

**Note: Conduct all individual tests at  $\alpha=0.05$  significance level, and all multiple comparisons at an experiment-wise error rate of  $\alpha_E = 0.05$ .**

Q.1. A study compared antioxidant activity of  $t = 8$  brands of craft beer in a 1-Way ANOVA. One response reported was DPPH radical scavenging activity. Each brand was had  $n = 3$  replicates measured.

Brand	Mean	SD
L	794.9	27.5
P	376	32.4
W	706.2	30.9
B9	586.5	17.7
N	864.6	42.4
R	670.6	19.9
T	1310.3	19
E	679.2	25

$$\sum_{i=1}^t (\bar{y}_{i.} - \bar{y}_{..})^2 = 508911.8 \quad \sum_{i=1}^t s_i^2 = 6253.88$$

p.1.a. Complete the following Analysis of Variance table used to test  $H_0: \mu_1 = \dots = \mu_8$

Source	df	SS	MS	F_obs	F(.05)
Brand					
Error					
Total					

p.1.b. Do we reject the null hypothesis, and conclude the population means differ among the brands? **Yes** or **No**

p.1.c. Compute Tukey's minimum significant difference and determine which brands are significantly different.

**P      B9      R      E      W      L      N      T**

Q.2. An experiment had 2 factors, each with 2 levels: Factor A:Instructional Method (Standard and Enhanced), and Factor B: Instructional Medium (Desktop and Mobile Device). There were a total of N = 88 subjects, with n = 22 receiving each treatment (combination of method and medium). The sample means and standard deviations of scores on a transfer test are given below.

Mean	Medium				SD	Medium	
Method	Desktop	Mobile	Overall		Method	Desktop	Mobile
Standard	2.58	2.36	2.47		Standard	1.93	1.76
Enhanced	4.04	4.34	4.19		Enhanced	3.04	2.67
Overall	3.31	3.35	3.33				

The model fit is a 2-Way fixed effects ANOVA with interaction.  $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$

p.2.a. Compute the sums of squares for: Method, Medium, and complete the following ANOVA table.

Source	df	SS	MS	F_obs	F(0.05)
Method					
Medium					
M*M		1.4872			
Error		487.053			
Total					

p.2.b. Do you reject the hypothesis:  $H_0^{AB} : (\alpha\beta)_{11} = (\alpha\beta)_{12} = (\alpha\beta)_{21} = (\alpha\beta)_{22} = 0$     Yes / No

p.2.c. Do you reject the hypothesis:  $H_0^A : \alpha_1 = \alpha_2 = 0$     Yes / No

p.2.d. Do you reject the hypothesis:  $H_0^B : \beta_1 = \beta_2 = 0$     Yes / No

Q.3. A study compared  $t = 4$  warm-up protocols in terms of vertical jump ability in dancers. There were  $b = 10$  dancers, each dancer was measured under each warm-up protocol and the experiment is a Randomized Block Design with dancers as blocks.

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} = \mu_i + \beta_j + \varepsilon_{ij}$$

The treatments and their means are: Static Stretch: 38.0    Dynamic Stretch: 41.4    Static&Dynamic Stretch: 41.0    Control: 37.8

p.3.a. Complete the following ANOVA table.

Source	df	SS	MS	F_obs	F(.05)
<b>Treatment (Warm-up)</b>					
<b>Block (Dancer)</b>		<b>850.6</b>			
<b>Error</b>		<b>42.0</b>			
<b>Total</b>					

p.3.b. Do you reject  $H_0 : \alpha_1 = \dots = \alpha_4 = 0$  ( $\mu_1 = \dots = \mu_4$ )?    **Yes** /    **No**

p.3.c. Compute the Relative Efficiency of the RCB to the Completely Randomized Design. How many subjects would be needed per treatment to have the same standard error of a treatment (warm-up protocol) in a CRD?

Relative Efficiency \_\_\_\_\_ # of Subjects per treatment \_\_\_\_\_

p.3.d. Compute Bonferroni's minimum significant difference and determine which treatments are significantly different.

Control

Static

Static&Dynamic

Dynamic

Q.4. An engineering experiment was conducted to measure variation in semiconductors for a particular measurement. A sample of 5 lots (batches) of semiconductors was selected. Within each lot, 2 wafers were sampled. Each wafer was measured at 9 random sites (these are replicates). Note that the wafers are nested within lots. Both lots and wafers are random effects.

$$Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad \alpha_i \sim N(0, \sigma_a^2) \quad \beta_{j(i)} \sim N(0, \sigma_b^2) \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

p.4.a. Complete the following Analysis of Variance table.

Source	df	SS	MS	F_obs	F(.05)
<b>Lot</b>		<b>1698.5</b>			
<b>Wafer(Lot)</b>		<b>272.2</b>			
<b>Error</b>		<b>1803.8</b>			
<b>Total</b>					

p.4.c. Do you reject the hypothesis:  $H_0^A : \sigma_a^2 = 0$ ?    **Yes** / **No**

p.4.d. Do you reject the hypothesis:  $H_0^B : \sigma_b^2 = 0$ ?    **Yes** / **No**

p.4.e. For the nested design, with random factors A, and B(A), we have:

$$E\{MSE\} = \sigma^2 \quad E\{MSB(A)\} = \sigma^2 + n\sigma_b^2 \quad E\{MSA\} = \sigma^2 + n\sigma_b^2 + bn\sigma_a^2$$

Obtain unbiased estimates for the 3 variances:

$$\hat{\sigma}^2 = \underline{\hspace{10em}} \quad \hat{\sigma}_b^2 = \underline{\hspace{10em}} \quad \hat{\sigma}_a^2 = \underline{\hspace{10em}}$$

Q.5. For the following problems, identify the factor(s), state whether they are fixed or random, give the analysis of variance table, with sources of variation and degrees of freedom, symbolic F-ratios and critical values for  $\alpha = 0.05$  significance level tests.

p.5.a. An experiment was conducted to determine the effects of 2 factors on growth of cucumbers in a greenhouse. Factor A was irrigation method (Furrow and Subsurface Drip) and Factor B was post-irrigation aeration level (0, 0.50, 0.75, 1). Due to the nature of the experimental set-up, the experiment was conducted as a Split-Plot Design with irrigation method as the Whole-plot factor, and aeration level as the Sub-plot factor. The experiment was conducted in 5 blocks. The response was cucumber length.

p.5.b. A steel experiment was conducted to compare 3 levels of titanium carbon content. Two blocking factors were included: sliding velocity and applied load (each with 3 levels). Each titanium carbon content was applied to each sliding velocity and to each applied load once. The response was wear rate.

p.5.c. A study compared the swimming speeds of male and female zebra-fish at 4 rearing temperatures. The experimenters raised 48 males and 48 females, 12 of each gender at each of the 4 temperatures (22C, 25, 28, and 31). The response is relative critical swimming speed.

Q.6. For each of the following scenarios compute the appropriate minimum significant difference for comparing (fixed) treatment means:

p.6.a. Two-way ANOVA: Factor A Fixed,  $a = 4$ , Factor B Random,  $b = 5$ ,  $n=3$  replicates per combination of factors A and B.

$$SSA = 600 \quad SSB = 1000 \quad SSAB = 600 \quad SSE = 1600$$

Compute Tukey's HSD for comparing levels of factor A.

p.6.b. Nested 2-Way ANOVA: Factor A Fixed,  $a=3$ , Factor B Fixed,  $b=3$ ,  $n=5$  replicates cell.

$$SSA = 120 \quad SSB(A) = 180 \quad SSE = 144$$

Compute Bonferroni's MSD for comparing levels of factor A.

Q.7. A study classified a sample of French Ski resorts into 3 classifications (large, medium, and small) based on their volume of business. The researchers obtained a measure of each resort's Luenberger Productivity Index (LPI) was obtained. The authors conducted a Kruskal-Wallis test to test whether population median LPI scores differ by resort size group. The numbers and rank sums for each resort size group are given below.

Size	n	RankSum
Large	16	428
Medium	31	932
Small	17	720

Test Statistic: \_\_\_\_\_ Rejection Region \_\_\_\_\_ P-value is  $> 0.05$  or  $< 0.05$