STA 6167 – Exam 2 – Spring 2017 – **PRINT** Name ______________________

For all significance tests, use α = 0.05 significance level.

Q.1. An experiment was conducted comparing various treatments (involving various hydrocolloids and amounts of wheat flower) with the goal of reducing oil content in a food product. The experiment was conducted in separate replicates (blocks). One response measured was Oil Content of the sample. The partial ANOVA table is given below.

Source	df	SS	MS	F	F(.05)
Treatments	12	261.146			
Blocks	2	0.523		#N/A	#N/A
Error		0.689		#N/A	#N/A
Total		262.358	#N/A	#N/A	#N/A

p.1.a. Complete the table. Is the P-value for testing H_0: No Treatment Effect > 0.05 or < 0.05

p.1.b. Give the number of Treatments and number of Blocks in the experiment. # Trts = _______ # Blks = ________

p.1.c. What is the estimated standard error of the difference between any 2 treatment means? $SE\left\{\overline{Y}_{i\bullet} - \overline{Y}_{j\bullet}\right\}$

p.1.d. Suppose we wish to use Scheffe's method to compare all pairs of treatment means. What would be the minimum significant difference?

Q.2. An experiment was conducted to determine the effect of g = 3 different food portion/container sizes on food intake in a Completely Randomized Design. There were a total of N = 90 subjects who were randomized so that 30 received each condition (each subject was observed in one of the 3 conditions). The conditions were: 1= medium portion/small container, 2 = medium portion/large container, 3 = large proportion/large continer. The response was food intake (Y, in grams) that the subject consumed while watching a television show. The model and summary statistics are given below.

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} = \mu_i + \varepsilon_{ij} \quad n_1 = 30,\ \overline{y}_{1\bullet} = 30,\ s_1 = 30 \quad n_2 = 30,\ \overline{y}_{2\bullet} = 69,\ s_2 = 44 \quad n_3 = 30,\ \overline{y}_{3\bullet} = 60,\ s_3 = 45$$

p.2.a. Compute the Between Treatment Sum of Squares (SST) and Within Treatment Sum of Squares (SSE).

SST = ____________________________________ SSE = ____________________________________

p.2.b. Test $H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$

Test Statistic: ________________________ Rejection Region ______________________ P-value > or < 0.05

p.2.c. Use Tukey's method to compare all pairs of treatments.

Tukey's W = ______________________ Trt1 Trt3 Trt2

Q.3. An experiment was conducted to compare a = 3 theories for the apparent modulus of elasticity (Y) of b = 3 apple varieties. The 3 theories were: Hooke's, Hertz's, and Boussineq's; the 3 apple varieties were: Golden Delicious, Red Delicious, and Granny Smith. The researchers determined the elasticity for r = 15 apples based on each combination of theory and variety. For the purposes of this experiment, each factor is fixed.

Model: $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim NID(0, \sigma^2) \quad \sum_{i=1}^{a} \alpha_i = \sum_{j=1}^{b} \beta_j = \sum_{i=1}^{a} (\alpha\beta)_{ij} = \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0$

$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk} - \overline{y}_{ij\bullet}\right)^2 = 17.095 \quad \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk} - \overline{y}_{\bullet\bullet\bullet}\right)^2 = 113.119 \quad bn\sum_{i=1}^{a}\left(\overline{y}_{i\bullet\bullet} - \overline{y}_{\bullet\bullet\bullet}\right)^2 = 57.987 \quad an\sum_{j=1}^{b}\left(\overline{y}_{\bullet j\bullet} - \overline{y}_{\bullet\bullet\bullet}\right)^2 = 35.779$

Cell Means	GoldenDelicious	RedDelicious	GrannySmith	Row Mean
Hooke	2.68	3.46	4.23	3.457
Hertz	2.44	3.06	3.84	3.113
Boussinesq	1.53	1.89	2.36	1.927
Column Mean	2.217	2.803	3.477	2.832

Complete the following Analysis of Variance Table, and test for interaction effects and main effects.

Source	df	SS	MS	F	F(.95)	P-value
Theory						>0.05 or <0.05
Variety						>0.05 or <0.05
Theory*Variety						>0.05 or <0.05
Error				#N/A	#N/A	#N/A
Total			#N/A	#N/A	#N/A	#N/A

Q.4. A latin square design was used to test for treatment effects among 5 mixes of concrete in terms of tensile strenth. There were 5 molds, and 5 workers who made and poured the concrete molds. The design is shown below.

	Worker1	Worker2	Worker3	Worker4	Worker5
Mold1	Mix1	Mix2	Mix3	Mix4	Mix5
Mold2	Mix2	Mix3	Mix4	Mix5	Mix1
Mold3	Mix3	Mix4	Mix5	Mix1	Mix2
Mold4	Mix4	Mix5	Mix1	Mix2	Mix3
Mold5	Mix5	Mix1	Mix2	Mix3	Mix4

p.4.a. The mix means are: 70, 65, 50, 80, and 75 for Mixes 1-5, respectively. The error sum of squares is *SSE* = 600. Use Bonferroni's method to compare all pairs of Mix means.

Bonferoni's MSD: ____________________ Mix3 Mix2 Mix1 Mix5 Mix4

p.4.b. The sums of squares for molds and workers are *SSR* = 1000 and *SSC* = 400, respectively. Compute the Relative Efficiency of the Latin Square design, relative to the Completely Randomized Design.

RE(LS,CR) = ____

Q.5. A 2-Way Random Effects model is fit, where a sample of a = 8 products were measured by a sample of b = 6 machinists, with r = 3 replicates per machinist per product. The model fit is as follows (independent random effects):

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \alpha_i \sim NID\left(0,\sigma_a^2\right) \quad \beta_j \sim NID\left(0,\sigma_b^2\right) \quad (\alpha\beta)_{ij} \sim NID\left(0,\sigma_{ab}^2\right) \quad \varepsilon_{ijk} \sim NID\left(0,\sigma^2\right)$$

You are given the following sums of squares: $SSA = 420 \quad SSB = 350 \quad SSAB = 140 \quad SSE = 210$

Give the test statistic and rejection region for the following 3 tests. Note for test 1, your rejection region will be symbolic, give the specific numerator and denominator degrees of freedom. Also give unbiased (ANOVA) estimates of each variance component.

1) $H_0^{AB}: \sigma_{ab}^2 = 0 \quad H_A^{AB}: \sigma_{ab}^2 > 0$ 2) $H_0^{A}: \sigma_{a}^2 = 0 \quad H_A^{A}: \sigma_{a}^2 > 0$ 3) $H_0^{B}: \sigma_{b}^2 = 0 \quad H_A^{B}: \sigma_{b}^2 > 0$

1: Test Stat: ____________________________ Rejection Region: _________________ Estimate: ________________

2: Test Stat: ____________________________ Rejection Region: _________________ Estimate: ________________

3: Test Stat: ____________________________ Rejection Region: _________________ Estimate: ________________

Q.6. A wildlife researcher is interested in comparing levels of a chemical in the water among the 4 lakes in a state park. The lakes are broken into many subsections based on a survey. She samples 3 subsections from each lake at random, and takes water measurements at 8 sites within each subsection. A laboratory measures the chemical in each of the water specimens. The lake means are: 76, 72, 60, and 64, respectively. The model is:

$$Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk} \quad \sum_{i=1}^{a} \alpha_i = 0 \quad \beta_{j(i)} \sim N\left(0, \sigma_b^2\right) \quad \varepsilon_{ijk} \sim N\left(0, \sigma^2\right)$$

p.6.a. Compute the sum of squares, degrees of freedom, and mean square for lakes (factor A).

SSA = ______________________ df_A = ________________________ MSA = ____________________

The sum of squares for subsections (factor B) nested within lakes (factor A) is SSB(A) = 640, and the error sum of squares is SSE = 2520.

p.6.b. Test $H_0 : \alpha_1 = ... = \alpha_4 = 0$

Test Statistic: _________________________ Rejection Region: ______________________ P-value < or > .05

p.6.c. Test $H_0 : \sigma_b^2 = 0 \quad H_A : \sigma_b^2 > 0$

Test Statistic: _________________________ Rejection Region: ______________________ P-value < or > .05

Q.7. Consider the following 3 scenarios for a (Fixed Effects) Completely Randomized Design.

$$y_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,2,3;\ j = 1,\dots,n \quad \varepsilon_{ij} \sim N\left(0,\sigma^2\right)$$

$a)\ \mu_1 = 80, \mu_2 = 100, \mu_3 = 120, \sigma = 20, n = 5 \quad b)\ \mu_1 = 90, \mu_2 = 100, \mu_3 = 110, \sigma = 10, n = 3 \quad c)\ \mu_1 = 95, \mu_2 = 100, \mu_3 = 105, \sigma = 5, n = 7$

Rank the from smallest to largest in terms of $\dfrac{E\{MST\}}{E\{MSE\}}$

Smallest: __________________________ Middle: _______________________ Largest: ______________________

Q.8. A delivery company is considering buying one of 3 drones for deliveries. They fly each drone 12 times, measuring the distance from the landing point to the target. Due to the skewed distribution of the distances, they use the non-parametric Kruskal-Wallis procedure to test for differences among the drones' true medians. The rank sums are 200, 218, and 248 for the 3 drones. Test $H_0: M_1 = M_2 = M_3$.

Test Statistic: _________________________ Rejection Region: ______________________ P-value < or > .05

Q.9. A researcher is using a latin square design to compare 4 brands of car tires in terms of miles driven before reaching a given wear level. For one blocking factor they use tire position (Driver Front, Passenger Front, Driver Rear, Passenger Rear). They choose to use 12 cars as the other blocking factor. Note that each brand will be on each car once, and on each tire position 3 times. What will be the error degrees of freedom?

Q.10. Unless the number of treatments is 2, Tukey's HSD (W) will always be smaller that Bonferroni's MSD (B) for a given set of data. **True / False**

Q.11. A 2-Way (crossed) ANOVA is used to measure the effects of 2 factors, each at 3 levels. There are 4 replicates for each treatment (combination of levels of factors A and B). There is a significant interaction between the 2 factors, so the researchers choose to use Tukey's method to compare all pairs of treatment means. Give Tukey's W for comparing all pairs of means, with MSE = 100.