Problem 6.34

Group	Initial	year1	year2	year3
1	87.3	86.9	86.7	75.5
1	59	60.2	60	53.6
1	76.7	76.5	75.7	69.5
1	70.6	76.1	72.1	65.3
1	54.9	55.1	57.2	49
1	78.2	75.3	69.1	67.6
1	73.7	70.8	71.8	74.6
1	61.8	68.7	68.2	57.4
1	85.3	84.4	79.2	67
1	82.3	86.9	79.4	77.4
1	68.6	65.4	72.3	60.8
1	67.8	69.2	66.3	57.9
1	66.2	67	67	56.2
1	81	82.3	86.8	73.9
1	72.3	74.6	75.3	66.1
2	83.8	85.5	86.2	81.2
2	65.3	66.9	67	60.6
2	81.2	79.5	84.5	75.2
2	75.4	76.7	74.3	66.7
2	55.3	58.3	59.1	54.2
2	70.3	72.3	70.6	68.6
2	76.5	79.9	80.4	71.6
2	66	70.9	70.3	64.1
2	76.7	79	76.9	70.3
2	77.2	74	77.8	67.9
2	67.3	70.7	68.9	65.9
2	50.3	51.4	53.6	48
2	57.7	57	57.5	51.5
2	74.3	77.7	72.6	68
2	74	74.7	74.5	65.7
2	57.3	56	64.7	53

Group 1/n vector

0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.066667
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625
0.0625

year	xbar1	xbar2
0	72.38	69.2875
1	73.29333	70.65625
2	72.47333	71.18125
3	64.78667	64.53125

S1

92.11886	86.11057	73.36229	74.589
86.11057	89.07638	72.95552	71.77276
73.36229	72.95552	71.89067	63.59176
74.589	71.77276	63.59176	75.4441

Spooled

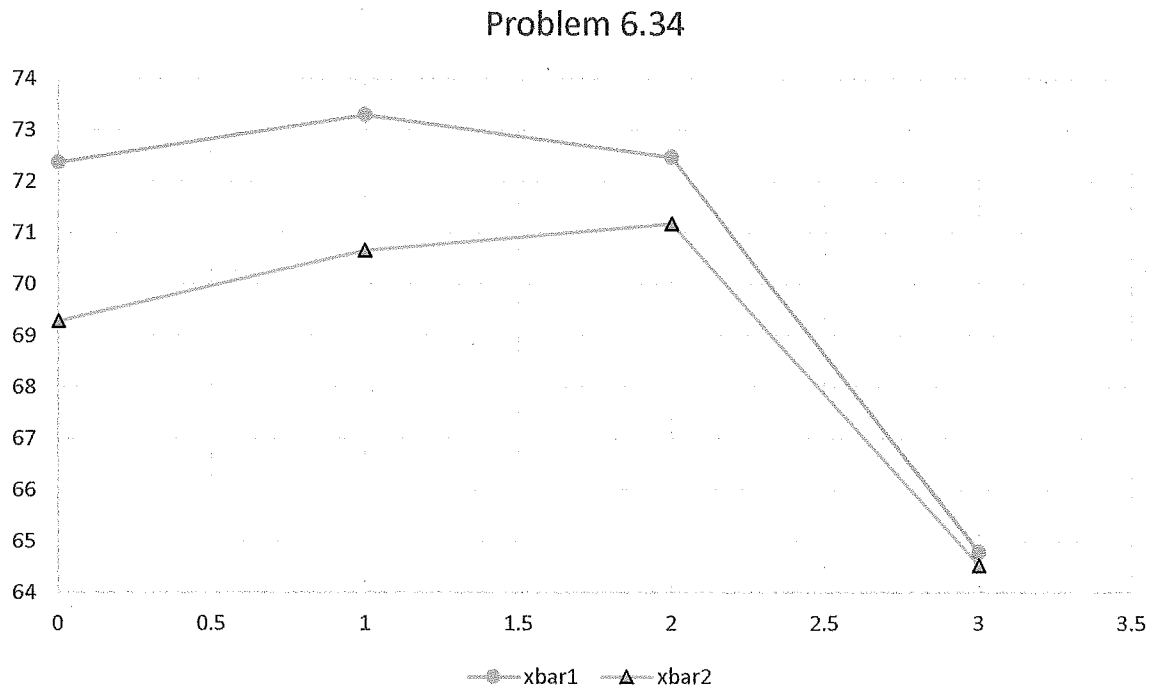
93.21516	88.4104	81.94445	78.28262
88.4104	90.28613	80.32385	76.87318
81.94445	80.32385	80.90426	72.3943
78.28262	76.87318	72.3943	76.3006

S2

94.23838	90.5569	89.95448	81.73
90.5569	91.41524	87.20095	81.63357
89.95448	87.20095	89.31695	80.61
81.73	81.63357	80.61	77.1

W

2703.24	2563.902	2376.389	2270.196
2563.902	2618.298	2329.392	2229.322
2376.389	2329.392	2346.224	2099.435
2270.196	2229.322	2099.435	2212.717



B

1	0
1	1
1	2
1	3

B'INV(sp)B

0.013740355	0.054058975
0.054058975	0.979001873

B'S^-1xb1	B'S^-1xb2
0.901211972	0.909330751
2.111577929	2.855606806

Beta-hat1	Beta-hat2
72.95136092	69.88632844
-1.87139362	-0.94216006

Group	Initial	year1	year2	year3	Xhat0	Xhat1	Xhat2	Xhat3	Res0	Res1	Res2	Res3
1	87.3	86.9	86.7	75.5	72.95136	71.07997	69.20857	67.33718	14.34864	15.82003	17.49143	8.16282
1	59	60.2	60	53.6	72.95136	71.07997	69.20857	67.33718	-13.9514	-10.88	-9.20857	-13.7372
1	76.7	76.5	75.7	69.5	72.95136	71.07997	69.20857	67.33718	3.748639	5.420033	6.491426	2.16282
1	70.6	76.1	72.1	65.3	72.95136	71.07997	69.20857	67.33718	-2.35136	5.020033	2.891426	-2.03718
1	54.9	55.1	57.2	49	72.95136	71.07997	69.20857	67.33718	-18.0514	-15.98	-12.0086	-18.3372
1	78.2	75.3	69.1	67.6	72.95136	71.07997	69.20857	67.33718	5.248639	4.220033	-0.10857	0.26282
1	73.7	70.8	71.8	74.6	72.95136	71.07997	69.20857	67.33718	0.748639	-0.27997	2.591426	7.26282
1	61.8	68.7	68.2	57.4	72.95136	71.07997	69.20857	67.33718	-11.1514	-2.37997	-1.00857	-9.93718
1	85.3	84.4	79.2	67	72.95136	71.07997	69.20857	67.33718	12.34864	13.32003	9.991426	-0.33718
1	82.3	86.9	79.4	77.4	72.95136	71.07997	69.20857	67.33718	9.348639	15.82003	10.19143	10.06282
1	68.6	65.4	72.3	60.8	72.95136	71.07997	69.20857	67.33718	-4.35136	-5.67997	3.091426	-6.53718
1	67.8	69.2	66.3	57.9	72.95136	71.07997	69.20857	67.33718	-5.15136	-1.87997	-2.90857	-9.43718
1	66.2	67	67	56.2	72.95136	71.07997	69.20857	67.33718	-6.75136	-4.07997	-2.20857	-11.1372
1	81	82.3	86.8	73.9	72.95136	71.07997	69.20857	67.33718	8.048639	11.22003	17.59143	6.56282
1	72.3	74.6	75.3	66.1	72.95136	71.07997	69.20857	67.33718	-0.65136	3.520033	6.091426	-1.23718
2	83.8	85.5	86.2	81.2	69.88633	68.94417	68.00201	67.05985	13.91367	16.55583	18.19799	14.14015
2	65.3	66.9	67	60.6	69.88633	68.94417	68.00201	67.05985	-4.58633	-2.04417	-1.00201	-6.45985
2	81.2	79.5	84.5	75.2	69.88633	68.94417	68.00201	67.05985	11.31367	10.55583	16.49799	8.140152
2	75.4	76.7	74.3	66.7	69.88633	68.94417	68.00201	67.05985	5.513672	7.755832	6.297992	-0.35985
2	55.3	58.3	59.1	54.2	69.88633	68.94417	68.00201	67.05985	-14.5863	-10.6442	-8.90201	-12.8598
2	70.3	72.3	70.6	68.6	69.88633	68.94417	68.00201	67.05985	0.413672	3.355832	2.597992	1.540152
2	76.5	79.9	80.4	71.6	69.88633	68.94417	68.00201	67.05985	6.613672	10.95583	12.39799	4.540152
2	66	70.9	70.3	64.1	69.88633	68.94417	68.00201	67.05985	-3.88633	1.955832	2.297992	-2.95985
2	76.7	79	76.9	70.3	69.88633	68.94417	68.00201	67.05985	6.813672	10.05583	8.897992	3.240152
2	77.2	74	77.8	67.9	69.88633	68.94417	68.00201	67.05985	7.313672	5.055832	9.797992	0.840152
2	67.3	70.7	68.9	65.9	69.88633	68.94417	68.00201	67.05985	-2.58633	1.755832	0.897992	-1.15985
2	50.3	51.4	53.6	48	69.88633	68.94417	68.00201	67.05985	-19.5863	-17.5442	-14.402	-19.0598
2	57.7	57	57.5	51.5	69.88633	68.94417	68.00201	67.05985	-12.1863	-11.9442	-10.502	-15.5598
2	74.3	77.7	72.6	68	69.88633	68.94417	68.00201	67.05985	4.413672	8.755832	4.597992	0.940152
2	74	74.7	74.5	65.7	69.88633	68.94417	68.00201	67.05985	4.113672	5.755832	6.497992	-1.35985
2	57.3	56	64.7	53	69.88633	68.94417	68.00201	67.05985	-12.5863	-12.9442	-3.30201	-14.0598

W_lin

2772.915828	2625.375897	2310.867	2381.998
2625.375897	2876.393122	2538.996	2174.016
2310.866783	2538.99577	2623.315	1845.017
2381.99835	2174.01558	1845.017	2477.329

|W|	|W_lin|	Lambda*
28759213721	1.66577E+11	0.172648

N	p	q	g
31	4	1	2

N-0.5(p-q+g)	(p-q-1)g	TestStat	X2(4,.01)
28.5	4	50.06025	13.2767

Reject the hypothesis that linear growth is adequate.

Problem 6.35

R Program

```
data1 <- read.table("http://www.stat.ufl.edu/~winner/sta4702/data/wichern/T6-
5.dat",
      header=F, col.names=c("X11","X12","X13","X14"))

data2 <- read.table("http://www.stat.ufl.edu/~winner/sta4702/data/wichern/T6-
6.dat",
      header=F, col.names=c("X21","X22","X23","X24"))

X1 <- cbind(data1$X11,data1$X12, data1$X13, data1$X14)
X2 <- cbind(data2$X21,data2$X22, data2$X23, data2$X24)

X <- rbind(X1,X2)

n <- nrow(X); p <- ncol(X)
I_n <- diag(n);  J_n <- matrix(rep(1,n^2),n,n)

(xbar <- (1/n) * (t(X) %*% rep(1,n)))
(S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n) * J_n) %*% X))

B <- matrix(c(1,0,0,1,1,1,1,2,4,1,3,9),byrow=T,ncol=3)
q <- ncol(B)-1

(beta.hat <- solve(t(B) %*% solve(S) %*% B) %*% t(B) %*% solve(S) %*% xbar)
(cov.beta.hat <- (((n-1)*(n-2)) / ((n-1-p+q)*(n-p+q)*n)) *  solve(t(B) %*% solve(S)
%*% B))
se.beta.hat <- sqrt(diag(cov.beta.hat))
t.beta <- beta.hat / se.beta.hat

beta.out <- cbind(beta.hat, se.beta.hat, t.beta)
colnames(beta.out) <- c("beta-hat", "std. error", "t")

round(beta.out,3)

(X.hat1 <- B %*% beta.hat)
X.hat <- matrix(rep(0,n*p),ncol=p)

for (i1 in 1:n) {
  for (i2 in 1:p) {
  X.hat[i1,i2] <- X.hat1[i2,1]
  }
}

X.err <- X - X.hat

W_quad <- t(X.err) %*% X.err
W <- (n-1) * S

Lambda.star <- det(W) / det(W_quad)
(Test.Stat <- -(n-0.5*(p-q+1))*log(Lambda.star))
df.TS <- p-q-1
(CV.TS <- qchisq(.95,df.TS))
```

R Output

```
> (xbar <- (1/n) * (t(X) %*% rep(1,n)))
         [,1]
[1,] 70.78387
[2,] 71.93226
[3,] 71.80645
[4,] 64.65484
> (S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n) * J_n) %*% X))
         [,1]     [,2]     [,3]     [,4]
[1,] 94.54406 90.79620 80.00811 78.06758
[2,] 90.79620 93.66159 78.99645 77.77251
[3,] 80.00811 78.99645 77.15462 70.03663
[4,] 78.06758 77.77251 70.03663 75.93189

> (beta.hat <- solve(t(B) %*% solve(S) %*% B) %*% t(B) %*% solve(S) %*% xbar)
          [,1]
[1,] 71.603889
[2,]  3.867281
[3,] -1.940367
> (cov.beta.hat <- (((n-1)*(n-2)) / ((n-1-p+q)*(n-p+q)*n)) *  solve(t(B) %*% solve(S) %*% B))
             [,1]       [,2]         [,3]
[1,]  3.189363308 -0.2066245  0.002762725
[2,] -0.206624499  0.3214991 -0.100343499
[3,]  0.002762725 -0.1003435  0.037189773

> round(beta.out,3)
     beta-hat std. error       t
[1,]   71.604      1.786  40.094
[2,]    3.867      0.567   6.820
[3,]   -1.940      0.193 -10.062

> (X.hat1 <- B %*% beta.hat)
         [,1]
[1,] 71.60389
[2,] 73.53080
[3,] 71.57698
[4,] 65.74242

> (Test.Stat <- -(n-0.5*(p-q+1))*log(Lambda.star))
[1] 7.892599
> df.TS <- p-q-1
> CV.TS <- qchisq(.05,df.TS)
>
> (CV.TS <- qchisq(.95,df.TS))
[1] 3.841459
```

- The Fit is not good. ($p < .01$)

Problem 6.36

R Program

```
p <- 2; g <- 2
n1 <- 45; n2 <- 55

S1 <- matrix(c(13825.3, 23823.4, 23823.4, 73107.4),2,2)
S2 <- matrix(c(8632.0, 19616.7, 19616.7, 55964.5), 2,2)

Sp <- ((n1-1)*S1 + (n2-1)*S2) / (n1+n2-2)

M <- (n1+n2-2)*log(det(Sp)) - ((n1-1)*log(det(S1)) + (n2-1)*log(det(S2)))
u1 <- (1/(n1-1) + 1/(n2-1)) - 1/(n1+n2-2)
u2 <- (2*p^2+3*p-1) / (6*(p+1)*(g-1))
u <- u1*u2

(Box.C <- (1-u)*M)
(Box.df <- p*(p+1)*(g-1)/2)
(Box.CV <- qchisq(.95,Box.df))
(Box.PV <- 1-pchisq(Box.C, Box.df))
```

R Output

```
>
> (Box.C <- (1-u)*M)
[1] 18.93306
> (Box.df <- p*(p+1)*(g-1)/2)
[1] 3
> (Box.CV <- qchisq(.95,Box.df))
[1] 7.814728
> (Box.PV <- 1-pchisq(Box.C, Box.df))
[1] 0.0002822514
```

Problem 6.37

R Program

```
turtle <- read.table("http://www.stat.ufl.edu/~winner/sta4702/data/wichern/T6-
9.dat",
   header=F, col.names=c("length","width","height","gender"))
attach(turtle)

length.f <- length[gender == "female"]
width.f <- width[gender == "female"]
height.f <- height[gender == "female"]

length.m <- length[gender == "male"]
width.m <- width[gender == "male"]
height.m <- height[gender == "male"]

X_f <- cbind(length.f, width.f, height.f)
X_m <- cbind(length.m, width.m, height.m)
n_f <- length(length.f); n_m <- length(length.m)
I_f <- diag(n_f); J_f <- matrix(rep(1,n_f^2),n_f,n_f)
I_m <- diag(n_m); J_m <- matrix(rep(1,n_m^2),n_m,n_m)

(S_f <- (1/(n_f-1)) * (t(X_f) %*% (I_f - (1/n_f) * J_f) %*% X_f))
(S_m <- (1/(n_m-1)) * (t(X_m) %*% (I_m - (1/n_m) * J_m) %*% X_m))
(S_p <- ((n_f-1)*S_f + (n_m-1)*S_m) / (n_f+n_m-2))

p <- ncol(X_f); g <- 2

M <- (n_f+n_m-2)*log(det(Sp)) - ((n_f-1)*log(det(S_f)) + (n_m-
1)*log(det(S_m)))
u1 <- (1/(n_f-1) + 1/(n_m-1)) - 1/(n_f+n_m-2)
u2 <- (2*p^2+3*p-1) / (6*(p+1)*(g-1))
u <- u1*u2

(Box.C <- (1-u)*M)
(Box.df <- p*(p+1)*(g-1)/2)
(Box.CV <- qchisq(.95,Box.df))
```

R Output

```
> (S_f <- (1/(n_f-1)) * (t(X_f) %*% (I_f - (1/n_f) * J_f) %*% X_f))
          length.f  width.f  height.f
length.f 451.5199 270.9746 165.95471
width.f  270.9746 171.7319 101.84420
height.f 165.9547 101.8442  64.73732
> (S_m <- (1/(n_m-1)) * (t(X_m) %*% (I_m - (1/n_m) * J_m) %*% X_m))
           length.m  width.m height.m
length.m 138.76630 79.14674 37.37500
width.m   79.14674 50.04167 21.65399
height.m  37.37500 21.65399 11.25906
> (S_p <- ((n_f-1)*S_f + (n_m-1)*S_m) / (n_f+n_m-2))
          length.f   width.f  height.f
length.f 295.1431 175.06069 101.66486
width.f  175.0607 110.88678  61.74909
height.f 101.6649  61.74909  37.99819


> (Box.C <- (1-u)*M)
[1] 478.6941
> (Box.df <- p*(p+1)*(g-1)/2)
[1] 6
> (Box.CV <- qchisq(.95,Box.df))
[1] 12.59159
> (Box.PV <- 1-pchisq(Box.C, Box.df))
[1] 0
```