STA 4211 – Exam 3 – Spring 2015   PRINT Name ____

Conduct all tests at α = 0.05 significance level.

Q.1. A stability study considered the potency of tablets from 3 (fixed) batches of pharmaceutical tablets. The study included a covariate (time since tablet was produced, in days). Two dummy variables were created for Batch, and the covariate was centered by subtracting off the mean number of days. There were a total of $n_T = 31$ sampled tablets.

$$Y_{ij} \equiv \text{Potency of } j^{th} \text{ Tablet from batch } i \quad x_{ij} = X_{ij} - \overline{X}_{\bullet\bullet} \quad I_{ij1} = \begin{cases} 1 \text{ if Batch} = 1 \\ 0 \text{ if Batch} = 2 \\ -1 \text{ if Batch} = 3 \end{cases} \quad I_{ij2} = \begin{cases} 1 \text{ if Batch} = 2 \\ 0 \text{ if Batch} = 1 \\ -1 \text{ if Batch} = 3 \end{cases}$$

Model 1: $Y_{ij} = \mu_{\bullet} + \gamma x_{ij} + \varepsilon_{ij}$

Model 2: $Y_{ij} = \mu_{\bullet} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$

Model 3: $Y_{ij} = \mu_{\bullet} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$

The error sums of squares for the 3 models are: $SSE_1 = 18.06$ $SSE_2 = 17.46$ $SSE_3 = 17.15$

p.1.a. Test whether the association (slope) between Potency and Day is common among batches: $H_0 : \beta_1 = \beta_2 = 0$

Test Statistic: ____________________________ Reject H_0 if the test statistic falls in the range _____________________

p.1.b. Assuming no interaction, test whether there are potency differences among batches, controlling for Day:

H_0: ________________________ H_A: _____________________

Test Statistic: ____________________________ Reject H_0 if the test statistic falls in the range _____________________

Q.2. A study was conducted to compare 3 treatments for depression (Sudarshan Kriya Yoga (SKY), Electroconvulsive Therapy (ET), and Imipramine (IMN)) in subjects suffering from melancholia. A sample of 45 subjects was obtained and randomized so that 15 received SKY, 15 received ET, and 15 received IMN. One response measured was the patients' scores on the 17-item Hamilton Depression Scale (HRSD). Patients were each measured at 5 time points.

p.2.a. Complete the ANOVA table.

Source	df	SS	MS	F_obs	F(0.05)
Treatment		184			
Subject(Trt)				#N/A	#N/A
Time		11119			
Trt*Time		577			
Error		4805		#N/A	#N/A
Total		23630	#N/A	#N/A	#N/A

p.2.b. Is there a significant treatment by time interaction? **Yes** or **No**

p.2.c. The standard error of the difference between two treatment means at the same time point, and estimated degrees of freedom are:

$$s\left\{\overline{Y}_{\bullet jk} - \overline{Y}_{\bullet j'k}\right\} = \sqrt{\frac{2\left(MSS(A) + (b-1)MSB.S(A)\right)}{bs}}$$

with approximate df:

$$df = \frac{\left[(b-1)MSB.S(A) + MSS(A)\right]^2}{\left[\frac{\left[(b-1)MSB.S(A)\right]^2}{a(b-1)(s-1)} + \frac{\left[MSS(A)\right]^2}{a(s-1)}\right]}$$

The means for the 3 treatments at the final time point are: ECT = 2.5, IMN = 6.3, SKY = 8.3. Use Bonferroni's method to compare all pairs of treatments at the final time point. Note that low scores mean lower depression.

ECT IMN SKY

Q.3. Based on the 2014 WNBA season, we have the point totals (Y) by game **Location** (Home/Away) for a **sample** of 10 **Players**. Each player played 17 home games and 17 away games. Consider the (restricted) model:

$$Y_{ijk} = \mu_{\bullet\bullet} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \qquad i=1,...,a \quad j=1,...,b \quad k=1,...,n \quad \sum_{i=1}^{a}\alpha_i = 0 \quad \beta_j \sim N\left(0,\sigma_b^2\right) \quad \left\{(\alpha\beta)_{ij}\right\} \sim N\left(0,\frac{a-1}{a}\sigma_{\alpha\beta}^2\right) \quad \varepsilon_{ijk} \sim N\left(0,\sigma^2\right)$$

ANOVA					
Source	df	SS	MS	F	F(0.05)
Player		3879.30			
Location		1.30			
P*L		323.67			
Error		15787.29		#N/A	#N/A
Total		19991.56	#N/A	#N/A	#N/A

p.3.a. Complete the partial ANOVA table.

p.3.b. Test whether there is an interaction between Player and Location (Home). $H_0: \sigma_{\alpha\beta}^2 = 0$

Test Stat: _____________ Reject H_0 if Test Stat is in the range ________________ P-value > **or** < .05?

p.3.c. Test whether there is Location (Home vs Away) Main Effect. $H_0: \alpha_1 = \alpha_2 = 0$

Test Stat: _____________ Reject H_0 if Test Stat is in the range ________________ P-value > **or** < .05?

p.3.d. Test whether there is Player Main Effect. $H_0: \sigma_\beta^2 = 0$

Test Stat: _____________ Reject H_0 if Test Stat is in the range ________________ P-value > **or** < .05?

p.3.e. Give unbiased estimates of each of the variance components:

$s_{ab}^2 =$ __________________________ $s_b^2 =$ __________________________ $s^2 =$ __________________________

Q.4. A study is conducted to compare math scores among graduating Math majors among all of the colleges in the power 5 athletic conferences. From each college, a sample of 8 graduating math majors is selected, each student given a standardized exam. The five conferences (number of schools) are: ACC (15), Big Ten (14), Big 12 (10), Pac 12 (12), and SEC (14).

p.4.a. In this setting, would Conference and College(Conference) be best described as **Fixed** or **Random**? _______________

p.4.b. Give the degrees of freedom for the ANOVA for Conference ________ College(Conference)_________ Error ___________

Q.5. Among a population of pumpkin farmers, mean pumpkin weights are normally distributed. Approximately 95% of farmers' mean weights are between 8 and 12 pounds. Within farmers, individual pumpkin weights are normally distributed with approximately 95% weighing within 3 pounds of that farmer's mean. Based on this information, give: $\mu_{\bullet}, \quad \sigma_{\mu}^2, \quad \sigma^2, \quad \rho_I$

$\mu_{\bullet} =$ ________________ $\sigma_{\mu}^2 =$ ________________ $\sigma^2 =$ ________________ $\rho_I =$ ________________

Q.6. A study was conducted to measure the reliability of collegiate gymnastics judges, and variation in gymnast skills. A sample of 8 judges was selected, and a sample of 4 gymnasts was selected. Each gymnast was filmed on 3 occasions, and each judge rated the 3 videos. Give the statistical model, and give the degrees of freedom for each source of variation and whether it is fixed or random.

p.6.a. Model

p.6.b. ANOVA

Source	df	Fixed/Random

Have an Excellent Summer!

Studentized Range (0.05 Upper-tail)

df\#trts	2	3	4	5	6	7	8	9	10
1	17.969	26.976	32.819	37.082	40.408	43.119	45.397	47.357	49.071
2	6.085	8.331	9.798	10.881	11.734	12.435	13.027	13.539	13.988
3	4.501	5.910	6.825	7.502	8.037	8.478	8.852	9.177	9.462
4	3.926	5.040	5.757	6.287	6.706	7.053	7.347	7.602	7.826
5	3.635	4.602	5.218	5.673	6.033	6.330	6.582	6.801	6.995
6	3.460	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493
7	3.344	4.165	4.681	5.060	5.359	5.606	5.815	5.997	6.158
8	3.261	4.041	4.529	4.886	5.167	5.399	5.596	5.767	5.918
9	3.199	3.948	4.415	4.755	5.024	5.244	5.432	5.595	5.738
10	3.151	3.877	4.327	4.654	4.912	5.124	5.304	5.460	5.598
11	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.486
12	3.081	3.773	4.199	4.508	4.750	4.950	5.119	5.265	5.395
13	3.055	3.734	4.151	4.453	4.690	4.884	5.049	5.192	5.318
14	3.033	3.701	4.111	4.407	4.639	4.829	4.990	5.130	5.253
15	3.014	3.673	4.076	4.367	4.595	4.782	4.940	5.077	5.198
16	2.998	3.649	4.046	4.333	4.557	4.741	4.896	5.031	5.150
17	2.984	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108
18	2.971	3.609	3.997	4.276	4.494	4.673	4.824	4.955	5.071
19	2.960	3.593	3.977	4.253	4.468	4.645	4.794	4.924	5.037
20	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.895	5.008
21	2.941	3.565	3.942	4.213	4.424	4.597	4.743	4.870	4.981
22	2.933	3.553	3.927	4.196	4.405	4.577	4.722	4.847	4.957
23	2.926	3.542	3.914	4.180	4.388	4.558	4.702	4.826	4.935
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
25	2.913	3.523	3.890	4.153	4.358	4.526	4.667	4.789	4.897
26	2.907	3.514	3.880	4.141	4.345	4.511	4.652	4.773	4.880
27	2.902	3.506	3.870	4.130	4.333	4.498	4.638	4.758	4.864
28	2.897	3.499	3.861	4.120	4.322	4.486	4.625	4.745	4.850
29	2.892	3.493	3.853	4.111	4.311	4.475	4.613	4.732	4.837
30	2.888	3.486	3.845	4.102	4.301	4.464	4.601	4.720	4.824
31	2.884	3.481	3.838	4.094	4.292	4.454	4.591	4.709	4.812
32	2.881	3.475	3.832	4.086	4.284	4.445	4.581	4.698	4.802
33	2.877	3.470	3.825	4.079	4.276	4.436	4.572	4.689	4.791
34	2.874	3.465	3.820	4.072	4.268	4.428	4.563	4.680	4.782
35	2.871	3.461	3.814	4.066	4.261	4.421	4.555	4.671	4.773
36	2.868	3.457	3.809	4.060	4.255	4.414	4.547	4.663	4.764
37	2.865	3.453	3.804	4.054	4.249	4.407	4.540	4.655	4.756
38	2.863	3.449	3.799	4.049	4.243	4.400	4.533	4.648	4.749
39	2.861	3.445	3.795	4.044	4.237	4.394	4.527	4.641	4.741
40	2.858	3.442	3.791	4.039	4.232	4.388	4.521	4.634	4.735
48	2.843	3.420	3.764	4.008	4.197	4.351	4.481	4.592	4.690
60	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646
80	2.814	3.377	3.711	3.947	4.129	4.277	4.402	4.509	4.603
120	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560
240	2.786	3.335	3.659	3.887	4.063	4.205	4.324	4.427	4.517
Inf	2.772	3.314	3.633	3.858	4.030	4.170	4.286	4.387	4.474

Bonferroni t-table (2-sided, α =0.05)

df\#Comparisons	1	3	6	10	15	21	28	36	45
1	12.706	38.188	76.390	127.321	190.984	267.379	356.506	458.366	572.957
2	4.303	7.649	10.886	14.089	17.277	20.457	23.633	26.805	29.975
3	3.182	4.857	6.232	7.453	8.575	9.624	10.617	11.563	12.471
4	2.776	3.961	4.851	5.598	6.254	6.847	7.392	7.900	8.376
5	2.571	3.534	4.219	4.773	5.247	5.666	6.045	6.391	6.713
6	2.447	3.287	3.863	4.317	4.698	5.030	5.326	5.594	5.840
7	2.365	3.128	3.636	4.029	4.355	4.636	4.884	5.107	5.310
8	2.306	3.016	3.479	3.833	4.122	4.370	4.587	4.781	4.957
9	2.262	2.933	3.364	3.690	3.954	4.179	4.374	4.549	4.706
10	2.228	2.870	3.277	3.581	3.827	4.035	4.215	4.375	4.518
11	2.201	2.820	3.208	3.497	3.728	3.923	4.091	4.240	4.373
12	2.179	2.779	3.153	3.428	3.649	3.833	3.992	4.133	4.258
13	2.160	2.746	3.107	3.372	3.584	3.760	3.912	4.045	4.164
14	2.145	2.718	3.069	3.326	3.530	3.699	3.845	3.973	4.086
15	2.131	2.694	3.036	3.286	3.484	3.648	3.788	3.911	4.021
16	2.120	2.673	3.008	3.252	3.444	3.604	3.740	3.859	3.965
17	2.110	2.655	2.984	3.222	3.410	3.565	3.698	3.814	3.917
18	2.101	2.639	2.963	3.197	3.380	3.532	3.661	3.774	3.874
19	2.093	2.625	2.944	3.174	3.354	3.503	3.629	3.739	3.837
20	2.086	2.613	2.927	3.153	3.331	3.477	3.601	3.709	3.804
21	2.080	2.601	2.912	3.135	3.310	3.453	3.575	3.681	3.775
22	2.074	2.591	2.899	3.119	3.291	3.432	3.552	3.656	3.749
23	2.069	2.582	2.886	3.104	3.274	3.413	3.531	3.634	3.725
24	2.064	2.574	2.875	3.091	3.258	3.396	3.513	3.614	3.703
25	2.060	2.566	2.865	3.078	3.244	3.380	3.495	3.595	3.684
26	2.056	2.559	2.856	3.067	3.231	3.366	3.480	3.578	3.666
27	2.052	2.552	2.847	3.057	3.219	3.353	3.465	3.563	3.649
28	2.048	2.546	2.839	3.047	3.208	3.340	3.452	3.549	3.634
29	2.045	2.541	2.832	3.038	3.198	3.329	3.440	3.535	3.620
30	2.042	2.536	2.825	3.030	3.189	3.319	3.428	3.523	3.607
40	2.021	2.499	2.776	2.971	3.122	3.244	3.347	3.436	3.514
50	2.009	2.477	2.747	2.937	3.083	3.201	3.300	3.386	3.461
60	2.000	2.463	2.729	2.915	3.057	3.173	3.270	3.353	3.426
70	1.994	2.453	2.715	2.899	3.039	3.153	3.248	3.330	3.402
80	1.990	2.445	2.705	2.887	3.026	3.138	3.232	3.313	3.383
90	1.987	2.440	2.698	2.878	3.016	3.127	3.220	3.299	3.369
100	1.984	2.435	2.692	2.871	3.007	3.118	3.210	3.289	3.358
110	1.982	2.431	2.687	2.865	3.001	3.110	3.202	3.280	3.349
120	1.980	2.428	2.683	2.860	2.995	3.104	3.195	3.273	3.342
130	1.978	2.425	2.679	2.856	2.990	3.099	3.190	3.267	3.335
140	1.977	2.423	2.676	2.852	2.986	3.095	3.185	3.262	3.330
150	1.976	2.421	2.674	2.849	2.983	3.091	3.181	3.258	3.325
160	1.975	2.419	2.671	2.846	2.980	3.087	3.177	3.254	3.321
170	1.974	2.418	2.669	2.844	2.977	3.084	3.174	3.251	3.317
180	1.973	2.417	2.668	2.842	2.975	3.082	3.171	3.247	3.314
190	1.973	2.415	2.666	2.840	2.973	3.079	3.169	3.245	3.311
200	1.972	2.414	2.665	2.839	2.971	3.077	3.166	3.242	3.309
400	1.966	2.404	2.651	2.823	2.953	3.058	3.145	3.220	3.285
600	1.964	2.401	2.647	2.817	2.947	3.051	3.138	3.212	3.277
800	1.963	2.399	2.645	2.815	2.944	3.048	3.134	3.208	3.273
1000	1.962	2.398	2.644	2.813	2.942	3.046	3.132	3.206	3.270
inf	1.960	2.394	2.638	2.807	2.935	3.038	3.124	3.197	3.261

Critical Values for t, χ^2, and F Distributions
F Distributions Indexed by Numerator Degrees of Freedom

df	$t_{.95}$	$t_{.975}$	$\chi^2_{.95}$	$F_{.95,1}$	$F_{.95,2}$	$F_{.95,3}$	$F_{.95,4}$	$F_{.95,5}$	$F_{.95,6}$	$F_{.95,7}$	$F_{.95,8}$
1	6.314	12.706	3.841	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883
2	2.920	4.303	5.991	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371
3	2.353	3.182	7.815	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845
4	2.132	2.776	9.488	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041
5	2.015	2.571	11.070	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818
6	1.943	2.447	12.592	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147
7	1.895	2.365	14.067	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726
8	1.860	2.306	15.507	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438
9	1.833	2.262	16.919	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230
10	1.812	2.228	18.307	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072
11	1.796	2.201	19.675	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948
12	1.782	2.179	21.026	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849
13	1.771	2.160	22.362	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767
14	1.761	2.145	23.685	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699
15	1.753	2.131	24.996	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641
16	1.746	2.120	26.296	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591
17	1.740	2.110	27.587	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548
18	1.734	2.101	28.869	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510
19	1.729	2.093	30.144	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477
20	1.725	2.086	31.410	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447
21	1.721	2.080	32.671	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420
22	1.717	2.074	33.924	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397
23	1.714	2.069	35.172	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375
24	1.711	2.064	36.415	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355
25	1.708	2.060	37.652	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337
26	1.706	2.056	38.885	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321
27	1.703	2.052	40.113	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305
28	1.701	2.048	41.337	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291
29	1.699	2.045	42.557	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278
30	1.697	2.042	43.773	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266
40	1.684	2.021	55.758	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180
50	1.676	2.009	67.505	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130
60	1.671	2.000	79.082	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097
70	1.667	1.994	90.531	3.978	3.128	2.736	2.503	2.346	2.231	2.143	2.074
80	1.664	1.990	101.879	3.960	3.111	2.719	2.486	2.329	2.214	2.126	2.056
90	1.662	1.987	113.145	3.947	3.098	2.706	2.473	2.316	2.201	2.113	2.043
100	1.660	1.984	124.342	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032
110	1.659	1.982	135.480	3.927	3.079	2.687	2.454	2.297	2.182	2.094	2.024
120	1.658	1.980	146.567	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016
130	1.657	1.978	157.610	3.914	3.066	2.674	2.441	2.284	2.169	2.081	2.010
140	1.656	1.977	168.613	3.909	3.061	2.669	2.436	2.279	2.164	2.076	2.005
150	1.655	1.976	179.581	3.904	3.056	2.665	2.432	2.274	2.160	2.071	2.001
160	1.654	1.975	190.516	3.900	3.053	2.661	2.428	2.271	2.156	2.067	1.997
170	1.654	1.974	201.423	3.897	3.049	2.658	2.425	2.267	2.152	2.064	1.993
180	1.653	1.973	212.304	3.894	3.046	2.655	2.422	2.264	2.149	2.061	1.990
190	1.653	1.973	223.160	3.891	3.043	2.652	2.419	2.262	2.147	2.058	1.987
200	1.653	1.972	233.994	3.888	3.041	2.650	2.417	2.259	2.144	2.056	1.985
∞	1.645	1.960	---	3.841	2.995	2.605	2.372	2.214	2.099	2.010	1.938