Example 2 Suppose the true cell means were the following:

	Fertilizer		Row	Main row
Manure	Low	High	Average	effect
Low	12.0	14.0	13.0	−2.0
High	16.0	18.0	17.0	2.0
Column average	14.0	16.0	15.0	
Main col. effect	−1.0	1.0		

2nd definition of additivity

In this special case, the factor effects are **additive.** This means, for instance, that each cell mean can be found by adding the respective row and column effects to the overall mean $\mu_{..}$.

$$\mu_{11} = \mu_{..} + \alpha_1 + \beta_1 = 15 + (-2) + (-1) = 12 ✓$$

$$\mu_{12} = \mu_{..} + \alpha_1 + \beta_2 = 15 + (-2) + 1 = 14 ✓$$

μ_{21}

$\vdots$

μ_{22}

$\mu_{ij} = \mu_{..} + (\mu_{i.} - \mu_{..}) + (\mu_{.j} - \mu_{..})$

$= \mu_{i.} + \mu_{.j} - \mu_{..}$

This also means that $\mu_{ij} = \mu_{i.} + \mu_{.j} - \mu_{..}$.

This is an example of an **additive** model. We also say there is **no interaction** between the two factors.

Example 2

Manure	Low	High
Low	12.0	14.0
High	16.0	18.0

Compare *simple effects* of Manure at all levels of Fertilizer. Is $\mu_{21} - \mu_{11} = \mu_{22} - \mu_{12}$? Answer is "Yes, $4 = 4$." So, by our first definition of interaction, there is no interaction between Manure and Fertilizer in this case. This is an *additive model*.

Another definition of additivity is based on using the main effects and grand mean. Does $\mu_{ij} = \mu_{..} + \alpha_i + \beta_j$ for every pair (i, j)? If yes, then the model is additive. Here, again, the answer is "Yes."

The *interaction plot* is a useful plot of the cell means which enables us to easily spot whether there is any interaction, and to diagnose the nature of any interaction that exists.

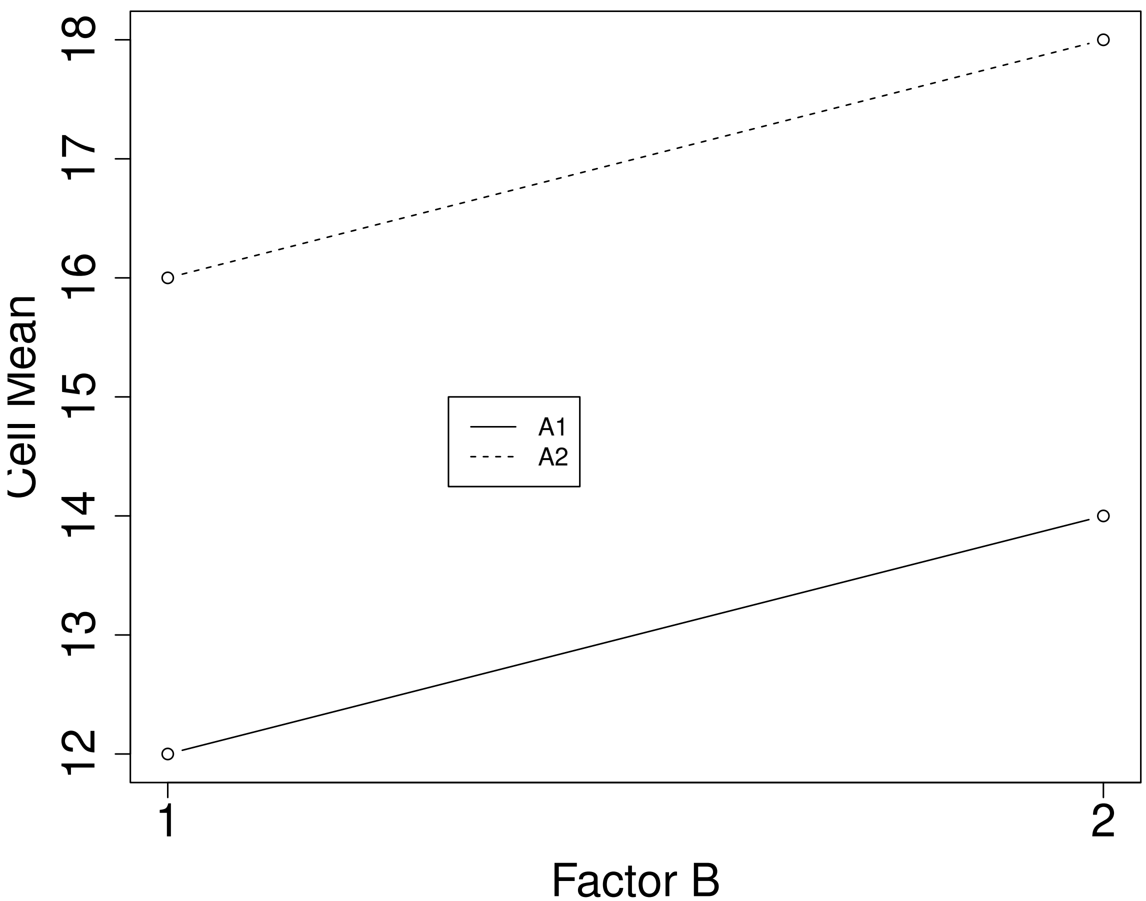
Example 2

	Fertilizer	
Manure	Low	High
Low	12.0	14.0
High	16.0	18.0

Interaction plot (true cell means)

Interaction Plot is of **treatment** means with levels of one factor represented on the horizontal axis, and treatment means for the levels of the other factor connected by line segments.

Interaction plot, additive model (Example 2):

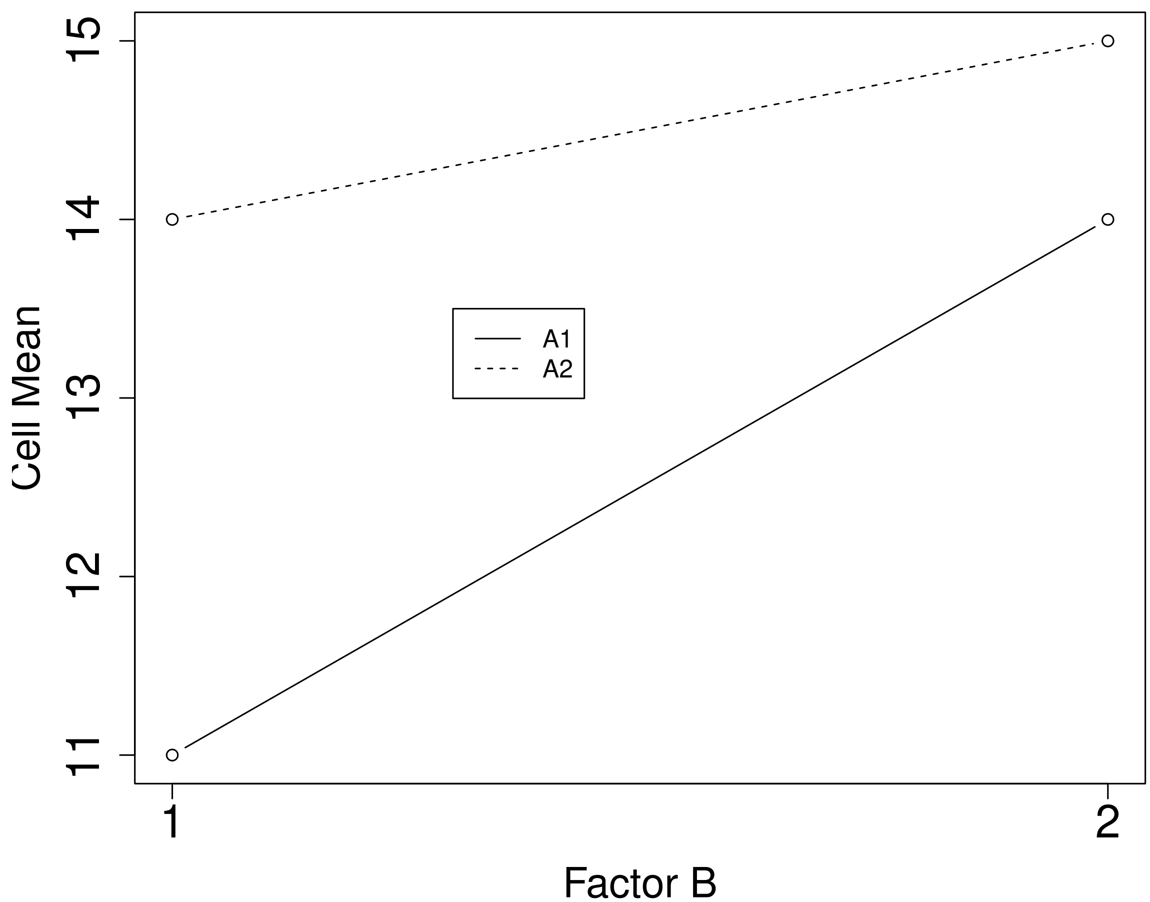


Interaction Plot, Example 2

The lines are parallel. This means the effect of Factor A is constant for all levels of Factor B.

So, Factors A and B do not interact.

Interaction plot, model with interaction (Example 1):



Interaction plot, model with interaction (Example 1):
The lines are not parallel. So, we do have interaction between Factors A and B in their effects on the response Y.

Let $\alpha_i = \mu_{i.} - \mu_{..}$ represent the i^{th} row effect, and $\beta_j = \mu_{.j} - \mu_{..}$ represent the j^{th} column effect:

If for all i, j, the cell mean $\mu_{ij} = \mu_{..} + \alpha_i + \beta_j$, then the factor effects are additive. Otherwise, the factor effects are interacting.

Interaction of i^{th} level of Factor A with j^{th} level of Factor B is defined to be

$$(\alpha\beta)_{ij} = \mu_{ij} - (\mu_{..} + \alpha_i + \beta_j)$$

Example (p. 111)

	Fertilizer		Row	Main row
Manure	Low	High	Average	effect
Low	11.0	14.0	12.5	-1.0
High	14.0	15.0	14.5	1.0
Column avg.	12.5	14.5	$\mu_{..} = 13.5$	
Main col. effects	-1.0	1.0		

$$(\alpha\beta)_{11} = 11.0 - (13.5 + (-1.0) + (-1.0)) = 11.0 - 11.5 = -.5$$

$$(\alpha\beta)_{12} = 14.0 - (13.5 + (1.0) + (1.0)) = 14.0 - 13.5 = .5$$

$(\alpha\beta)_{21}$ = .5

$\vdots$

$(\alpha\beta)_{22}$ = $-.5$