Analysis of Covariance Problems

Q.1. A study is conducted to compare the effects of 5 methods of practicing to play the trombone among college band trombone players. A sample of 30 trombone players is obtained, 6 are assigned at random to each of the 5 methods of practicing. Baseline measures of ability (X) are obtained as well as a post-practice score (Y). The researchers find no interaction between the effects of baseline score and method of practicing. The estimated regression equation is:

$$\hat{Y} = 7.4 + 0.7X + 4.5M_1 + 1.9M_2 + 6.7M_3 + 2.8M_4 \qquad SSE = 118$$

where $M_1,\ldots,M_4$ are dummy variables for methods 1, 2, 3, and 4, respectively.

- Give the adjusted means for methods 1 and 5, where the overall mean practice score is 24.2

- How large would SSE have to be for the model $E(Y)=\alpha+\beta X$, for us to conclude that the methods of practicing effects are not all equal?

Q.2. An Analysis of Covariance is conducted to compare three exercise regimens with respect to conditioning. Each participant is given a test to measure their baseline strength prior to training (X). Out of the 30 participants, 10 are assigned to method 1 ($Z_1=1$, $Z_2=0$), 10 are assigned to method 2 ($Z_1=0$, $Z_2=1$), and the remaining 10 receive method 3 ($Z_1=0$, $Z_2=0$). After training, each participant is given a test of their strength (Y) .

- Model 1: $E(Y) = \alpha + \beta X$ $\qquad R_1^2 = 0.204$
- Model 2: $E(Y) = \alpha + \beta X + \beta_1 Z_1 + \beta_2 Z_2$ $\qquad R_2^2 = 0.438$
- Model 3: $E(Y) = \alpha + \beta X + \beta_1 Z_1 + \beta_2 Z_2 + \gamma_1 XZ_1 + \gamma_2 XZ_2$ $\qquad R_3^2 = 0.534$

a) Test whether the slopes relating Y to X differ for the three exercise regimen.
i. Null /Alternative Hypotheses:
ii. Test Statistic:
iii. Rejection Region/Conclusion:
b) Based on Model 2, test whether the three regimens differ, after controlling for X
i. Null / Alternative Hypotheses:
ii. Test Statistic:
iii. Rejection Region/Conclusion:
c) Give the adjusted means for the 3 regimens for Model 2. (X-bar=29)

Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	18.088	7.382		2.450	.021
	X	.676	.252	.452	2.683	.012
2	(Constant)	17.258	7.639		2.259	.032
	X	.721	.238	.482	3.030	.005
	Z1	3.224	2.590	.228	1.245	.224
	Z2	-4.650	2.503	-.328	-1.857	.075
3	(Constant)	9.456	11.772		.803	.430
	X	.970	.373	.649	2.604	.016
	Z1	28.333	15.588	2.002	1.818	.082
	Z2	-12.390	17.222	-.875	-.719	.479
	XZ1	-.885	.526	-1.735	-1.684	.105
	XZ2	.299	.577	.606	.519	.609

a. Dependent Variable: Y

Q.3. An Analysis of Covariance is conducted to compare the readability of 3 newspaper writers' articles. The researcher samples 4 articles each from the 3 journalists (thus, 12 total articles), and assesses a readability index (Y) to each article. As a covariate, the length of the article (X, in 100s of words) is obtained. The following 3 models are fit, where Z_1=1 if writer 1, 0 otherwise; and Z_2=1 if writer 2, 0 otherwise:

- Model 1: $E(Y) = \alpha + \beta X$ $\quad R_1^2 = 0.10$
- Model 2: $E(Y) = \alpha + \beta X + \beta_1 Z_1 + \beta_2 Z_2$ $\quad R_2^2 = 0.60$ $\qquad \hat{Y} = 20 - 0.2X + 10Z_1 - 5Z_2$
- Model 3: $E(Y) = \alpha + \beta X + \beta_1 Z_1 + \beta_2 Z_2 + \gamma_1 XZ_1 + \gamma_2 XZ_2$ $\quad R_3^2 = 0.66$

p.3.a. Test H_0: No interaction between writer and article length ($\gamma_1 = \gamma_2 = 0$)

p.3.a.i. Test Statistic:

p.3.a.ii. Reject H_0 if the test statistic falls in the range ______________________________

p.3.b. Assuming no interaction, Test H_0: No writer effect, controlling for article length ($\beta_1 = \beta_2 = 0$)

p.3.b.i. Test Statistic:

p.3.b.ii. Reject H_0 if the test statistic falls in the range ______________________________

p.3.c. Based on model 2, give the adjusted means for each writer (X-bar = 10)

p.3.c.i. Writer 1 ____________ p.3.c.ii. Writer 2 ______________ p.3.c.iii. Writer 3 ____________

Q.4. A study was conducted to compare Y=Average Daily Weight Gain (Kg) under 2 Grazing Conditions (Z=1 if Continuous, 0 if Rotated), adjusting for a Covariate X = Stock Rate (Animals/hectare). Three regression models were fit:

Model 1: $E\{Y\} = \beta_0 + \beta_1 X$ $\qquad$ Model 2: $E\{Y\} = \beta_0 + \beta_1 X + \gamma_1 Z$ $\qquad$ Model 3: $E\{Y\} = \beta_0 + \beta_1 X + \gamma_1 Z + \delta_1 XZ$

The results are given below. Answer the following questions:

ANOVA	Model1		ANOVA	Model2		ANOVA	Model3	
	df	*SS*		*df*	*SS*		*df*	*SS*
Regressior	1	0.1254	Regressior	2	0.1344	Regressior	3	0.1354
Residual	13	0.0338	Residual	12	0.0248	Residual	11	0.0238
Total	14	0.1592	Total	14	0.1592	Total	14	0.1592
	Coefficients	*Standard Error*		*Coefficients*	*Standard Error*		*Coefficients*	*Standard Error*
Intercept	1.1372	0.1042	Intercept	1.1876	0.0959	Intercept	1.1463	0.1166
X=SR	-0.0658	0.0095	X=SR	-0.0677	0.0085	X=SR	-0.0639	0.0104
			Z=CG	-0.0505	0.0241	Z=CG	0.0856	0.2081
						XZ	-0.0125	0.0190

p.4.a. Overall sample size = ______ p.4.b. Proportion of variation in Y explained by model 3 = ________________

p.4.c. Bivariate correlation between Y and X = ____

p.4.d. Test whether Grazing Conditions are significantly different, controlling for Stock Rate: H_0: ________ H_A: ______

Test Statistic: ________________________ Reject H_0 if the test statistic falls in the range __________________

Q.5. An Analysis of Covariance is conducted to compare two methods (treatment and control) of sight-singing among 4th grade children. Each participant is given a test to measure their baseline skill prior to training (X). Out of the 40 participants, 20 are assigned to experimental treatment (Z_1=1), 20 are assigned to control (Z_1=0). Post-training scores were obtained for each child (Y) . The total sum of squares is TSS = 10522.

- Model 1: $E(Y) = \alpha + \beta X$ $\qquad R_1^2 = .267$
- Model 2: $E(Y) = \alpha + \beta X + \beta_1 Z_1$ $\qquad R_2^2 = .363$
- Model 3: $E(Y) = \alpha + \beta X + \beta_1 Z_1 + \gamma_1 X Z_1$ $\qquad R_3^2 = .432$

p.5.a. Test whether the slopes relating Y to X differ for the treatment conditions. H_0: ___________________

Test Statistic __________________________________ Rejection Region: ______________________ Reject H_0 ? ______

p.5.b. Based on Model 2, test whether the two treatments differ, after controlling for X H_0: ___________________

Test Statistic __________________________________ Rejection Region: ______________________ Reject H_0 ? ______

p.5.c. Based on Model 2, give the adjusted means for the 2 treatments. (X-bar=8.0)

	Coefficients	tandard Err	t Stat
Intercept	7.38	3.23	2.28
X	0.59	0.15	3.89
Z	10.06	4.26	2.36

Experimental ___________________________ Control ________________________

Q.6. An experiment was conducted as an Analysis of Covariance to measure the effect of an experimental treatment on creative performance. The experimental group received the Theory of Inventive Problem Solving (TRIZ) approach, and the control group received traditional problem solving approach. Baseline creativity scores of Novelty (X) were obtained, as well as post-treatment scores (Y). The authors report the following partial ANOVA table, there were 61 students in the experimental group, and 60 in the control group. The model is written as:

$E\{Y_i\} = \alpha + \beta X_i + \beta_1 Z_{1i}$ where $Z_{1i} = 1$ if Subject i is in the Experimental Group, 0 if in Control Group

p.6.a. Complete the following ANOVA table. For the P-value, state whether it is > 0.05 or < 0.05. Note that these are the partial sums of squares (Groups given Pre-Test and Pre-Test given groups).

Source	df	SS	MS	F	F(.05)	P-value
Pre-Test		2574.301		#N/A	#N/A	#N/A
Groups		58.36				
Error		340.98		#N/A	#N/A	#N/A

p.6.b. The fitted equation and overall mean for pre-test scores are: $\hat{Y} = -120 + 6.7X + 5.6Z$ $\quad \overline{X} = 21.00$
Compute the Adjusted means for each group.

Experimental: ________________________ Control: ________________________________

p.6.c. The Pre-Test Means for each group are 20.76 for Experimental, and 21.24 for Control groups, respectively. Compute the Unadjusted means for each group.

Experimental: ________________________ Control: ________________________________

Q.7. An experiment was conducted to measure the effect of a new program (Moodle course) to teach school children about table tennis skills (the children had experience playing prior to the experiment). There were a total of 32 children that were randomized so that $n_T = 16$ received Treatment (Moodle) and $n_C = 16$ received Control (no course). Pre-treatment scores (X) were obtained on each child, as well as Post-treatment scores (Y). The model, pre-treatment and post-treatment means are given below, along with the partial ANOVA table. The response measured was a knowledge skills score from the TTKT test.

$E\{Y_i\} = \alpha + \beta X_i + \beta_1 Z_{1i}$ where $Z_{1i} = 1$ if Subject i is in the Experimental Group, 0 if in Control Group

Mean	Trt	Control		Source	df	SS	MS	F
Pre	2.50	2.56		X		6.3		
Post	7.30	3.06		Trt(Adj)		139.2		
				Error				
				Total	31	165.0		

p.7.a. Complete the ANOVA table.

p.7.b. Test for a Treatment effect after controlling for Pre-treatment score. H_0 ______________ H_A ____________

Test Statistic ______________________ Rejection Region ___________________ P-value ___________________

p.7.c. The difference in Adjusted means is 4.17. What is the difference in the Unadjusted Means? _______________

p.7.d. Based on this information, the regression coefficient for X must be (circle one option):

Large and positive Small and positive Large and Negative Small and Negative

Q.8. In the Analysis of Covariance with 2 treatment groups and a single numeric covariate (X), the unadjusted and adjusted means (within each group) will be the same if the groups have the same mean value for the covariate.

True / False

Q.9. An experiment was conducted to study the effect of a Facebook-based instructional course on pre-service teachers. The teachers were assigned at random to receive either the Facebook-based couse (Z=1) or a control (no course) condition (Z=0). Pre-treatment scores were obtained (X) and the response were post-treatment scores (Y). Consider the following 3 models. There were n_F=20 Facebook instructors and n_C=20 control instructors, and relevant summary statistics are given below.

Model 1: $E\{Y\} = \alpha + \beta X \qquad \hat{Y} = 31.73 + 0.28X \quad SSE_1 = 6442$

Model 2: $E\{Y\} = \alpha + \beta X + \gamma Z \qquad \hat{Y} = 28.80 + 0.27X + 6.60Z \quad SSE_2 = 6006$

Model 3: $E\{Y\} = \alpha + \beta X + \gamma Z + \delta XZ \qquad \hat{Y} = 24.25 + 0.46X + 15.25Z - 0.37XZ \quad SSE_3 = 5939$

$\overline{x}_F = 23.82 \quad \overline{x}_c = 23.35 \quad \overline{x} = 23.58 \qquad \overline{y}_F = 41.74 \quad \overline{y}_c = 35.02 \quad TSS = 6602$

p.9.a. Test if there is a course effect (Facebook versus Control) after controlling for Pre-treatment score, based on the additive (common slopes) model.

H_0:

Test Statistic ____________________________ Rejection Region: ____________________ P > or < .05

p.9.b. Based on Model 2, give the adjusted means for the Facebook and Control groups, and their difference. Confirm the difference as a regression coefficient from the model.

Facebook (Adj) Mean _________ Contol (Adj) Mean _________ Differnce _________ Regression Coefficient _______

p.9.c. Test whether any of the predictors (X, Z, XZ) are associated with post-treatment score (Y).

H_0:

Test Statistic ____________________________ Rejection Region: ____________________ P > or < .05

Q.10. An experiment was conducted in elderly women to measure the effect a treatment to improve store fitting room accessibility. The response was physical cost (Y, higher values mean more difficulty)The treatment factor had 2 levels: Experimental and Control. There was a total of 72 women, half were placed in the Experimental fitting room, half in the Control fitting room. Further, a Covariate, Physical Competence, was measured on each subject. The model fit is given below.

$E\{Y_i\} = \alpha + \beta X_i + \beta_1 Z_{1i}$ where $Z_{1i} = 1$ if Subject i is in the Experimental Group, 0 if in Control Group

p.10.a. Complete the ANOVA table.

Source	df	SS	MS	F	F(.05)
Trt Grp		58.53			
Competence		3.46			
Error					
Total		124.8			

p.10.b. Test for a Treatment effect after controlling for Competence score. H_0 ______________ H_A ____________

Test Statistic ______________________ Rejection Region ____________________ P-value ____________________

p.10.c. The Unadjusted and Adjusted means are: $\overline{Y}_E = 1.30$ $\overline{Y}_C = 3.52$ $\overline{Y}_E^{Adj} = 1.51$ $\overline{Y}_C^{Adj} = 3.31$
Based on this information, and assuming the regression coefficient for X is negative (controlling for treatment, women with higher competence scores have lower Physical cost, which statement is appropriate regarding mean competency?

Experimental Group is higher than Control   Experimental Group is lower than Control   They are equal