j \sim N\left(0,\sigma_b^2\right) \quad (\alpha\beta)_{ij} \sim N\left(0,\sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$$

p.9.a. The sample means for the 4 varieties are: $\overline{y}_A = 4.0 \quad \overline{y}_B = 7.0 \quad \overline{y}_C = 3.0 \quad \overline{y}_D = 6.0$

Complete the following ANOVA table:

Source	df	SS	MS	F	F(.05)
Variety					
Rater		24.01			
V*R		3.83			
Error				#N/A	#N/A
Total		152.05	#N/A	#N/A	#N/A

p.9.b. Test H_0: No Interaction between variety and rater

p.9.b.i. Test Stat: _______ p.9.b.ii. Reject H_0 if Test Stat is in the range _________ p.9.b.iii. P-value > **or** < .05?

p.9.c. Compute Tukey's minimum significant difference (W) and Bonferroni's minimum significant difference (B) when we wish to test for differences among variety effects.

Tukey's W: ______________________________ Bonferroni's B ___________________________________

QD.10. A research paper reported the following Analysis of Variance, based on raw data presented earlier in the paper. There were 3 factors (A, B, C), each at 2 levels. There were 3 blocks, each block being made up of one observation for each of the 2(2)(2)=8 combinations of factors A, B, and C. Note that the **Block and Corrected Total Sums of Squares are correct** in the table. **Ignore BC and Residual for this table.**

Response 3 Efficiency

ANOVA for selected factorial model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Block	7.07	2	3.53			
Model	19.37	4	4.84	11.69	< 0.0001	significant
A-Charge Voltage	*3.25*	*1*	*3.25*	*7.85*	*0.0123*	
B-Armature Mass	*8.94*	*1*	*8.94*	*21.60*	*0.0002*	
C-Starting Location	*5.25*	*1*	*5.25*	*12.69*	*0.0024*	
BC	*1.92*	*1*	*1.92*	*4.64*	*0.0459*	
Residual	7.04	17	0.41			
Cor Total	33.47	23				

q.10.a. Suppose we want to obtain the sums of squares for each of the 3 main effects (recall that each has 2 levels). How many observations are each of these means based on? That is, the mean of all observations when factor A is at its low level is based on how many observations (This will be the same as A_{High}, B_{Low}, B_{High}, C_{Low}, C_{High})?

q.10.b. The means for each of the levels for A, B, and C are given below, as well as the overall mean. Compute the sums of squares for A, B, and C, respectively. How do they compare to those given by the authors (who were probably using more decimal places internally).

Group	Mean
A_Low	5.03
A_High	4.89
B_Low	5.57
B_High	4.35
C_Low	5.43
C_High	4.49
overall	4.96

Actual SSA = ____________________ SSB = ____________________ SSC = ____________________

QD.11. An experiment was conducted to determine the effects of viewing a magical film versus a non-magical film in children. Samples of 32 6-year olds, and 32 8-year-olds were selected, and randomly assigned such that 16 of each age-group viewed the magical film and 16 of each age-group viewed the non-magical film (both age and film category are treated as fixed effects). The following table gives the means (SDs) for each treatment. The response (y) was a score on an imagination scale (rating of a child acting out an object or animal).

Model: $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad i=1,2 \; j=1,2 \; k=1,...,16 \quad \sum_{i=1}^{2}\alpha_i = \sum_{j=1}^{2}\beta_j = \sum_{i=1}^{2}(\alpha\beta)_{ij} = \sum_{j=1}^{2}(\alpha\beta)_{ij} = 0 \quad \varepsilon_{ijk} \sim N(0,\sigma^2)$

p.11.a. The following table gives the means (SDs) for each treatment:

Age\Film	Non-Magical	Magical	Mean
6	17.0 (2.7)	21.6 (4.1)	19.3
8	18.7 (3.8)	22.7 (3.8)	20.7
Mean	17.85	22.15	20

Complete the following ANOVA table:

Source	df	SS	MS	F	F(.05)
Age					
Film					
A*F		1.44			
Error		794.7		#N/A	#N/A
Total			#N/A	#N/A	#N/A

p.11.b. Test H_0: No Interaction between Age and Film Type p.11.b.i. Test Stat: _______ p.11.b.ii. Reject H_0 if Test Stat is in the range _________ p.11.b.iii. P-value > **or** < .05?

p.11.c. Test H_0: No Age effect

p.11.c.i. Test Stat: _______ p.11.c.ii. Reject H_0 if Test Stat is in the range _________ p.11.c.iii. P-value > **or** < .05?

p.11.d. Test H_0: No Film Type effect

p.11.d.i. Test Stat: _________ p.11.d.ii. Reject H_0 if Test Stat is in the range ________ p.11.d.iii. P-value > **or** < .05?

QD.12. A study was conducted, measuring the effects of 3 electronic **Readers** and 4 **Illumination** levels on time for people to read a given text (100s of seconds). There were a total of 60 subjects, 5 each assigned to each combination of Reader/Illumination level. For this analysis, consider both Reader and Illumination level as fixed effects.

p.12.a Complete the following ANOVA table, and test for significant Reader/Illumination Interaction effects, as well as main effects for Reader and Illumination levels.

ANOVA						
Source	**df**	**SS**	**MS**	**F**	**F(0.05)**	**Significant Effects?**
Reader		70.70				
Illumination		148.11				
Read*Illum		2.15				
Error		365.02				
Total		585.98				

p.12.b Use Tukey's Method to make all pairwise comparisons among Readers.

p.12.c Use Bonferroni's method to make all pairwise comparisons among Illumination levels.

QD.13. An experiment had 2 factors, each with 2 levels: Factor A:Instructional Method (Standard and Enhanced), and Factor B: Instructional Medium (Desktop and Mobile Device). There were a total of N = 88 subjects, with n = 22 receiving each treatment (combination of method and medium). The sample means and standard deviations of scores on a transfer test are given below.

Mean	Medium				SD	Medium	
Method	Desktop	Mobile	Overall		Method	Desktop	Mobile
Standard	2.58	2.36	2.47		Standard	1.93	1.76
Enhanced	4.04	4.34	4.19		Enhanced	3.04	2.67
Overall	3.31	3.35	3.33				

The model fit is a 2-Way fixed effects ANOVA with interaction. $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$

p.13.a. Compute the sums of squares for: Method, Medium, and complete the following ANOVA table.

Source	df	SS	MS	F_obs	F(0.05)
Method					
Medium					
M*M		1.4872			
Error		487.053			
Total					

p.13.b. Do you reject the hypothesis: $H_0^{AB} : (\alpha\beta)_{11} = (\alpha\beta)_{12} = (\alpha\beta)_{21} = (\alpha\beta)_{22} = 0$ **Yes / No**

p.13.c. Do you reject the hypothesis: $H_0^{A} : \alpha_1 = \alpha_2 = 0$ **Yes / No**

p.13.d. Do you reject the hypothesis: $H_0^{B} : \beta_1 = \beta_2 = 0$ **Yes / No**

QD.14. Compute the appropriate minimum significant difference for comparing (fixed) treatment means.

Two-way ANOVA: Factor A Fixed, a = 4, Factor B Random, b = 5, n=3 replicates per combination of factors A and B.

$SSA = 600 \quad SSB = 1000 \quad SSAB = 600 \quad SSE = 1600$

Compute Tukey's HSD for comparing levels of factor A.

QD.15. An experiment was conducted to compare a = 3 theories for the apparent modulus of elasticity (Y) of b = 3 apple varieties. The 3 theories were: Hooke's, Hertz's, and Boussineq's; the 3 apple varieties were: Golden Delicious, Red Delicious, and Granny Smith. The researchers determined the elasticity for r = 15 apples based on each combination of theory and variety. For the purposes of this experiment, each factor is fixed.

Model: $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim NID(0, \sigma^2) \quad \sum_{i=1}^{a}\alpha_i = \sum_{j=1}^{b}\beta_j = \sum_{i=1}^{a}(\alpha\beta)_{ij} = \sum_{j=1}^{b}(\alpha\beta)_{ij} = 0$

$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk} - \overline{y}_{ij\bullet}\right)^2 = 17.095 \quad \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk} - \overline{y}_{\bullet\bullet\bullet}\right)^2 = 113.119 \quad bn\sum_{i=1}^{a}\left(\overline{y}_{i\bullet\bullet} - \overline{y}_{\bullet\bullet\bullet}\right)^2 = 57.987 \quad an\sum_{j=1}^{b}\left(\overline{y}_{\bullet j\bullet} - \overline{y}_{\bullet\bullet\bullet}\right)^2 = 35.779$

Cell Means	GoldenDelicious	RedDelicious	GrannySmith	Row Mean
Hooke	2.68	3.46	4.23	3.457
Hertz	2.44	3.06	3.84	3.113
Boussinesq	1.53	1.89	2.36	1.927
Column Mean	2.217	2.803	3.477	2.832

Complete the following Analysis of Variance Table, and test for interaction effects and main effects.

Source	df	SS	MS	F	F(.95)	P-value
Theory						>0.05 or <0.05
Variety						>0.05 or <0.05
Theory*Variety						>0.05 or <0.05
Error				#N/A	#N/A	#N/A
Total			#N/A	#N/A	#N/A	#N/A

QD.16. A 2-Way Random Effects model is fit, where a sample of a = 8 products were measured by a sample of b = 6 machinists, with r = 3 replicates per machinist per product. The model fit is as follows (independent random effects):

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \alpha_i \sim NID\left(0, \sigma_a^2\right) \quad \beta_j \sim NID\left(0, \sigma_b^2\right) \quad (\alpha\beta)_{ij} \sim NID\left(0, \sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim NID\left(0, \sigma^2\right)$$

You are given the following sums of squares: $SSA = 420 \quad SSB = 350 \quad SSAB = 140 \quad SSE = 210$

Give the test statistic and rejection region for the following 3 tests. Note for test 1, your rejection region will be symbolic, give the specific numerator and denominator degrees of freedom. Also give unbiased (ANOVA) estimates of each variance component.

1) $H_0^{AB}: \sigma_{ab}^2 = 0 \quad H_A^{AB}: \sigma_{ab}^2 > 0$ 2) $H_0^{A}: \sigma_{a}^2 = 0 \quad H_A^{A}: \sigma_{a}^2 > 0$ 3) $H_0^{B}: \sigma_{b}^2 = 0 \quad H_A^{B}: \sigma_{b}^2 > 0$

1: Test Stat: ____________________________ Rejection Region: _________________ Estimate: ________________

2: Test Stat: ____________________________ Rejection Region: _________________ Estimate: ________________

3: Test Stat: ____________________________ Rejection Region: _________________ Estimate: ________________

QD.17. A 2-Way (crossed) ANOVA is used to measure the effects of 2 factors, each at 3 levels. There are 4 replicates for each treatment (combination of levels of factors A and B). There is a significant interaction between the 2 factors, so the researchers choose to use Tukey's method to compare all pairs of treatment means. Give Tukey's W for comparing all pairs of means, with MSE = 100.

QD.18. An unbalanced two-way ANOVA was conducted to compare desire scores for a product (Y) among a sample of female consumers. The factors were Product (X_1=1 if **S**hoe, -1 if **I**nk toner) and Endorser (X_2=1 if **C**elebrity, -1 if **O**nline Consumee). The sample sizes were: S/C = 43, S/O = 44, I/C = 46, I/O =43. Four regressions models were fit:

Model 1: $E\{Y_{ijk}\} = \beta_0 + \beta_1 X_{1ijk} + \beta_2 X_{2ijk} + \beta_3 X_{1ijk} X_{2ijk}$ $SSE_1 = 169.1$ $\hat{Y}_1 = 2.478 - 0.183X_1 + 0.088X_2 + 0.148X_1X_2$

Model 2: $E\{Y_{ijk}\} = \beta_0 + \beta_1 X_{1ijk} + \beta_2 X_{2ijk}$ $SSE_2 = 172.9$ $\hat{Y}_2 = 2.472 - 0.181X_1 + 0.086X_2$

Model 3: $E\{Y_{ijk}\} = \beta_0 + \beta_2 X_{2ijk} + \beta_3 X_{1ijk} X_{2ijk}$ $SSE_3 = 174.9$ $\hat{Y}_3 = 2.479 + 0.092X_2 + 0.146X_1X_2$

p.18.a. Test whether there is an interaction between Product and Endorser. H_0: ____________ H_A: ___________

Test Statistic _______________________ Rejection Region _______________________ P-value **> 0.05** or **< 0.05**

p.18.b. Test whether there is main effect for Product. H_0: ____________ H_A: ___________

Test Statistic _______________________ Rejection Region _______________________ P-value **> 0.05** or **< 0.05**

p.18.c. Based on Model 1, give the predicted scores for all combinations of Product and Endorser.

Product\Endor	**Celebrity**	**Online Co**
Shoe		
InkToner		

QD.19. An experiment was conducted to compare rating scores assigned to wines. There were 5 wine producers (vineyards), there were 2 appelations (white and red wines), and 14 judges. Each judge rated each wine variety a single time. Note that each producer makes a white wine and a red wine. Give the sources of variation and their corresponding degrees of freedom, using the highest order interaction as the error term.

Source Degrees of Freedom

QD.20. An experiment is conducted as a 2-Way factorial design with a = 3 levels, b = 2 levels, and n = 4 replicates per treatment. MSE = 100. There is a significant interaction, so the researchers decide to use Tukey's method to compare all of the combinations of levels of Factors A and B. Compute Tukey's HSD for the researchers.

HSD = ______________________________

QD.21. A study was conducted to compare 3 methods of measuring blood pressure (Factor A, Fixed). There were 20 Human Subjects in the study (Factor B, Random). Each subject was measured twice by each method (n = 2).

Model: $Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_j \sim N(0, \sigma_\beta^2) \quad (\alpha\beta)_{ij} \sim N(0, \sigma_{\alpha\beta}^2) \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$

p.21.a. Complete the ANOVA table.

ANOVA			
Source	df	SS	MS
Method		4117	
Subject		20120	
Meth*Subj		1680	
Error		1666	
Total		27583	#N/A

p.21.b. Test $H_0^{AB}: \sigma_{\alpha\beta}^2 = 0$ (No interaction between methods and subjects) vs $H_A^{AB}: \sigma_{\alpha\beta}^2 > 0$

Note: $F(.05, df_1, df_2) = 1.603$

Test Statistic: _________________________ Rejection Region _________________________

p.21.c. Test $H_0^A: \alpha_1 = \alpha_2 = \alpha_3 = 0$ (No method effect differences) vs H_A^A: Not all $\alpha_i = 0$

Test Statistic: _________________________ Rejection Region _________________________

p.21.d. Test $H_0^B: \sigma_\beta^2 = 0$ (No subject effect differences) vs $H_A^B: \sigma_\beta^2 > 0$

Test Statistic: _________________________ Rejection Region _________________________

QD.22. An experiment was conducted to measure the energy efficiency of electric clothes dryer machines. Factor A was Clothing Type (1=Towels, 2=Jeans,3=Thermal Clothing) and Factor B was Dryer Type (1=Electric Dryer, 2=Bi-directional Electric dryer, 3=Town Gas-Fired Dryer, 4=LPG-Fired dryer). There were n = 3 replicates per combination of Clothing and Dryer Types. The model is given below, with Factors A and B both being Fixed factors.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim N(0, \sigma^2) \quad \sum_i \alpha_i = \sum_j \beta_j = \sum_i \alpha\beta_{ij} = \sum_j \alpha\beta_{ij} = 0$$

Source	sumsq	df	MS	F	F(.05)
Clothing Type					
Dryer Type	5.2296				
CxD	0.0362				
Error				#N/A	#N/A
Total	5.6011		#N/A	#N/A	#N/A

i	1	2	3	All
Mean	1.5408	1.7118	1.6141	1.6222
j	1	2	3	4
Mean	1.3031	1.3437	2.2250	1.5871

p.22.a. Complete the Analysis of Variance Table.

p.22.b. For each of the following three null hypotheses, are they rejected?

$H_0^{AB}: \alpha\beta_{11} = ... = \alpha\beta_{34} = 0$ ________ $H_0^{A}: \alpha_1 = ... = \alpha_3 = 0$ ________ $H_0^{B}: \beta_1 = ... = \beta_4 = 0$ ________

p.22.c. Compute Tukey's HSD for comparing the all pairs among the 3 clothing types (use MSE and df_E directly from the ANOVA table). Which pairs (if any) are significantly different?

p.22.d. Compute Bonferroni's MSD for comparing the all pairs among the 4 dryer types (use MSE and df_E directly from the ANOVA table). Which pairs (if any) are significantly different?

QD.23. An unbalanced 2-Factor study was conducted to test for gender and movie type effects on brand recall of products placed in movies. Factor A was gender with a=2 levels (female/male) and Factor B was movie type with b=3 levels (action/comedy/drama). The response was Y (number of correct recalls of items, 0-6). The model was fit as a linear regression and set up as follows. There were a total of n = 137 subjects in the study.

$$X_1 = \begin{cases} 1 \text{ if Female} \\ -1 \text{ if Male} \end{cases} \quad X_2 = \begin{cases} 1 \text{ if Action} \\ -1 \text{ if Drama} \\ 0 \text{ if Comedy} \end{cases} \quad X_3 = \begin{cases} 1 \text{ if Comedy} \\ -1 \text{ if Drama} \\ 0 \text{ if Action} \end{cases}$$

Model 1 contains all main effects and interactions, Model 2 contains all main effects, Models 3 and 4 contain only Factor A and B main effects, respectively.

Model 1: $E\{Y\} = \mu + \alpha_1 X_1 + \beta_1 X_2 + \beta_2 X_3 + \alpha\beta_{11} X_1 X_2 + \alpha\beta_{12} X_1 X_3 \quad SSE_1 = 231.66 \quad df_{E1} = 131$

Model 2: $E\{Y\} = \mu + \alpha_1 X_1 + \beta_1 X_2 + \beta_2 X_3 \quad SSE_2 = 232.04 \quad df_{E2} = 133$

Model 3: $E\{Y\} = \mu + \alpha_1 X_1 \quad SSE_3 = 243.62 \quad df_{E3} = 135$ Model 4: $E\{Y\} = \mu + \beta_1 X_2 + \beta_2 X_3 \quad SSE_4 = 236.44 \quad df_{E4} = 134$

p.23.a. Use Models 1 and 2 to show that the gender/movie type interaction is not significant. $H_0^{AB}: \alpha\beta_{11} = \alpha\beta_{12} = 0$

Test Statistic ________________________ Rejection Region ___________________________ P > or < .05

p.23.b. Use Models 2, 3, and 4 to test whether gender and/or movie type main effects are significant.
$H_0^A: \alpha_1 = 0 \qquad H_0^B: \beta_1 = \beta_2 = 0$

Factor A: Test Statistic _____________________ Rejection Region ________________________ P > or < .05

Factor B: Test Statistic _____________________ Rejection Region ________________________ P > or < .05

QD.24. In the Broiler Chicken study, Factor A is base diet with a = 2 levels (sorghum and corn), and Factor B is methionine with b = 2 levels (absent and present). There were n = 60 chickens per treatment in a Completely Randomized Design. The model fit is given below, along with treatment means. The Error sum of squares is *SSE* = 167560.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim N\left(0, \sigma^2\right) \quad \sum_i \alpha_i = \sum_j \beta_j = \sum_i \alpha\beta_{ij} = \sum_j \alpha\beta_{ij} = 0$$

	Meth-	Meth+	RowMean
Sorghum	46	35	
Corn	39	42	
ColMean			

p.24.a. Compute SSA, SSB, and SSAB, and complete the following Analysis of Variance Table.

Source	df	SS	MS	F	F(.05)	Significant?
A						Yes / No
B						Yes / No
AB						Yes / No
Error				#N/A	#N/A	#N/A
Total			#N/A	#N/A	#N/A	#N/A

p.24.b. Assuming the interaction effect is significant, compute Tukey's HSD and Bonferroni's MSD among the 4 treatments. (Hint: for Bonferroni's method there are 4(4-1)/2 = 6 pairs of treatments. Based on Tukey's HSD identify which pairs of treatments are not significantly different by joining lines or letter superscripts.

Tukey HSD ___________ Bonferroni MSD ___________ Sorg/Meth+ Corn/Meth- Corn/Meth+ Sorg/Meth-

QD.25. A wine making experiment was conducted to determine the effects of two factors on Color index (Y, chroma) of Chardonnay Wine. Factor A was pressing method (aerobic/inert) and factor B was handling condition (oxidative/reductive). There were n = 3 replicates for each combination of factors A and B. The treatment means are given below (Note that this would be considered a Fixed Effects model).

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \sum_i \alpha_i = \sum_j \beta_j = \sum_i (\alpha\beta)_{ij} = \sum_j (\alpha\beta)_{ij} = 0 \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

Mean			
A\B	Oxidative	Reductive	Row Mean
Aerobic	5.88	6.37	6.125
Inert	6.97	7.46	7.215
Column Mean	6.425	6.915	6.67

p.25.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)
A					
B					
AB					
Error		1.2456		#N/A	#N/A
Total			#N/A	#N/A	#N/A

p.25.b. What are your conclusions based on the following tests?

$H_0^{AB} : (\alpha\beta)_{11} = (\alpha\beta)_{ab} = 0$ ____________________

$H_0^{A} : \alpha_1 = \ldots = \alpha_a = 0$ ______________________ $H_0^{B} : \beta_1 = \ldots = \beta_b = 0$ ______________________

QD.26. An experiment was conducted to study the effects of maturation time (Factor A: **1**,**2**,**3** months) and type of storage (Factor B: **G**lass, Plastic with **L**ight **T**oast chips, Plastic with **M**edium **T**oast Chips, Plastic with **D**ark **T**oast Chips, Oak **B**arrel). The model is fit as an additive 2-factor, crossed Fixed Effects Model. The response was Bitterness (Y, in IBUs) and there were n = 3 replicates per treatment.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk} \quad \sum_i \alpha_i = \sum_j \beta_j = 0 \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

The marginal means and Error Sum of Squares are given below.

$SSE = 16.6$ Maturation Time: $\overline{y}_{1\bullet\bullet} = 18.1$ $\overline{y}_{2\bullet\bullet} = 20.1$ $\overline{y}_{3\bullet\bullet} = 18.9$

Storage: $\overline{y}_{\bullet 1 \bullet} = 18.5$ $\overline{y}_{\bullet 2 \bullet} = 19.4$ $\overline{y}_{\bullet 3 \bullet} = 19.0$ $\overline{y}_{\bullet 4 \bullet} = 19.9$ $\overline{y}_{\bullet 5 \bullet} = 18.3$

p.26.a.Use Tukey's method to compare all pairs of Maturation Times.

1 Month 3 Months 2 Months

p.26.b. Use Bonferroni's to compare all pairs of Storage Types.

B G MT LT DT

QD.27. A mixed model was fit relating measured blood alcohol content (Y=100*g/210L) among Factor A (a=6 Models of Instruments, Fixed factor) and Factor B (b=3 Subjects who consumed alcohol, Random factor), with n=10 measurements made by each instrument on each subject. The model fit is as follows (independent random effects):

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_j \sim NID\left(0, \sigma_b^2\right) \quad (\alpha\beta)_{ij} \sim NID\left(0, \sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim NID\left(0, \sigma^2\right)$$

p.27.a. Complete the following ANOVA table and test for main effects and interactions.

$$H_0^{AB}: \sigma_{ab}^2 = 0 \qquad H_0^A: \alpha_1 = \ldots = \alpha_a = 0 \qquad H_0^B: \sigma_b^2 = 0$$

Source	df	SS	MS	F	F(.05)	P-value
Instrument (A)		20.58				> < .05
Subject (B)		210.5				> < .05
Inst x Subj (AB)		3.52				> < .05
Error		8.89		#N/A	#N/A	#N/A
Total		243.49	#N/A	#N/A	#N/A	#N/A

p.27.b. Obtain point estimates for $\sigma^2, \sigma_{ab}^2, \sigma_b^2$

$s^2 =$ ____________________ $s_{ab}^2 =$ ____________________ $s_b^2 =$ ____________________

p.27.c. Compute Tukey's HSD (W) for comparing all pairs of instruments.

Tukey's HSD (W) = ____

QD.28. An unbalanced experiment was conducted to study the effects of Selenium (X_1=1 if Present, 0 if Absent) and N-acetyl-cysteine (X_2=1 if Present, 0 if Absent) on sperm count in infertile men. The response was sperm count (Y). The sample sizes in the 4 cells were slightly unbalanced due to the nature of the study, with a total sample size of N = 420. The following 4 models are fit.

Model 1: $E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 \quad \hat{Y} = 60.80 + 5.20X_1 + 3.95X_2 + 1.25X_1X_2 \quad SSE_1 = 98074 \quad R_1^2 = .3985$

Model 2: $E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \quad \hat{Y} = 60.80 + 5.19X_1 + 3.94X_2 \quad SSE_2 = 98730 \quad R_2^2 = .3914$

Model 3: $E\{Y\} = \beta_0 + \beta_1 X_1 \quad \hat{Y} = 60.78 + 5.19X_1 \quad SSE_2 = 105263 \quad R_3^2 = .3117$

Model 4: $E\{Y\} = \beta_0 + \beta_2 X_2 \quad \hat{Y} = 60.78 + 3.94X_2 \quad SSE_2 = 110061 \quad R_4^2 = .2367$

p.28.a. Test whether there is an interaction between Selenium and N-acetyl-cystine on sperm count.

H_0: __________________ Test Stat: ____________________ Rej Region: ____________________ P > or < .05

p.28.b. Given the additive model (Model 2), test whether there is a Selenium effect.

H_0: __________________ Test Stat: ____________________ Rej Region: ____________________ P > or < .05

p.28.c. Given the additive model (Model 2), test whether there is a N-acetyl-cystine effect.

H_0: __________________ Test Stat: ____________________ Rej Region: ____________________ P > or < .05

Part E: Nested Designs

QE.1. A study is conducted to compare 3 types of traffic signal settings (pre-timed, semi-actuated, and fully actuated). A sample of 30 intersections in a large city are obtained, and 10 are assigned to each of the 3 settings at random. Measurements are obtained at each signal at 20 “points” in time, where Y=traffic delay (seconds/vehicle). Write out the sources of variation and degrees of freedom for the ANOVA table. Would these factors each be best described as fixed or random? What would be the appropriate error term for testing for signal effects? What would be the degrees of freedom?

QE.2. A study is conducted to compare pH levels in rivers in 3 geographic areas. Random samples of 5 rivers were sele within each of the geographic areas, and 4 replicates were obtained within each river.

p.2.a. Complete the following Analysis of Variance table.

Source	df	SS	MS	F	F(.05)
Area		4000			
River w/in Area		2400			
Error		2250			
Total					

p.2.b. Compute Bonferroni’s B to be used to compare all pairs of geographic areas.

QE.3. An experiment was conducted to compare 3 traffic light **types** (Factor A). A random sample of 9 **intersections** (Factor B) were selected, and 3 were assigned to each traffic light **type** at random. **Types** are treated as **fixed**, and **intersections** are to be treated as **random**. Measurements of average waiting times are made at each intersection over **r** = 8 time periods. Set up the ANOVA table, giving all sources of variation, degrees of freedom, F-statistics (symbolically by specifying appropriate Mean squares), and critical F-values.

Source	df	F_obs = MS1/MS2	F(.05)

QE.4. An experiment was conducted to compare 5 machines in terms of strain-readings (y) of glass-cathode supports. The engineer had 4 “heads” from which the glass was formed for each machine (that is, the 4 “heads” for machine 1 differ from those from machine 2, etc..., implying “heads” are nested under machines). Each “head” is measured 4 times (replicates) to obtain a strain reading. Note that these are the only 5 machines of interest (fixed effects), but the “heads” used are a sample from a larger population of “heads” (random effects).

Model: $y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad i=1,...,5 \; j=1,...,4 \; k=1,...,4 \quad \sum_{i=1}^{5} \alpha_i = 0 \quad \beta_{j(i)} \sim N\left(0,\sigma_b^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$

p.4.a. The 5 machine **total** strain-readings are: 93, 81, 82, 88, and 58, respectively. Compute the machine and overall means (hint: how many measurements are taken from each machine):

$\overline{y}_1 =$ __________ $\overline{y}_2 =$ __________ $\overline{y}_3 =$ __________ $\overline{y}_4 =$ __________ $\overline{y}_5 =$ __________ $\overline{y} =$ __________

p.4.b. Complete the following partial ANOVA table:

Source	df	SS	MS	F	F(.05)
Machine					
Head(M)		282.88			
Error		642		#N/A	#N/A
Total			#N/A	#N/A	#N/A

p.4.c.. Test H$_0$: differences among "head" effects $\left(\sigma_b^2 = 0\right)$ vs H$_A$: Differences among "head" effects $\left(\sigma_b^2 > 0\right)$

p.4.c.i. Test Stat: _______ p.4.c.ii. Reject H$_0$ if Test Stat is in the range _________ p.4.c.iii. P-value > **or** < .05?

p.4.c. Compute Tukey's minimum significant difference (W) and Bonferroni's minimum significant difference (B) when we wish to test for differences among machine effects.

Tukey's W: ______________________________ Bonferroni's B ___________________________________

QE.5. An experiment was conducted to compare 6 batches of auto body side panels in terms of deviations from nominal position (y). The engineer samples 2 "groups" of body panels from each batch (that is, the 2 "groups" for batch 1 differ from those from batch 2, etc..., implying "groups" are nested under batches). Each "group" has 3 individual body panels selected and measured (replicates) for y. Note that these are a random sample of batches (random effects), and the "groups" used are a sample from a larger population of "groups" (random effects).

Model: $y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad i=1,...,6 \; j=1,2 \; k=1,2,3 \quad \alpha_i \sim N\left(0,\sigma_a^2\right) \quad \beta_{j(i)} \sim N\left(0,\sigma_b^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$

p.5.a. The 6 batch **mean** y-values are given below. Compute the overall mean, and obtain the sum of squares for batches.

$\overline{y}_1 = 4.000 \quad \overline{y}_2 = 2.017 \quad \overline{y}_3 = -4.567 \quad \overline{y}_4 = -1.117 \quad \overline{y}_5 = 4.050 \quad \overline{y}_6 = -1.100 \quad \overline{y} =$ ____________________

p.5.b. Complete the following partial ANOVA table:

Source	df	SS	MS	F	F(.05)
Batch					
Grp(B)		62.05			
Error		438.57		#N/A	#N/A
Total			#N/A	#N/A	#N/A

p.5.c. Test H_0: differences among "group" effects $\left(\sigma_b^2 = 0\right)$ vs H_A: Differences among "group" effects $\left(\sigma_b^2 > 0\right)$

p.5.c.i. Test Stat: _______ p.5.c.ii. Reject H_0 if Test Stat is in the range _________ p.5.c.iii. P-value > **or** < .05?

p.5.d. Test H_0: differences among "batch" effects $\left(\sigma_a^2 = 0\right)$ vs H_A: Differences among "batch" effects $\left(\sigma_a^2 > 0\right)$

p.5.d.i. Test Stat: _______ p.5.d.ii. Reject H_0 if Test Stat is in the range _________ p.5.d.iii. P-value > **or** < .05?

p.5.e. The Expected Mean Squares for Batches, Groups within Batches, and Error are:

$$E\{MSBatches\} = \sigma^2 + 3\sigma_b^2 + 3(2)\sigma_a^2 \qquad E\{MSGrp(B)\} = \sigma^2 + 3\sigma_b^2 \qquad E\{MSE\} = \sigma^2$$

Give unbiased estimates of each of the variance components:

$\hat{\sigma}_a^2 =$ ________________________ $\hat{\sigma}_b^2 =$ ________________________ $\hat{\sigma}^2 =$ ________________________

QE.6. Researchers conducted an experiment measuring acoustic metric values in 3 habitats (1=Cliff, 2=Mud, 3=Gravel) in 3 patches, nested within each habitat, with replicates representing 5 sites within each patch (N=3(3)(5)=45). The habitats are considered to be fixed levels, while patches within habitats are considered to be random. The response measured was snap amplitude.

p.6.a Complete the following ANOVA table, and test for habitat effects (H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$) and for patch effects (H_0: $\sigma_{ab}^2 = 0$).

ANOVA						
Source	**df**	**SS**	**MS**	**F**	**F(.05)**	**Reject H0?**
Habitat		**403.3**				
Patch(Hab)		**304.7**				
Error		**386.6**				
Total		**1094.6**				

p.6.b Compute Bonferroni's minimum significant difference for comparing pairs of habitat means.

p.6.c Obtain point estimate for σ_{ab}^2 and σ^2

QE.7. An engineering experiment was conducted to measure variation in semiconductors for a particular measurement. A sample of 5 lots (batches) of semiconductors was selected. Within each lot, 2 wafers were sampled. Each wafer was measured at 9 random sites (these are replicates). Note that the wafers are nested within lots. Both lots and wafers are random effects.

$$Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \qquad \alpha_i \sim N\left(0, \sigma_a^2\right) \qquad \beta_{j(i)} \sim N\left(0, \sigma_b^2\right) \qquad \varepsilon_{ijk} \sim N\left(0, \sigma^2\right)$$

p.7.a. Complete the following Analysis of Variance table.

Source	df	SS	MS	F_obs	F(.05)
Lot		1698.5			
Wafer(Lot)		272.2			
Error		1803.8			
Total					

p.7.b. Do you reject the hypothesis: $H_0^A : \sigma_a^2 = 0$? **Yes** / **No**

p.7.c. Do you reject the hypothesis: $H_0^B : \sigma_b^2 = 0$? **Yes** / **No**

p.7.d. For the nested design, with random factors A, and B(A), we have:

$$E\{MSE\} = \sigma^2 \quad E\{MSB(A)\} = \sigma^2 + n\sigma_b^2 \quad E\{MSA\} = \sigma^2 + n\sigma_b^2 + bn\sigma_a^2$$

Obtain unbiased estimates for the 3 variances:

$\hat{\sigma}^2 =$ ______________________ $\hat{\sigma}_b^2 =$ ______________________ $\hat{\sigma}_a^2 =$ ______________________

QE.8. For the following scenario compute the appropriate minimum significant difference for comparing (fixed) treatment means:

Nested 2-Way ANOVA: Factor A Fixed, a=3, Factor B Fixed, b=3, n=5 replicates cell.

$$SSA = 120 \quad SSB(A) = 180 \quad SSE = 144$$

Compute Bonferroni's MSD for comparing levels of factor A.

QE.9. A wildlife researcher is interested in comparing levels of a chemical in the water among the 4 lakes in a state park. The lakes are broken into many subsections based on a survey. She samples 3 subsections from each lake at random, and takes water measurements at 8 sites within each subsection. A laboratory measures the chemical in each of the water specimens. The lake means are: 76, 72, 60, and 64, respectively. The model is:

$$Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_{j(i)} \sim N(0, \sigma_b^2) \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

p.9.a. Compute the sum of squares, degrees of freedom, and mean square for lakes (factor A).

SSA = _____________________ df_A = _______________________ MSA = ___________________

The sum of squares for subsections (factor B) nested within lakes (factor A) is SSB(A) = 640, and the error sum of squares is SSE = 2520.

p.9.b. Test $H_0 : \alpha_1 = ... = \alpha_4 = 0$

Test Statistic: ________________________ Rejection Region: ____________________ P-value $<$ or $>$ .05

p.9.c. Test $H_0 : \sigma_b^2 = 0 \quad H_A : \sigma_b^2 > 0$

Test Statistic: ________________________ Rejection Region: ____________________ P-value $<$ or $>$ .05

QE.10. An experiment was conducted to compare 2 methods of constructing blue jeans (Manually and with Laser Beams). Samples of 20 pairs of jeans were constructed by each method, and 3 measurements were made on each pair of jeans. Note that the blue jeans (random) are nested within the method (fixed) by which they were constructed. The statistical model is as follows (the response is the extension of the blue jeans).

$$Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad i = 1,2;\ j = 1,\ldots,20;\ k = 1,2,3 \quad \alpha_1 + \alpha_2 = 0 \quad \beta_{j(i)} \sim N\left(0, \sigma_\beta^2\right) \quad \varepsilon_{ijk} \sim N\left(0, \sigma^2\right)$$

p.10.a. Complete the following ANOVA table.

ANOVA			
Source	df	SS	MS
Method		1185	
Jeans(Method)		5851	
Error		1837	
Total		8873	#N/A

p.10.b. Test whether there is a difference in the two methods. $H_0 : \alpha_1 = \alpha_2 = 0$

Test Statistic __________________________ Rejection Region ___________________________ p-value > 0.05 or < 0.05

p.10.c. Obtain point estimates for σ_β^2 and σ^2

$\hat{\sigma}_\beta^2 =$ ________________________________ $\sigma^2 =$ ________________________________

p.10.d. The sample means for the two methods are $\overline{Y}_{1\bullet\bullet} = 65.1$ and $\overline{Y}_{2\bullet\bullet} = 71.3$. Compute a 95% Confidence Interval for $\alpha_1 - \alpha_2$

95% CI: ____

QE.11. An experiment was conducted to determine the effect of Twitter use (based on academic and co-curricular discussions). The experiment consisted of 8 class sections, 4 sections were placed in the Experimental condition (Twitter based course activities) and the remaining 4 sections were placed in the Control condition (no course based Twitter activities). There were n = 18 students (replicates) in each section. Note that the Twitter (Experimental/Control) factor would be considered Fixed, while the Class Sections are Random. The student's course grade on 4 point scale is Y.

The model is: $Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_{j(i)} \sim N\left(0, \sigma_b^2\right) \quad \varepsilon_{ijk} \sim N\left(0, \sigma^2\right)$

p.11.a. Compute the sum of squares, degrees of freedom, and mean square for Twitter conditions (factor A).

$\bar{y}_{1\bullet\bullet} = 2.78 \quad \bar{y}_{2\bullet\bullet} = 2.32$

SSA = ____________________ df_A = ______________________ MSA = __________________

The sum of squares for sections (factor B) nested within Twitter conditions (factor A) is SSB(A) = 5.00, and the error sum of squares is SSE = 130.5.

p.11.b. Test $H_0 : \alpha_1 = ... = \alpha_a = 0$

Test Statistic: _______________________ Rejection Region: ____________________ P-value < or > .05

p.11.c. Test $H_0 : \sigma_b^2 = 0 \quad H_A : \sigma_b^2 > 0$

Test Statistic: _______________________ Rejection

Part F: Split-Plot Designs

QF.1. An experiment is conducted to compare five formulations of cookies and 4 cooking temperatures in an oven. Due to the nature of the experiment and time constraints, it was decided that on each of 7 days, there would be 4 cooking periods (one at each of the 4 temperatures), with each formulation being prepared in each cooking period. Give the Analysis of Variance table, including all sources of variation, degrees of freedom, and appropriate F-statistics. The response is a measure of cookie quality.

QF.2. A split-plot experiment is to be conducted to compare 4 grass varieties and 3 fertilizers on yield. Due to the nature of planting the grass varieties, they need to be planted on "large" strips of land, while fertilizer can be applied to "smaller" strips of land. Thus, a split-plot experiment will be conducted, with grass variety as the "whole plot" factor and fertilizer as the "subplot" factor. The experiment will be conducted on 5 strips (blocks) on a university's agricultural fields. The following model is to be fit (with grass and fertilizer as fixed factors, block as random):

$$Y_{ijk} = \mu + \alpha_i + b_j + (ab)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + \varepsilon_{ijk} \quad i = 1,...,4 \quad j = 1,...,5 \quad k = 1,2,3$$

$$\sum_{i=1}^{4} \alpha_i = \sum_{k=1}^{3} \gamma_k = \sum_{i=1}^{4} (\alpha\gamma)_{ik} = \sum_{k=1}^{3} (\alpha\gamma)_{ik} = 0 \quad b_j \sim N(0, \sigma_b^2) \quad (ab)_{ij} \sim N(0, \sigma_{ab}^2) \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

p.2.a. You are given the following "schematic diagram" of the layout, as well as random numbers **to be used for randomizing treatments to plots**. Fill in which treatments would be assigned to each position, where A1, would represent Grass A/Fertilizer 1.

	Treatment											
Block1												
Block2												
Block3												
Block4												
Block5												

	Grass			
Block1	**0.802**	**0.961**	**0.436**	**0.282**
Block2	**0.042**	**0.038**	**0.205**	**0.803**
Block3	**0.532**	**0.816**	**0.931**	**0.046**
Block4	**0.721**	**0.919**	**0.312**	**0.715**
Block5	**0.624**	**0.448**	**0.124**	**0.751**

	Fertilizer											
Block1	**0.060**	**0.059**	**0.501**	**0.058**	**0.637**	**0.960**	**0.899**	**0.612**	**0.619**	**0.457**	**0.968**	**0.044**
Block2	**0.431**	**0.594**	**0.141**	**0.881**	**0.546**	**0.298**	**0.077**	**0.307**	**0.408**	**0.761**	**0.157**	**0.830**
Block3	**0.052**	**0.677**	**0.377**	**0.704**	**0.482**	**0.256**	**0.956**	**0.249**	**0.317**	**0.401**	**0.409**	**0.745**
Block4	**0.743**	**0.972**	**0.385**	**0.040**	**0.309**	**0.535**	**0.051**	**0.543**	**0.585**	**0.507**	**0.287**	**0.738**
Block5	**0.226**	**0.086**	**0.496**	**0.901**	**0.463**	**0.909**	**0.290**	**0.153**	**0.561**	**0.797**	**0.846**	**0.519**

p.2.b. You are about to go into the field for a data collection mission. You decide to leave your advisor a set-up of the ANOVA in such a form that even he/she can't mess it up. Fill out following ANOVA table as simply as possible for him/her (assume he/she does know how to obtain the correct sums of squares):

Source	df	SS	MS	F	F(.05)
Grass		SSG			
Block		SSB			#N/A
G*B		SSGB			#N/A
Fert		SSF			
G*F		SSGF			
Error2		SSE2		#N/A	#N/A
Total		SSTot	#N/A	#N/A	#N/A

QF3. A split-plot experiment is to be conducted to compare 4 **nitrogen** sources and 3 time lengths of **thatch** accumulation on chlorophyll content (Y) of grass. Due to the nature of planting the nitrogen sources, they need to be planted on "large" strips of grass, while time of thatch accumulation can be applied to "smaller" strips of grass. Thus, a split-plot experiment will be conducted, with nitrogen source as the "whole plot" factor and time of thatch accumulation as the "subplot" factor. The experiment will be conducted on 2 constructed putting greens (blocks) on a university's agricultural fields. The following model is to be fit (with nitrogen source and time of thatch accumulation as fixed factors, block as random). Also note, this is not repeated measures, as different sub-plots are observed at the 3 time lengths.

$$Y_{ijk} = \mu + \alpha_i + b_j + (ab)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + \varepsilon_{ijk} \quad i = 1,\ldots,4 \quad j = 1,2 \quad k = 1,2,3$$

$$\sum_{i=1}^{4}\alpha_i = \sum_{k=1}^{3}\gamma_k = \sum_{i=1}^{4}(\alpha\gamma)_{ik} = \sum_{k=1}^{3}(\alpha\gamma)_{ik} = 0 \quad b_j \sim N\left(0,\sigma_b^2\right) \quad (ab)_{ij} \sim N\left(0,\sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$$

p.3.a. You are given the following “schematic diagram” of the layout, as well as random numbers **to be used for randomizing treatments to plots**. Fill in which treatments would be assigned to each position, where A1, would represent Nitrogen A/Thatch 1.

Treatmen	Cell1	Cell2	Cell3	Cell4	Cell5	Cell6	Cell7	Cell8	Cell9	Cell10	Cell11	Cell12
Block1												
Block2												

Nitrogen	A	B	C	D
Block1	0.057	0.865	0.135	0.524
Block2	0.340	0.514	0.198	0.807

Thatch	1	2	3	1	2	3	1	2	3	1	2	3
Block1	0.004	0.631	0.759	0.939	0.175	0.216	0.070	0.959	0.025	0.259	0.831	0.457
Block2	0.973	0.929	0.821	0.091	0.600	0.964	0.754	0.359	0.470	0.586	0.527	0.646

p.3.b. Complete the following ANOVA table:

Source	df	SS	MS	F	F(.05)
Nitrogen		37.32			
Block		0.51		#N/A	#N/A
N*B		1.26		#N/A	#N/A
Thatch		3.82			
N*T		4.15			
Error2				#N/A	#N/A
Total		48.78	#N/A	#N/A	#N/A

p.3.c. The p-values for testing for the various effects are:

Nitrogen: > 0.05 or < 0.05 Thatch: > 0.05 or < 0.05 Nit*Thatch: > 0.05 or < 0.05

p.3.d. Obtain Tukey’s HSD for comparing all Nitrogen Sources, and for comparing all Thatch Time Lengths:

p.3.d.i. Nitrogen Sources:

p.3.d.ii. Thatch Time Lengths:

QF.4. A split-plot experiment is conducted to compare 4 cooking conditions (combinations of temperature/time) and 3 recipes for quality of taste of cupcakes. Because of the logistics of the experiment, each of the 4 cooking conditions can be conducted once per day (in random order). The recipes are randomly assigned to the slots in the oven (each recipe is observed once in each cooking condition). The experiment is conducted on 5 different days (blocks). Give the Analysis of Variance (sources and degrees of freedom and critical F-values), assuming no interaction between blocks and subplot units. The response is an average taste rating among a panel of judges.

Source	Label	df	Error df	F(.05)
Whole Plot Factor				
Blocks			#N/A	#N/A
Error1			#N/A	#N/A
Sub Plot Factor				
WP*SP Interaction				
Error2			#N/A	#N/A
Total			#N/A	#N/A

QF.5. An ergonomic study was conducted as a Split-Plot design in Randomized Blocks. The response was lowest EMG signal for the Right Deltoid when a handwheel valve was being turned. The **Whole-plot** factor was the **height** of the handwheel with **4** levels (Overhead, Shoulder, Elbow, Knee). **Blocks** were **15 subjects** who turned each handwheel at each angle. The **Sub-Plot** factor was **angle** with **3** levels (90◦, 45◦,0◦). Within each subject, the heights were assigned in random order, and each angle was measured in random order for that height. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)
Height(WP)		11959			
Subj(BLK)		#N/A	#N/A	#N/A	#N/A
H*S(Err1)		39651		#N/A	#N/A
Angle(SP)		10328			
H*A(WPxSP)		117737			
Error2	112	91461		#N/A	#N/A
Total	179	#N/A	#N/A	#N/A	#N/A

The P-values for Interactions and Main effects are:

HeightxAngle: < 0.05 or > 0.05 Angle < 0.05 or > 0.05 Height < 0.05 or > 0.05

QF.6. An experiment was conducted as a Split Plot design in Randomized blocks. There were 3 recipes for cake mix, 4 cooking conditions (combinations of temperature/time), and the experiment was run on 5 days (blocks). The restriction on the randomization was that due to timing, only 4 cooking conditions could be conducted in a day. Thus, the 4 cooking conditions were randomly assigned to the order 1,2,3,4 on a given day, and all recipes were cooked together. The recipes and cooking conditions are fixed factors, while day is considered random. The response measured was an overall quality rating from a panel of judges (average across judges).

p.6.a. Which factor is the Whole-Plot Factor? ________________ The Sub-Plot Factor? ________________

p.6.b. Complete the following ANOVA Table.

Source	SumSq	df	MS	F	F(.05)
WP	679.1				
Block	3291.3			#N/A	#N/A
WP*Blk	496.2			#N/A	#N/A
SP	1589.7				
WP*SP	311.9				
Error	731.9			#N/A	#N/A
Total	7100.1	59	#N/A	#N/A	#N/A

p.6.c. Assuming the interaction is not significant, use Tukey's method to compare the 3 recipe means.

Recipe 1: $\bar{y}_1 = 56.19$ Recipe 2: $\bar{y}_2 = 54.06$ Recipe 3: $\bar{y}_3 = 44.36$

p.6.d. Assuming the interaction is not significant, use Tukey's method to compare the 4 cooking means.

Condition 1: $\bar{y}_1 = 56.65$ Condition 2: $\bar{y}_2 = 52.46$ Condition 3: $\bar{y}_3 = 48.72$ Condition 4: $\bar{y}_4 = 48.32$

QF.7. An grain study was conducted as a Split-Plot design in Randomized Blocks. The response was grain yield (kg/ha). The **Whole-plot** factor was the **nitrogen** fertilizing rate with **5** levels (0,45,90,135,180). **Blocks** were **3 Years** that the experiment was conducted in. The **Sub-Plot** factor was **rice straw** incorporation with **2** levels (absent, present).

p.7.a. Complete the following ANOVA table.

ANOVA					
Source	df	SS	MS	F	F(.05)
WP (Nit)		5693.63			
Block (Year)		216.82			
WP*Block		60.18			
SP (RiceStraw)		110.82			
WP*SP		0.96			
Error		16.53			
Total		6098.94			

p.7.b. The P-values for Interactions and Main effects are:

HeightxAngle: < 0.05 or > 0.05 Angle < 0.05 or > 0.05 Height < 0.05 or > 0.05

Nitrogen	Mean	Rice Straw	Mean
0	48.65	0	73.02
45	75.19	1	76.87
90	79.07		
135	85.85		
180	85.96		

p.7.c. Use Tukey's HSD to compare all pairs of Nitrogen levels on yield.

p.7.d. Obtain a 95% CI for the difference in true means for Rice Straw Present - Absent

Part G: Repeated Measures Designs

QG.1. A researcher is interested in comparing 4 diet plans. She selects 160 subjects and randomly assigns 40 subjects to each diet. She will measure their weight loss at 3 time points over the course of the year. Her analysis of variance will have the following sources of variation. Give her degrees of freedom for each source (actual numbers, not symbols)

Source	Degrees of freedom
Diets	
Subjects(Diet) --- Error1	
Time Points	
Diets*Time	
Time*Subjects(Diet) --- Error2	
Total	

QG.2. A repeated measures experiment was conducted to compare two treatments (zylkene and placebo) for cat anxiety. A total of 34 cats with anxiety were obtained, and randomized such that 17 received zylkene and 17 received placebo. Each cat was observed at 5 time points, and a global score of emotional state was observed (high scores are better). The following model is fit:

$$y_{ijk} = \mu + \alpha_i + b_{j(i)} + \tau_k + (\alpha\tau)_{ik} + \varepsilon_{ijk} \quad i=1,2 \ j=1,...,17 \ k=1,...,5$$

p.2.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)
Trt		383			
Cat(Trt)		2132			
Time		324			
Time*Trt		51			
Error2					
Total		3278			

p.2.a.i. The p-value for testing no time*treatment interaction is < 0.05 or > 0.05

p.2.a.ii. The p-value for testing no time main effect is < 0.05 or > 0.05

p.2.a.iii. The p-value for testing no treatment main effect is < 0.05 or > 0.05

p.2.b. Ignoring any potential interaction, obtain a 95% Confidence Interval for the difference between Zyrtec and placebo effects (their means are 13.59 and 10.59, respectively).

QG.3. A repeated measures experiment was conducted to compare three battery recycling promotion strategies (humorous, factual, and control (no promotion)) for battery recycling (y=percent change from pre-ad recycling levels). A total of 21 stores were obtained, and randomized such that 7 received each strategy. Each store was observed at 8 time points, and a y was observed (negative scores imply lower levels than pre-treatment). The following model is fit:

$$y_{ijk} = \mu + \alpha_i + b_{j(i)} + \tau_k + (\alpha\tau)_{ik} + \varepsilon_{ijk} \quad i = 1,2,3 \; j = 1,...,7 \; k = 1,...,8 \quad \text{with: } \overline{y}_1 = -7.4 \quad \overline{y}_2 = 35.8 \quad \overline{y}_3 = -11.9, \quad \overline{y} = 5.5$$

p.3.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)
Strategy					
Store(Strategy)		10000			
Time		10780			
Time*Strategy		42000			
Error2		252000			
Total					

p.3.a.i. The p-value for testing no time*strategy interaction is < 0.05 or > 0.05

p.3.a.ii. The p-value for testing no time main effect is < 0.05 or > 0.05

p.3.a.iii. The p-value for testing no strategy main effect is < 0.05 or > 0.05

p.3.b. Ignoring any potential interaction, obtain Bonferroni's Minimum significant difference, and compare all pairs of strategies.

QG.4. A study compared doses of a drug on female rats' activity levels in a maze. A sample of 91 rats were selected, and randomized such that 21 rats received the Control Dose, 25 received Low Dose, 24 received Medium Dose, and 21 received High Dose. Each rat's activity levels were observed at 4 time points after dosing (15, 30, 45, and 60 minutes). Hint: There are a total of 21+25+24+21=91 rats in the study.

p.4.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)	Significant?
Dose		16890				
Rat(Dose)		176677		#N/A	#N/A	#N/A
Time		176765				
Dose*Time		2161				
Error2		92826		#N/A	#N/A	#N/A
Total	363	465319	#N/A	#N/A	#N/A	#N/A

p.4.b. The means for each dose/time combination are given below. Use Bonferroni's method to compare all pairs of doses.

Trt	n_dose	Time1	Time2	Time3	Time4	Mean
Control	21	165.9	136.3	121.4	103.4	131.8
LowDose	25	167.8	148.4	123.7	118.4	139.6
MidDose	24	168.3	140.3	117.2	109.8	133.9
HighDose	21	184.4	156.2	136.3	122.7	149.9
Sum/Mean	91	171.3	145.3	124.4	113.7	138.7

Note: When comparing doses *i* and *j*, the standard error of the difference between dose means is:

$$SE\left\{\overline{Y}_{i\bullet\bullet} - \overline{Y}_{j\bullet\bullet}\right\} = \sqrt{MS_{Subjects(Trts)}\left(\frac{1}{n_i t} + \frac{1}{n_j t}\right)}$$

Values of $\left(\frac{1}{n_i t} + \frac{1}{n_j t}\right)$ are given below

i,j	1,2	1,3	1,4	2,3	2,4	3,4
1/tni+1/tnj	0.0219	0.0223	0.0238	0.0204	0.0219	0.0223

QG.5. A repeated measures design was used to compare the effects of Zylkene and Selgian Anipryl in dogs with anxiety disorders. There were 38 dogs, randomly assigned to the treatments (19 dogs per treatment). Each dog was measured at 5 time points, with a scale that has lower scores are better outcomes than higher scores).

p.5.a. Complete the following table.

Source	df	SS	MS	F	F(.05)	Significant?
Treatment		8.85				
Dog(Treatment)		2020.42		#N/A	#N/A	#N/A
Time		1573.45				
Time*Treatment		20.44				
Error2 (Time*Dog(Trt))		1212.11		#N/A	#N/A	#N/A
Total		4835.27	#N/A	#N/A	#N/A	#N/A

p.5.b. Assuming no significant interaction, obtain a 95% Confidence Interval for $\mu_Z - \mu_{SA}$. Note that the sample means are 19.99 and 19.56 for Zylkene and Selgian Anipryl, respectively.

p.5.c. Assuming no significant interaction, use Bonferroni's method to compare all pairs of Time Means. The sample means are: $Time_1 = 24.58$ $Time_2 = 21.21$ $Time_3 = 19.00$ $Time_4 = 17.63$ $Time_5 = 16.45$

$Time_5$ $Time_4$ $Time_3$ $Time_2$ $Time_1$

QG.6. A repeated measures design was conducted to compare 3 treatments for dry skin (Placebo, WPLC-O, WPLC-P). The study had a total of 60 subjects who were randomized so that 20 subjects received Placebo, 20 received WPLC-O, and 20 received WPLC-P. Each subject was measured on 3 days (Days 15, 30, 60) for the respons skin hydration.

p.6.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)
Trts		3475			
Subj(Trt)		3245		#N/A	#N/A
Time		575			
TrtxTime		128			
Error2	114	4868		#N/A	#N/A
Total	179	12291	#N/A	#N/A	#N/A

p.6.b Are the following effects significant? TrtxTime Interaction __________ Trts _________ Time _______

p.6.c. The means for the Treatments are: Placebo: 47.4 WPLC-O: 57.3 WPLC-P: 55.9. Use Bonferroni's method to compare all pairs of treatments (assuming no interaction).

Bonferroni MSD: ______________________________ Placebo WPLC-P WPLC-O

QG.7. A study was conducted among obese Thai subjects on the effect of drinking green tea on weight. There were a total of 60 subjects, and were randomized so that 30 drank green tea and 30 received a placebo. Each subject's weight was measured in kilograms at 4, 8, and 12 weeks after intervention. The model was conducted as a Repeated Measures design with 2 treatments, 30 subjects per treatment, and 3 time points.

p.7.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)
Treatment		617.1605			
Subject(Trt)		11388.05		#N/A	#N/A
Time		67.357			
Trt*Time		47.221			
Error2		3796.017		#N/A	#N/A
Total		15915.81	#N/A	#N/A	#N/A

p.7.b. Is there a significant treatment by time interaction? Yes / No

p.7.c. Is there a significant treatment main effect? Yes / No

p.7.d. Is there a significant time main effect? Yes / No

p.7.e. The mean weights for the green tea and placebo groups across time points are $\overline{y}_g = 66.34$ $\overline{y}_p = 70.04$

Compute a 95% Confidence Interval for the difference in their effects on weight.

<u>Part H: General Design Identification Problems</u>

QH.1. For the following problems, identify the factor(s), state whether they are fixed or random, give the analysis of variance table, with sources of variation and degrees of freedom, symbolic F-ratios and critical values (just give degrees of freedom) for relevant significance tests for treatment factors.

p.1.a. An ergonomic study was conducted to compare 6 car seat designs in terms of an overall comfort index (Y). A sample of 12 subjects was selected, and each subject rated each car seat one time.

p.1.b. A food preference study was interested in the main effects and interactions among two factors on subjects' ratings of attractiveness of a plate of food. The factors under study were plate color (monochrome, color) and balance of food placement on the plate (symmetric (balanced), asymmetric (unbalanced)). A sample of 68 subjects were selected, and randomized such that 17 received each combination of color and balance. Each person only rated one plate.

p.1.c. A study was conducted to compare 6 models of bread machines on quality of baked bread. There were 6 varieties of bread, and 6 chefs, and each variety was made by each machine once, and each chef used each machine once. The response was an overall quality rating based by a panel of judges (which was combined to a single rating).

p.1.d. A study was conducted to measure the reliability of collegiate gymnastics judges, and variation in gymnast skills. A sample of 8 judges was selected, and a sample of 4 gymnasts was selected. Each gymnast was filmed on 3 occasions, and each judge rated the 3 videos.

QH.2. For the following problems, identify the factor(s), state whether they are fixed or random, give the analysis of variance table, with sources of variation and degrees of freedom, symbolic F-ratios and critical values for $\alpha = 0.05$ significance level tests.

p.2.a. A researcher is interested in studying the variation in laboratories measuring the levels of nutrient in batches of raw materials. Her department has contracts with 6 laboratories, and she obtains a random sample of 5 batches of the raw material, dividing each batch into 24 sub-batches. She sends each laboratory 4 randomly chosen sub-batches from each of the 5 batches and has each laboratory measure the nutrient levels in each of their 20 assigned sub-batches.

p.2.b. A study measured emulsion properties when different types of plant oils (soybean, hazelnut, canola, sunflower, corn, cotton, and olive) are applied to different types of meat (chicken, beef, and turkey). Each type of oil was applied to each type of meat, and there were 2 replicates per treatment (combination of plant oil and meat type).

p.2.c. An experiment is conducted in a field to measure the effects among 5 seeding rates in an experimental field that is set up on plots set in a 5x5 array with 5 rows and 5 columns. The rates are applied to the field such that each rate is applied once on each row and once on each column.

p.2.d. A study compares 4 popular diets on weight loss. A sample of 160 overweight subjects are obtained and assigned at random, such that each diet has 40 subjects. Weight loss over 30 days is measured.

QH.3. For the following problems, identify the factor(s), state whether they are fixed or random, give the analysis of variance table, with sources of variation and degrees of freedom, symbolic F-ratios and critical values for $\alpha = 0.05$ significance level tests.

p.3.a. An experiment was conducted to determine the effects of 2 factors on growth of cucumbers in a greenhouse. Factor A was irrigation method (Furrow and Subsurface Drip) and Factor B was post-irrigation aeration level (0, 0.50, 0.75, 1). Due to the nature of the experimental set-up, the experiment was conducted as a Split-Plot Design with irrigation method as the Whole-plot factor, and aeration level as the Sub-plot factor. The experiment was conducted in 5 blocks. The response was cucumber length.

p.3.b. A steel experiment was conducted to compare 3 levels of titanium carbon content. Two blocking factors were included: sliding velocity and applied load (each with 3 levels). Each titanium carbon content was applied to each sliding velocity and to each applied load once. The response was wear rate.

p.3.c. A study compared the swimming speeds of male and female zebra-fish at 4 rearing temperatures. The experimenters raised 48 males and 48 females, 12 of each gender at each of the 4 temperatures (22C, 25, 28, and 31). The response is relative critical swimming speed.

QH.4. For the following scenarios, give the sources of variation, their degrees of freedom, appropriate F-ratios, and critical values for the F-tests.

p.4.a. A mock jury experiment was conducted among college students to study the effects of 2 factors on judgments of witness effectiveness (Y). The factors were: Defendant's group identity (Factor A: In-group (similar background to students), Neutral (no information given), Out-Group (member of a radical political group)) and Eyewitness Testimony (Factor B: Consistent during cross-examiniation, Inconsistent). There were a total on N = 180 students in the experiment, with them randomized so that 30 were assigned to each of 6 combinations of levels of Factors A and B in a Completely Randomized Design (each subject was in exactly one condition).

Source	DF	F-ratio	F(.05)

p.4.b. An engineer is interested in variation in products and operator measurements in a factory. She samples 5 parts and 4 employees within her factory and has each engineer measure each part 3 times, in random order. The response is the measurement of the part.

Source	DF	F-ratio	F(.05)

p.4.c. A large juice producer makes 3 varieties of juice (Orange, Lemon, and Apple). For each variety, the company has many farms that provide fruit of that specific variety (no farm produces fruits of more than one variety). They are interested in measuring sugar concentration in fruit (Y). Random samples of 4 farms are selected from each variety, with 20 fruits being sampled from each farm, and sugar concentration is measured on each fruit.

Source	DF	F-ratio	F(.05)