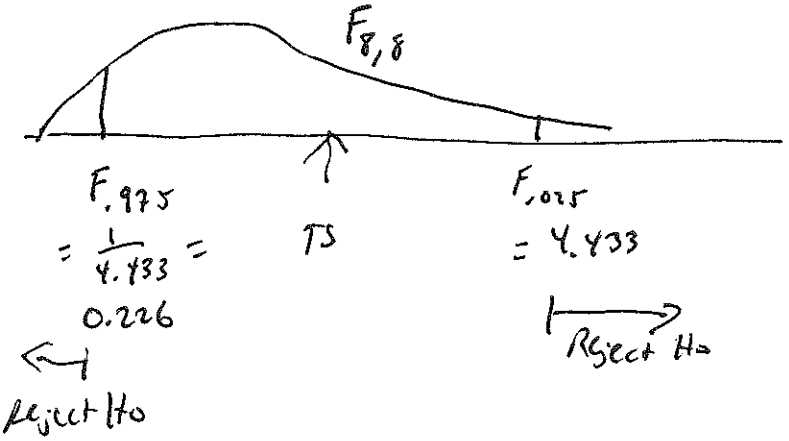
Inference Concerning Variances

Q.1. $n_1 = n_2 = 9$ $\quad s_1^2 = 1^2 = 1$ $\quad s_2^2 = 0.8^2 = 0.64$

T.S. $F_{obs} = \dfrac{s_1^2}{s_2^2} = \dfrac{1}{0.64} = 1.5625$



Do not reject H_0, $\sigma_A^2 = \sigma_B^2$

Q.2. False

Q.3. $n_1 = n_2 = 9$ $\quad s_1 = 10$ $\quad s_2 = 7$

T.S. $F_{obs} = \dfrac{10^2}{7^2} = \dfrac{100}{49} = 2.041$

RR: $F_{obs} \le 0.226$ or $F_{obs} \ge 4.433$ (see Q.1)

Q.4. $n_1 = n_2 = 7$ $\quad s_1 = 2.4$ $\quad s_2 = 3.2$

T.S. $F_{obs} = \dfrac{2.4^2}{3.2^2} = 0.5625$

RR: $F_{obs} \le F_{.975,6,6} = \dfrac{1}{F_{.025,6,6}} = \dfrac{1}{5.820} = 0.172$ or $F_{obs} \ge F_{.025,6,6} = 5.820$

Do not reject H_0: $\sigma_A^2 = \sigma_B^2$

Q.5. $n=18 \quad s=0.8$

95% CI for σ^2: $\left[\frac{(n-1)s^2}{\chi^2_{.025,n-1}}, \frac{(n-1)s^2}{\chi^2_{.975,n-1}}\right]$

$(n-1)s^2 = 17(0.8)^2 = 10.88 \quad \chi^2_{.025,17} = 30.191 \quad \chi^2_{.975,17} = 7.564$

$\Rightarrow \left[\frac{10.88}{30.191}, \frac{10.88}{7.564}\right] \equiv [0.36, 1.44]$

Q.6. $n_1=10, \; n_2=9 \quad s_1=70 \quad s_2=49$

T.S. $F_{obs} = \frac{70^2}{49^2} = 2.041$

RR: $F_{obs} \leq \chi^2_{.975,10-1,9-1} = \frac{1}{\chi^2_{.025,9-1,10-1}} = \frac{1}{4.102} = 0.244$

or $F_{obs} \geq \chi^2_{.025,10-1,9-1} = 4.357$

P-value $> .05$

Q.7. $n=93 \quad s=7.1 \quad \chi^2_{.975,92} \approx 67.362$ (Interpolating) $\quad \chi^2_{.025,92} \approx 120.421$ "

95% CI for σ^2: $\left[\frac{(93-1)(7.1^2)}{120.421}, \frac{(93-1)(7.1^2)}{67.362}\right] \equiv (38.51, 68.85)$

95% CI for σ: $(6.21, 8.30)$

Q. 8. $H_0: \sigma \leq 0.5 \quad H_A: \sigma > 0.5$

Q. 9. $H_0: \sigma_1^2 = \sigma_2^2 \quad H_A: \sigma_1^2 \neq \sigma_2^2$

$n_1 = n_2 = 8 \qquad s_1 = 1.24 \qquad s_2 = 0.64$

T.S. $F_{obs} = \frac{s_1^2}{s_2^2} = \frac{1.24^2}{0.64^2} = 3.75$

RR: $F_{obs} \leq F_{.975;\,8-1,8-1} = \frac{1}{F_{.025,8-1,8-1}} = \frac{1}{4.995} = 0.200$

or $F_{obs} \geq F_{.025,8-1,8-1} = 4.995$

P-value > 0.05 — Do not reject H_0.

Q. 10. $n_A = n_B = 7 \qquad s_A^2 = 5.0 \qquad s_B^2 = 1.25$

$H_0: \sigma_A^2 \leq \sigma_B^2 \qquad H_A: \sigma_A^2 > \sigma_B^2$

T.S. $F_{obs} = \frac{s_A^2}{s_B^2} = \frac{5.0}{1.25} = 4.00$

RR: $F_{obs} \geq F_{.05,7-1,7-1} = 4.284$

P-value > 0.05