

## R Material for Chapter 11

```
> dbp.data          ## Our Data
  age dbp
1  27  73
2  21  66
3  22  63
4  24  75
5  25  71
6  23  70
7  20  65
8  20  70
9  29  79
10 24  72
11 25  68
12 28  67
13 26  79
14 38  91
15 32  76
16 33  69
17 31  66
18 34  73
19 37  78
20 38  87
21 33  76
22 35  79
23 30  73
24 31  80
25 37  68
26 39  75
27 46  89
28 49 101
29 40  70
30 42  72
31 43  80
32 46  83
33 43  75
34 44  71
35 46  80
36 47  96
37 45  92
38 49  80
39 48  70
40 40  90
41 42  85
42 55  76
43 54  71
44 57  99
45 52  86
46 53  79
47 56  92
48 52  85
49 50  71
50 59  90
51 50  91
52 52 100
53 58  80
54 57 109

> attach(dbp.data)
> lm.dbp <- lm(dbp~age)
```

```

> summary(lm.dbp)

Call:
lm(formula = dbp ~ age)

Residuals:
    Min       1Q   Median       3Q      Max
-16.47859  -5.78765  -0.07844   5.61173  19.78132

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 56.15693    3.99367  14.061 < 2e-16 ***
age          0.58003    0.09695   5.983 2.05e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.146 on 52 degrees of freedom
Multiple R-squared: 0.4077,    Adjusted R-squared: 0.3963
F-statistic: 35.79 on 1 and 52 DF,  p-value: 2.050e-07

```

### Weighted Least Squares:

```

> res <- residuals(lm.dbp)    ## residuals from the fit
> plot(age,res)              ## scatter plot of the residuals vs age
> abres <- abs(res)          ## absolute values of the residuals
> reg2 <- lm(abres~age)      ## regressing the absolute values on age
> fit <- fitted(reg2)        ## the fitted values from that regression
> wt <- fit^-2               ## weights equal to 1/(square of fitted value)
> wreg <- lm(dbp~age,weights=wt)  ## weighted least squares regression
> summary(wreg)

Call:
lm(formula = dbp ~ age, weights = wt)

Residuals:
    Min       1Q   Median       3Q      Max
-2.0230 -0.9939 -0.0327  0.9250  2.2008

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 55.56577    2.52092  22.042 < 2e-16 ***
age          0.59634    0.07924   7.526 7.19e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.213 on 52 degrees of freedom
Multiple R-squared: 0.5214,    Adjusted R-squared: 0.5122
F-statistic: 56.64 on 1 and 52 DF,  p-value: 7.187e-10

```

```

> bfat.data
  tric thigh midarm bfat    ## the data
1 19.5 43.1 29.1 11.9
2 24.7 49.8 28.2 22.8
3 30.7 51.9 37.0 18.7
4 29.8 54.3 31.1 20.1
5 19.1 42.2 30.9 12.9
6 25.6 53.9 23.7 21.7
7 31.4 58.5 27.6 27.1
8 27.9 52.1 30.6 25.4
9 22.1 49.9 23.2 21.3
10 25.5 53.5 24.8 19.3
11 31.1 56.6 30.0 25.4
12 30.4 56.7 28.3 27.2
13 18.7 46.5 23.0 11.7
14 19.7 44.2 28.6 17.8
15 14.6 42.7 21.3 12.8
16 29.5 54.4 30.1 23.9
17 27.7 55.3 25.7 22.6
18 30.2 58.6 24.6 25.4
19 22.7 48.2 27.1 14.8
20 25.2 51.0 27.5 21.1

> attach(bfat.data)

> regbf <- lm(bfat~tric+thigh+midarm)

> summary(regbf)

Call:
lm(formula = bfat ~ tric + thigh + midarm)

Residuals:
    Min       1Q   Median       3Q      Max
-3.7263 -1.6111  0.3923  1.4656  4.1277

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  117.085     99.782   1.173   0.258
tric           4.334      3.016   1.437   0.170
thigh        -2.857      2.582  -1.106   0.285
midarm       -2.186      1.595  -1.370   0.190

Residual standard error: 2.48 on 16 degrees of freedom
Multiple R-squared: 0.8014,    Adjusted R-squared: 0.7641
F-statistic: 21.52 on 3 and 16 DF,  p-value: 7.343e-06

> library(MASS)

> ridgereg <- lm.ridge(bfat~tric+thigh+midarm,lambda=seq(0,.05,.001))
> ridgereg
      tric      thigh      midarm    ## coefficients corresponding to different
0.000 117.08469 4.334092 -2.8568479 -2.1860603    ## values of c
0.001 108.76152 4.082244 -2.6412701 -2.0533640
0.002 101.44532 3.860862 -2.4517737 -1.9367189
0.003  94.96379 3.664733 -2.2838958 -1.8333783
0.004  89.18181 3.489769 -2.1341369 -1.7411885
0.005  83.99195 3.332719 -1.9997141 -1.6584368
0.006  79.30768 3.190965 -1.8783865 -1.5837444
0.007  75.05856 3.062378 -1.7683298 -1.5159882
0.008  71.18666 2.945203 -1.6680432 -1.4542448
0.009  67.64388 2.837985 -1.5762812 -1.3977476
0.010  64.38999 2.739508 -1.4920015 -1.3458552
0.011  61.39103 2.648744 -1.4143247 -1.2980264

```

```
0.012 58.61818 2.564820 -1.3425044 -1.2538019
0.013 56.04681 2.486992 -1.2759025 -1.2127888
0.014 53.65573 2.414619 -1.2139701 -1.1746494
0.015 51.42662 2.347147 -1.1562331 -1.1390920
0.016 49.34357 2.284093 -1.1022791 -1.1058628
0.017 47.39269 2.225038 -1.0517482 -1.0747402
0.018 45.56178 2.169613 -1.0043248 -1.0455300
0.019 43.84011 2.117493 -0.9597309 -1.0180612
0.020 42.21820 2.068391 -0.9177207 -0.9921824
0.021 40.68762 2.022053 -0.8780759 -0.9677594
0.022 39.24086 1.978250 -0.8406024 -0.9446726
0.023 37.87124 1.936781 -0.8051267 -0.9228153
0.024 36.57274 1.897464 -0.7714931 -0.9020917
0.025 35.33997 1.860136 -0.7395618 -0.8824158
0.026 34.16806 1.824649 -0.7092070 -0.8637100
0.027 33.05262 1.790870 -0.6803147 -0.8459043
0.028 31.98966 1.758679 -0.6527817 -0.8289352
0.029 30.97556 1.727967 -0.6265144 -0.8127450
0.030 30.00704 1.698633 -0.6014275 -0.7972812
0.031 29.08110 1.670587 -0.5774432 -0.7824961
0.032 28.19497 1.643747 -0.5544905 -0.7683457
0.033 27.34617 1.618035 -0.5325042 -0.7547902
0.034 26.53237 1.593383 -0.5114246 -0.7417927
0.035 25.75145 1.569725 -0.4911969 -0.7293193
0.036 25.00147 1.547003 -0.4717703 -0.7173391
0.037 24.28063 1.525163 -0.4530983 -0.7058232
0.038 23.58725 1.504154 -0.4351378 -0.6947452
0.039 22.91980 1.483929 -0.4178489 -0.6840806
0.040 22.27686 1.464446 -0.4011946 -0.6738065
0.041 21.65710 1.445664 -0.3851407 -0.6639019
0.042 21.05928 1.427546 -0.3696554 -0.6543472
0.043 20.48227 1.410058 -0.3547089 -0.6451241
0.044 19.92500 1.393166 -0.3402737 -0.6362157
0.045 19.38648 1.376842 -0.3263239 -0.6276060
0.046 18.86577 1.361057 -0.3128356 -0.6192803
0.047 18.36201 1.345785 -0.2997862 -0.6112248
0.048 17.87438 1.331001 -0.2871547 -0.6034264
0.049 17.40212 1.316681 -0.2749213 -0.5958731
0.050 16.94452 1.302806 -0.2630676 -0.5885534
```

```
> plot(lm.ridge(bfat~tric+thigh+midarm,lambda=seq(0,.05,.001))) ## plot of coefficients
```

```
> select(lm.ridge(bfat~tric+thigh+midarm,lambda=seq(0,.05,.001)))
modified HKB estimator is 0.008505093
modified L-W estimator is 0.3098511
smallest value of GCV at 0.019          ## selected value of c
```

## The SAS System

Obs	_RIDGE_	tric	thigh	midarm
4	0.000	4.26370	-2.92870	-1.56142
7	0.002	1.44066	-0.41129	-0.48127
10	0.004	1.00632	-0.02484	-0.31487
13	0.006	0.83002	0.13142	-0.24716
16	0.008	0.73433	0.21576	-0.21030
19	0.010	0.67417	0.26841	-0.18703
22	0.012	0.63279	0.30431	-0.17095
25	0.014	0.60254	0.33029	-0.15912
28	0.016	0.57942	0.34990	-0.15003
31	0.018	0.56115	0.36519	-0.14279
34	0.020	0.54633	0.37740	-0.13687
37	0.022	0.53405	0.38736	-0.13193
40	0.024	0.52369	0.39560	-0.12772
43	0.026	0.51482	0.40252	-0.12408
46	0.028	0.50712	0.40839	-0.12090
49	0.030	0.50038	0.41341	-0.11808
52	0.032	0.49441	0.41775	-0.11556
55	0.034	0.48908	0.42152	-0.11328
58	0.036	0.48429	0.42480	-0.11122
61	0.038	0.47995	0.42769	-0.10933
64	0.040	0.47600	0.43024	-0.10758
67	0.042	0.47239	0.43249	-0.10597
70	0.044	0.46906	0.43448	-0.10447
73	0.046	0.46598	0.43626	-0.10307
76	0.048	0.46312	0.43784	-0.10175
79	0.050	0.46046	0.43924	-0.10051
82	0.052	0.45797	0.44050	-0.09933
85	0.054	0.45563	0.44162	-0.09822
88	0.056	0.45343	0.44263	-0.09716
91	0.058	0.45136	0.44352	-0.09615
94	0.060	0.44939	0.44432	-0.09519
97	0.062	0.44753	0.44503	-0.09426
100	0.064	0.44576	0.44566	-0.09337
103	0.066	0.44407	0.44622	-0.09252
106	0.068	0.44246	0.44671	-0.09169
109	0.070	0.44092	0.44715	-0.09089
112	0.072	0.43944	0.44753	-0.09012
115	0.074	0.43802	0.44786	-0.08938
118	0.076	0.43665	0.44814	-0.08865
121	0.078	0.43534	0.44838	-0.08795
124	0.080	0.43407	0.44858	-0.08726
127	0.082	0.43285	0.44874	-0.08660
130	0.084	0.43166	0.44887	-0.08595
133	0.086	0.43051	0.44897	-0.08531
136	0.088	0.42940	0.44904	-0.08469
139	0.090	0.42832	0.44909	-0.08409
142	0.092	0.42728	0.44911	-0.08350
145	0.094	0.42626	0.44910	-0.08292
148	0.096	0.42526	0.44907	-0.08235
151	0.098	0.42430	0.44903	-0.08179
154	0.100	0.42335	0.44896	-0.08125

## The SAS System

Obs	_RIDGE_	tric	thigh	midarm
2	0.000	708.843	564.343	104.606
5	0.002	50.559	40.448	8.280
8	0.004	16.982	13.725	3.363
11	0.006	8.503	6.976	2.119
14	0.008	5.147	4.305	1.624
17	0.010	3.486	2.981	1.377
20	0.012	2.543	2.231	1.236
23	0.014	1.958	1.764	1.146
26	0.016	1.570	1.454	1.086
29	0.018	1.299	1.238	1.043
32	0.020	1.103	1.081	1.011
35	0.022	0.956	0.963	0.986
38	0.024	0.843	0.872	0.966
41	0.026	0.754	0.801	0.949
44	0.028	0.683	0.744	0.935
47	0.030	0.626	0.697	0.923
50	0.032	0.578	0.658	0.913
53	0.034	0.539	0.626	0.904
56	0.036	0.505	0.599	0.896
59	0.038	0.477	0.575	0.888
62	0.040	0.453	0.555	0.881
65	0.042	0.432	0.538	0.875
68	0.044	0.413	0.522	0.869
71	0.046	0.397	0.509	0.863
74	0.048	0.383	0.497	0.858
77	0.050	0.370	0.486	0.853
80	0.052	0.359	0.476	0.848
83	0.054	0.349	0.467	0.844
86	0.056	0.340	0.459	0.839
89	0.058	0.332	0.452	0.835
92	0.060	0.324	0.445	0.831
95	0.062	0.318	0.439	0.827
98	0.064	0.311	0.434	0.823
101	0.066	0.306	0.428	0.819
104	0.068	0.300	0.423	0.815
107	0.070	0.296	0.419	0.811
110	0.072	0.291	0.415	0.807
113	0.074	0.287	0.411	0.804
116	0.076	0.283	0.407	0.800
119	0.078	0.280	0.403	0.797
122	0.080	0.276	0.400	0.793
125	0.082	0.273	0.397	0.790
128	0.084	0.270	0.394	0.787
131	0.086	0.267	0.391	0.783
134	0.088	0.265	0.388	0.780
137	0.090	0.262	0.385	0.777
140	0.092	0.260	0.383	0.774
143	0.094	0.258	0.380	0.771
146	0.096	0.255	0.378	0.767
149	0.098	0.253	0.376	0.764
152	0.100	0.252	0.373	0.761

```

> autoins.data
  prem age price
1   221  57 11804
2   448   8 12926
3   515   6 14054
4   632  12 17486
5    48  47  8700
6   289  30  8570
7   581  34 18982
8   202  30  9198
9   404  33 14986
10  283  59  8473
11  280  56 13891
12  565  13 16127
13 1105  10 29480
14  388  46 15868
15  435   2 10782
16  309  11  8645
17  322  17  9086
18  741  32 22559
19  500  34 14969
20 1051  34 29733
21  845   4 22893
22  278  59 15198
23  333  56 16696
24  650  34 20411
25  772  27 23128
26  947  19 16507
27  443  37 13704
28  692   3 16472
29  618  36 18422
30 1050   7 27110
31  643  45 22968
32  116  46  9177
33  269   9  8977
34  259  38 10514
35  491  16 13739

```

```

> reg <- lm(prem~age+price)

```

```

> summary(reg)

```

Call:

```
lm(formula = prem ~ age + price)
```

Residuals:

Min	1Q	Median	3Q	Max
-101.152	-50.447	-14.119	9.674	365.176

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	49.42378	51.07753	0.968	0.340
age	-4.89030	0.82872	-5.901	1.45e-06 ***
price	0.03788	0.00248	15.277	2.97e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 85.99 on 32 degrees of freedom

Multiple R-squared: 0.905, Adjusted R-squared: 0.8991

F-statistic: 152.4 on 2 and 32 DF, p-value: < 2.2e-16

```

> res <- residuals(reg)
> Me <- median(res)
> mad <- median(abs(res-Me))/(0.6745)
> estar <- res/mad
> aestar <- abs(estar)
> wt <- rep(1,35)
> for(i in 1:35) {if (aestar[i] > 1.345) wt[i] <- 1.345/aestar[i]}

> wt
 [1] 1.0000000 1.0000000 1.0000000 1.0000000 0.6906260 1.0000000 1.0000000
 [8] 1.0000000 1.0000000 0.3473276 1.0000000 1.0000000 1.0000000 1.0000000
[15] 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
[22] 1.0000000 0.9309111 1.0000000 1.0000000 0.1913004 1.0000000 1.0000000
[29] 1.0000000 1.0000000 1.0000000 1.0000000 0.9134579 1.0000000 1.0000000

> reg <- lm(prem~age+price,weights=wt)
> summary(reg)

Call:
lm(formula = prem ~ age + price, weights = wt)

Residuals:
    Min       1Q   Median       3Q      Max
-70.851 -36.897  -7.808  21.927 164.482

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 31.922691  32.496973   0.982   0.333
age         -4.941155   0.539822  -9.153 1.89e-10 ***
price         0.038341   0.001587  24.162 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 54.08 on 32 degrees of freedom
Multiple R-squared: 0.958, Adjusted R-squared: 0.9554
F-statistic: 365.1 on 2 and 32 DF, p-value: < 2.2e-16

```

```

> res <- residuals(reg)
> Me <- median(res)
> mad <- median(abs(res-Me))/(0.6745)
> estar <- res/mad
> aestar <- abs(estar)
> wt <- rep(1,35)
> for(i in 1:35) {if (aestar[i] > 1.345) wt[i] <- 1.345/aestar[i]}

```



```
> wt
 [1] 1.0000000 1.0000000 1.0000000 1.0000000 0.7399600 0.8222026 1.0000000
 [8] 1.0000000 1.0000000 0.2897309 1.0000000 1.0000000 1.0000000 1.0000000
[15] 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
[22] 1.0000000 1.0000000 1.0000000 1.0000000 0.1677550 0.9213300 1.0000000
[29] 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
```

```
> reg <- lm(prem~age+price,weights=wt)
```

```
> summary(reg)
```

```
Call:
```

```
lm(formula = prem ~ age + price, weights = wt)
```

```
Residuals:
```

```
    Min       1Q   Median       3Q      Max
-69.932 -33.906  -4.647   25.127 154.516
```

```
Coefficients:
```

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 27.845241  31.290246   0.890    0.38
age          -4.979054   0.518799  -9.597 6.14e-11 ***
price         0.038559   0.001532  25.165 < 2e-16 ***
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 52.12 on 32 degrees of freedom
```

```
Multiple R-squared:  0.961, Adjusted R-squared:  0.9586
```

```
F-statistic: 394.6 on 2 and 32 DF,  p-value: < 2.2e-16
```

```
> res <- residuals(reg)
```

```
> Me <- median(res)
```

```
> mad <- median(abs(res-Me))/(0.6745)
```

```
> estar <- res/mad
```

```
> aestar <- abs(estar)
```

```
> wt <- rep(1,35)
```

```
> for(i in 1:35) {if (aestar[i] > 1.345) wt[i] <- 1.345/aestar[i]}
```

```
> wt
```

```
 [1] 1.0000000 1.0000000 1.0000000 1.0000000 0.7854358 0.7974437 1.0000000
 [8] 1.0000000 1.0000000 0.2873614 1.0000000 1.0000000 1.0000000 1.0000000
[15] 1.0000000 1.0000000 1.0000000 1.0000000 0.9938708 1.0000000 1.0000000
[22] 1.0000000 1.0000000 1.0000000 1.0000000 0.1692565 0.8998252 1.0000000
[29] 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
```

```
> reg <- lm(prem~age+price,weights=wt)
```

```
> summary(reg)
```

```
Call:
```

```
lm(formula = prem ~ age + price, weights = wt)
```

```
Residuals:
```

```
    Min       1Q   Median       3Q      Max
-71.435 -33.316  -4.076  25.658 155.258
```

```
Coefficients:
```

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 27.243211  31.317609   0.870   0.391
age         -4.988303   0.518871  -9.614 5.89e-11 ***
price        0.038599   0.001533  25.181 < 2e-16 ***
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 52.15 on 32 degrees of freedom
```

```
Multiple R-squared:  0.9611,    Adjusted R-squared:  0.9587
```

```
F-statistic: 395.7 on 2 and 32 DF,  p-value: < 2.2e-16
```