# STA 7334: Limit Theory
# Fall 2002
# Additional Problems for Assignment 2

In addition to problems 3.P.4 and 3.P.8 in Serfling (pages 136–137), do the following exercises:

1. The key result in Slutsky's theorem is that if $X_n \xrightarrow{\text{d}} X$ and $Y_n \xrightarrow{P} c$, where $c$ is a finite constant, then $X_n + Y_n \xrightarrow{\text{d}} X + c$. This was proven in your probability class for real-valued random variables, but the result holds more generally. Prove that in particular it holds for random vectors taking values in $\mathbb{R}^k$. You may use Slutsky's theorem for real-valued random variables in your proof.

2. (a) Give an example where $X_n \xrightarrow{\text{d}} X$ and $Y_n \xrightarrow{\text{d}} Y$, but $(X_n, Y_n)$ does not converge in distribution.

   (b) If $X_n$ and $Y_n$ are independent random vectors, and $X_n \xrightarrow{\text{d}} X$ and $Y_n \xrightarrow{\text{d}} Y$, show that $(X_n, Y_n) \xrightarrow{\text{d}} (X, Y)$, where $X$ and $Y$ are independent.

3. (a) Show that if $R_n = O_p(1)$ as $n \to \infty$, then

$$\frac{1}{1 + o_p(R_n)} = 1 + o_p(R_n).$$

   (b) Show that if $R_n = o_p(1)$ as $n \to \infty$, then

$$\frac{1}{1 + O_p(R_n)} = 1 + O_p(R_n).$$

   (c) Show that (a) may not hold without the assumption $R_n = O_p(1)$. *Hint: Use deterministic sequences.*

   (d) Show that (b) may not hold if $R_n = O_p(1)$. *Hint: Use deterministic sequences.*