

STA 7334: Limit Theory
Fall 2002
Assignment 3

Do problems 2.P.8 and 3.P.8 in Serfling, plus the following exercises:

1. For a $N(\mu, \sigma^2)$ distribution, $\xi_{3/4} - \xi_{1/4} = c\sigma$, where $c = \Phi^{-1}(3/4) - \Phi^{-1}(1/4) \approx 1.35$ is the difference in quartiles for the $N(0, 1)$ distribution. Thus when sampling from a $N(\mu, \sigma^2)$ distribution,

$$\tilde{\sigma} = \frac{\hat{\xi}_{3/4} - \hat{\xi}_{1/4}}{c}$$

is a strongly consistent estimator of σ (because $\hat{\xi}_{3/4}$ and $\hat{\xi}_{1/4}$ are strongly consistent).

- (a) Find the asymptotic distribution of $\tilde{\sigma}$.
- (b) Find $\text{ARE}(\tilde{\sigma}, s)$, the asymptotic efficiency of $\tilde{\sigma}$ relative to the sample standard deviation, s , when sampling from a normal distribution. Noting that the ARE does not depend on μ or σ , give its approximate numerical value.
2. Suppose that F is in the *domain of attraction* of G , i.e., there exist constants $a_n > 0$ and b_n , $n \geq 1$, such that $F^n(a_n x + b_n) \xrightarrow{d} G(x)$ (of course G must then be one of the three types of max-stable distribution). Let $k \geq 1$ be a fixed integer. For $X_1, \dots, X_n \sim \text{i.i.d. } F$, derive the asymptotic distribution of $X_{(n-k)}$.
3. For each $n \geq 1$, suppose that X_{n1}, \dots, X_{nn} are independent Bernoulli random variables with success probabilities p_{n1}, \dots, p_{nn} , respectively. Let

$$S_n = \sum_{i=1}^n X_{ni}, \quad a_n = \sum_{i=1}^n p_{ni}, \quad b_n = \sum_{i=1}^n p_{ni}^2, \quad c_n = A_n - B_n,$$

and

$$T_n = \frac{S_n - a_n}{\sqrt{c_n}}.$$

Prove that if $c_n \rightarrow \infty$, then $T_n \xrightarrow{d} N(0, 1)$.