

For all significance tests, use $\alpha = 0.05$ significance level.

Q.1. A study was conducted to compare “likability” of 4 food packages (Treatments). A sample of 20 consumers (Blocks) rated the packages in random order.

p.1.a. Complete the ANOVA table.

Source	df	SS	MS	F_obs	F(0.05)
Treatments		90		20	
Blocks				#N/A	#N/A
Error				#N/A	#N/A
Total		400	#N/A	#N/A	#N/A

p.1.b. Do you conclude there are significant differences in terms of likability among the 4 packages? **Yes** or **No**

p.1.c. Compute the Relative Efficiency of the Randomized Block Design (relative to the Completely Randomized Design).

p.1.d. How many replicates per treatment and overall would be needed for a Completely Randomized Design to have equivalent precision for estimating the treatment means as this design has?

Per Package: _____ Overall: _____

Q.2. An unbalanced two-way ANOVA was conducted to compare ethics scores on an exam (Y) among members of the U.S. Coast Guard. The factors were Gender ($X_1=1$ if Male, -1 if Female) and Rank ($X_2=1$ if Officers, -1 if Enlisted). The sample sizes were: M/O = 72, M/E = 180, F/O = 15, F/E = 32. Four regressions models were fit:

Model 1: $E\{Y_{ijk}\} = \beta_0 + \beta_1 X_{1ijk} + \beta_2 X_{2ijk} + \beta_3 X_{1ijk} X_{2ijk}$ $SSE_1 = 42088$ $\hat{Y}_1 = 36.35 - 2.40X_1 + 3.85X_2 - 0.50X_1X_2$

Model 2: $E\{Y_{ijk}\} = \beta_0 + \beta_1 X_{1ijk} + \beta_2 X_{2ijk}$ $SSE_2 = 42122$ $\hat{Y}_2 = 36.23 - 2.21X_1 + 3.52X_2$

Model 3: $E\{Y_{ijk}\} = \beta_0 + \beta_2 X_{2ijk} + \beta_3 X_{1ijk} X_{2ijk}$ $SSE_3 = 42873$ $\hat{Y}_3 = 34.75 + 3.30X_2 + 0.39X_1X_2$

p.2.a. Test whether there is an interaction between Gender and Rank. H_0 : _____ H_A : _____

Test Statistic _____ Rejection Region _____ P-value > **0.05** or < **0.05**

p.2.b. Test whether there is main effect for Gender. H_0 : _____ H_A : _____

Test Statistic _____ Rejection Region _____ P-value > **0.05** or < **0.05**

p.2.c. Based on Model 2, give the predicted scores for all combinations of Gender and Rank.

Gender\Rank	Officer	Enlisted
Male		
Female		

Q.3. A study was conducted to measure oil holding capacity (Y) in bananas pre-treated with acid. There were 3 cultivars of bananas (Luvhele, Mabonde, M-red), 3 types of acid (Ascorbic, Citric, Lactic), and 3 doses of acid (10,15,20). There were 3 replicates per treatment. The researchers fit a 3-Way ANOVA. Complete the following table.

Source	df	SS	MS	F*	F(0.95)	Significant Effect?
Cultivar(A)		0.992				Yes / No
AcidType(B)		0.281	#N/A	#N/A	#N/A	#N/A
AcidDose©		0.356	#N/A	#N/A	#N/A	#N/A
AB		0.194				Yes / No
AC		0.463	#N/A	#N/A	#N/A	#N/A
BC		0.156	#N/A	#N/A	#N/A	#N/A
ABC		1.289				Yes / No
Error		14.078		#N/A	#N/A	#N/A
Total		17.808	#N/A	#N/A	#N/A	#N/A

Q.4. A taste preference experiment is to be conducted to compare 4 brands of seasoning (A,B,C,D). The experimenter plans to take a sample of consumers, and have each consumer taste meat cooked with each seasoning (separately). She will ask each consumer which of the 4 seasonings he/she preferred. The consumers will be “blind” to which brand is being tasted.

p.4.a. Assuming she wants each possible ordering to be applied to 3 consumers, how many consumers will be needed for the study? Note: An individual consumer will taste only one ordering of the 4 seasons.

p.4.b. Write out all orderings where Brand A is tasted first, and all orderings where A is tasted last.

Q.6. A study was conducted, measuring the effects of 3 electronic **Readers** and 4 **Illumination** levels on time for people to read a given text (100s of seconds). There were a total of 60 subjects, 5 each assigned to each combination of Reader/Illumination level.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad i = 1, 2, 3 \quad j = 1, 2, 3, 4 \quad k = 1, \dots, 5 \quad \sum_{i=1}^3 \alpha_i = \sum_{j=1}^4 \beta_j = \sum_{i=1}^3 (\alpha\beta)_{ij} = \sum_{j=1}^4 (\alpha\beta)_{ij} = 0 \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

p.6.a Complete the following ANOVA table, and test for significant Reader/Illumination Interaction effects, as well as main effects for Reader and Illumination levels.

$$\bar{Y}_{1..} = 12.5 \quad \bar{Y}_{2..} = 10.3 \quad \bar{Y}_{3..} = 10.2 \quad \bar{Y}_{...} = 11.0$$

ANOVA						
Source	df	SS	MS	F*	F(0.95)	Significant Effects?
Reader						
Illumination		148.11				
Read*Illum		2.15				
Error				#N/A	#N/A	#N/A
Total		585.98	#N/A	#N/A	#N/A	#N/A

p.6.b Use Tukey's Method to make all pairwise comparisons among Readers.

Tukey's HSD _____

R3 R2 R1

Q.7. Tukey's test for Non-additivity tests a specific assumption regarding interaction terms, which is the assumption:

a) $(\alpha\beta)_{ij} = D_1\alpha_i + D_2\beta_j$ b) $(\alpha\beta)_{ij} = D(\alpha_i + \beta_j)$ c) $(\alpha\beta)_{ij} = D\alpha_i\beta_j$ d) $(\alpha\beta)_{ij} = D\frac{\alpha_i}{\beta_j}$

Q.8. An experiment is to be conducted to compare 3 types of arthroscopic knee surgery (treatments) on a sample of 45 patients with a particular injury. After a pre-screening rating of severity of injury on a visual analogue scale, they decide to generate blocks containing: the patients with lowest 3 scores (block 1), next lowest 3 scores (block 2), ... highest 3 scores (block 15). The experiment is conducted and a post-surgery score of flexibility is obtained. If the Sum of Squares for the TreatmentxBlock interaction is 200, what will the Treatment Sum of Squares need to be (at least) for there to be a significant difference among the treatment means?

Q.9. If the $MSS < MSTR.S$ in a Repeated Measures Design, then The Relative Efficiency (E) of the Repeated Measures Design, relative to a Completely Randomized Design will be: a) $E = 1$ b) $E > 1$ c) $E < 1$

Q.10. A study is conducted, measuring Contingent Negative Variation (CNV) by EEG. There were 22 Subjects and 9 time periods (3 pre-smoking, 3 smoking, 3 post-smoking). There was one measurement per subject per period. The ANOVA is given below, along with the 1-degree of freedom partitioning of the Interaction sum of squares for Tukey's test for non-additivity. Test whether there is an interaction between smoker and time period.

H_0 : No Smoker by Period Interaction H_A : Smoker by Period Interaction

Source	df	SS
Period	8	104.76
Subject	21	1441.59
Period*Subject	168	790.35
Nonadditivity	1	81.10
Remainder	167	709.25
Total	197	2336.70

Test Statistic _____ Rejection Region _____ Reject H_0 ? **Yes** or **No**

Studentized Range (0.05 Upper-tail)

df\#trts	2	3	4	5	6	7	8	9	10
1	17.969	26.976	32.819	37.082	40.408	43.119	45.397	47.357	49.071
2	6.085	8.331	9.798	10.881	11.734	12.435	13.027	13.539	13.988
3	4.501	5.910	6.825	7.502	8.037	8.478	8.852	9.177	9.462
4	3.926	5.040	5.757	6.287	6.706	7.053	7.347	7.602	7.826
5	3.635	4.602	5.218	5.673	6.033	6.330	6.582	6.801	6.995
6	3.460	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493
7	3.344	4.165	4.681	5.060	5.359	5.606	5.815	5.997	6.158
8	3.261	4.041	4.529	4.886	5.167	5.399	5.596	5.767	5.918
9	3.199	3.948	4.415	4.755	5.024	5.244	5.432	5.595	5.738
10	3.151	3.877	4.327	4.654	4.912	5.124	5.304	5.460	5.598
11	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.486
12	3.081	3.773	4.199	4.508	4.750	4.950	5.119	5.265	5.395
13	3.055	3.734	4.151	4.453	4.690	4.884	5.049	5.192	5.318
14	3.033	3.701	4.111	4.407	4.639	4.829	4.990	5.130	5.253
15	3.014	3.673	4.076	4.367	4.595	4.782	4.940	5.077	5.198
16	2.998	3.649	4.046	4.333	4.557	4.741	4.896	5.031	5.150
17	2.984	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108
18	2.971	3.609	3.997	4.276	4.494	4.673	4.824	4.955	5.071
19	2.960	3.593	3.977	4.253	4.468	4.645	4.794	4.924	5.037
20	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.895	5.008
21	2.941	3.565	3.942	4.213	4.424	4.597	4.743	4.870	4.981
22	2.933	3.553	3.927	4.196	4.405	4.577	4.722	4.847	4.957
23	2.926	3.542	3.914	4.180	4.388	4.558	4.702	4.826	4.935
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
25	2.913	3.523	3.890	4.153	4.358	4.526	4.667	4.789	4.897
26	2.907	3.514	3.880	4.141	4.345	4.511	4.652	4.773	4.880
27	2.902	3.506	3.870	4.130	4.333	4.498	4.638	4.758	4.864
28	2.897	3.499	3.861	4.120	4.322	4.486	4.625	4.745	4.850
29	2.892	3.493	3.853	4.111	4.311	4.475	4.613	4.732	4.837
30	2.888	3.486	3.845	4.102	4.301	4.464	4.601	4.720	4.824
31	2.884	3.481	3.838	4.094	4.292	4.454	4.591	4.709	4.812
32	2.881	3.475	3.832	4.086	4.284	4.445	4.581	4.698	4.802
33	2.877	3.470	3.825	4.079	4.276	4.436	4.572	4.689	4.791
34	2.874	3.465	3.820	4.072	4.268	4.428	4.563	4.680	4.782
35	2.871	3.461	3.814	4.066	4.261	4.421	4.555	4.671	4.773
36	2.868	3.457	3.809	4.060	4.255	4.414	4.547	4.663	4.764
37	2.865	3.453	3.804	4.054	4.249	4.407	4.540	4.655	4.756
38	2.863	3.449	3.799	4.049	4.243	4.400	4.533	4.648	4.749
39	2.861	3.445	3.795	4.044	4.237	4.394	4.527	4.641	4.741
40	2.858	3.442	3.791	4.039	4.232	4.388	4.521	4.634	4.735
48	2.843	3.420	3.764	4.008	4.197	4.351	4.481	4.592	4.690
60	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646
80	2.814	3.377	3.711	3.947	4.129	4.277	4.402	4.509	4.603
120	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560
240	2.786	3.335	3.659	3.887	4.063	4.205	4.324	4.427	4.517
inf	2.772	3.314	3.633	3.858	4.030	4.170	4.286	4.387	4.474

Bonferroni t-table (2-sided, $\alpha = 0.05$)

df\#Comparisons	1	3	6	10	15	21	28	36	45
1	12.706	38.188	76.390	127.321	190.984	267.379	356.506	458.366	572.957
2	4.303	7.649	10.886	14.089	17.277	20.457	23.633	26.805	29.975
3	3.182	4.857	6.232	7.453	8.575	9.624	10.617	11.563	12.471
4	2.776	3.961	4.851	5.598	6.254	6.847	7.392	7.900	8.376
5	2.571	3.534	4.219	4.773	5.247	5.666	6.045	6.391	6.713
6	2.447	3.287	3.863	4.317	4.698	5.030	5.326	5.594	5.840
7	2.365	3.128	3.636	4.029	4.355	4.636	4.884	5.107	5.310
8	2.306	3.016	3.479	3.833	4.122	4.370	4.587	4.781	4.957
9	2.262	2.933	3.364	3.690	3.954	4.179	4.374	4.549	4.706
10	2.228	2.870	3.277	3.581	3.827	4.035	4.215	4.375	4.518
11	2.201	2.820	3.208	3.497	3.728	3.923	4.091	4.240	4.373
12	2.179	2.779	3.153	3.428	3.649	3.833	3.992	4.133	4.258
13	2.160	2.746	3.107	3.372	3.584	3.760	3.912	4.045	4.164
14	2.145	2.718	3.069	3.326	3.530	3.699	3.845	3.973	4.086
15	2.131	2.694	3.036	3.286	3.484	3.648	3.788	3.911	4.021
16	2.120	2.673	3.008	3.252	3.444	3.604	3.740	3.859	3.965
17	2.110	2.655	2.984	3.222	3.410	3.565	3.698	3.814	3.917
18	2.101	2.639	2.963	3.197	3.380	3.532	3.661	3.774	3.874
19	2.093	2.625	2.944	3.174	3.354	3.503	3.629	3.739	3.837
20	2.086	2.613	2.927	3.153	3.331	3.477	3.601	3.709	3.804
21	2.080	2.601	2.912	3.135	3.310	3.453	3.575	3.681	3.775
22	2.074	2.591	2.899	3.119	3.291	3.432	3.552	3.656	3.749
23	2.069	2.582	2.886	3.104	3.274	3.413	3.531	3.634	3.725
24	2.064	2.574	2.875	3.091	3.258	3.396	3.513	3.614	3.703
25	2.060	2.566	2.865	3.078	3.244	3.380	3.495	3.595	3.684
26	2.056	2.559	2.856	3.067	3.231	3.366	3.480	3.578	3.666
27	2.052	2.552	2.847	3.057	3.219	3.353	3.465	3.563	3.649
28	2.048	2.546	2.839	3.047	3.208	3.340	3.452	3.549	3.634
29	2.045	2.541	2.832	3.038	3.198	3.329	3.440	3.535	3.620
30	2.042	2.536	2.825	3.030	3.189	3.319	3.428	3.523	3.607
40	2.021	2.499	2.776	2.971	3.122	3.244	3.347	3.436	3.514
50	2.009	2.477	2.747	2.937	3.083	3.201	3.300	3.386	3.461
60	2.000	2.463	2.729	2.915	3.057	3.173	3.270	3.353	3.426
70	1.994	2.453	2.715	2.899	3.039	3.153	3.248	3.330	3.402
80	1.990	2.445	2.705	2.887	3.026	3.138	3.232	3.313	3.383
90	1.987	2.440	2.698	2.878	3.016	3.127	3.220	3.299	3.369
100	1.984	2.435	2.692	2.871	3.007	3.118	3.210	3.289	3.358
110	1.982	2.431	2.687	2.865	3.001	3.110	3.202	3.280	3.349
120	1.980	2.428	2.683	2.860	2.995	3.104	3.195	3.273	3.342
130	1.978	2.425	2.679	2.856	2.990	3.099	3.190	3.267	3.335
140	1.977	2.423	2.676	2.852	2.986	3.095	3.185	3.262	3.330
150	1.976	2.421	2.674	2.849	2.983	3.091	3.181	3.258	3.325
160	1.975	2.419	2.671	2.846	2.980	3.087	3.177	3.254	3.321
170	1.974	2.418	2.669	2.844	2.977	3.084	3.174	3.251	3.317
180	1.973	2.417	2.668	2.842	2.975	3.082	3.171	3.247	3.314
190	1.973	2.415	2.666	2.840	2.973	3.079	3.169	3.245	3.311
200	1.972	2.414	2.665	2.839	2.971	3.077	3.166	3.242	3.309
400	1.966	2.404	2.651	2.823	2.953	3.058	3.145	3.220	3.285
600	1.964	2.401	2.647	2.817	2.947	3.051	3.138	3.212	3.277
800	1.963	2.399	2.645	2.815	2.944	3.048	3.134	3.208	3.273
1000	1.962	2.398	2.644	2.813	2.942	3.046	3.132	3.206	3.270
inf	1.960	2.394	2.638	2.807	2.935	3.038	3.124	3.197	3.261

Critical Values for t , χ^2 , and F Distributions
F Distributions Indexed by Numerator Degrees of Freedom

df	$t_{.95}$	$t_{.975}$	$\chi^2_{.95}$	$F_{.95,1}$	$F_{.95,2}$	$F_{.95,3}$	$F_{.95,4}$	$F_{.95,5}$	$F_{.95,6}$	$F_{.95,7}$	$F_{.95,8}$
1	6.314	12.706	3.841	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883
2	2.920	4.303	5.991	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371
3	2.353	3.182	7.815	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845
4	2.132	2.776	9.488	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041
5	2.015	2.571	11.070	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818
6	1.943	2.447	12.592	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147
7	1.895	2.365	14.067	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726
8	1.860	2.306	15.507	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438
9	1.833	2.262	16.919	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230
10	1.812	2.228	18.307	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072
11	1.796	2.201	19.675	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948
12	1.782	2.179	21.026	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849
13	1.771	2.160	22.362	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767
14	1.761	2.145	23.685	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699
15	1.753	2.131	24.996	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641
16	1.746	2.120	26.296	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591
17	1.740	2.110	27.587	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548
18	1.734	2.101	28.869	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510
19	1.729	2.093	30.144	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477
20	1.725	2.086	31.410	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447
21	1.721	2.080	32.671	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420
22	1.717	2.074	33.924	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397
23	1.714	2.069	35.172	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375
24	1.711	2.064	36.415	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355
25	1.708	2.060	37.652	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337
26	1.706	2.056	38.885	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321
27	1.703	2.052	40.113	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305
28	1.701	2.048	41.337	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291
29	1.699	2.045	42.557	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278
30	1.697	2.042	43.773	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266
40	1.684	2.021	55.758	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180
50	1.676	2.009	67.505	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130
60	1.671	2.000	79.082	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097
70	1.667	1.994	90.531	3.978	3.128	2.736	2.503	2.346	2.231	2.143	2.074
80	1.664	1.990	101.879	3.960	3.111	2.719	2.486	2.329	2.214	2.126	2.056
90	1.662	1.987	113.145	3.947	3.098	2.706	2.473	2.316	2.201	2.113	2.043
100	1.660	1.984	124.342	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032
110	1.659	1.982	135.480	3.927	3.079	2.687	2.454	2.297	2.182	2.094	2.024
120	1.658	1.980	146.567	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016
130	1.657	1.978	157.610	3.914	3.066	2.674	2.441	2.284	2.169	2.081	2.010
140	1.656	1.977	168.613	3.909	3.061	2.669	2.436	2.279	2.164	2.076	2.005
150	1.655	1.976	179.581	3.904	3.056	2.665	2.432	2.274	2.160	2.071	2.001
160	1.654	1.975	190.516	3.900	3.053	2.661	2.428	2.271	2.156	2.067	1.997
170	1.654	1.974	201.423	3.897	3.049	2.658	2.425	2.267	2.152	2.064	1.993
180	1.653	1.973	212.304	3.894	3.046	2.655	2.422	2.264	2.149	2.061	1.990
190	1.653	1.973	223.160	3.891	3.043	2.652	2.419	2.262	2.147	2.058	1.987
200	1.653	1.972	233.994	3.888	3.041	2.650	2.417	2.259	2.144	2.056	1.985
∞	1.645	1.960	---	3.841	2.995	2.605	2.372	2.214	2.099	2.010	1.938