Lecture 8 Friday January 27, 2023
Example of two-sample pooled t-test, by hand
Iron deficiency in breast-fed vs. formula-fed infants

A study of iron deficiency among infants compared samples of infants following different feeding regimens. One group contained breast-fed infants, while the infants in another group were fed a standard baby formula without any iron supplements. Here are summary results on blood hemoglobin levels (in grams per deciliter or g/dl) at 12 months of age:

Group	n	$\bar{x}$	s
Breast-fed	23	13.3	1.7
Formula	19	12.4	1.8

Is there significant evidence that the mean hemoglobin level is higher among breast-fed babies? In carrying out this test of hypothesis, follow the steps listed below:

(i) Define the parameter of interest.

(ii) State the null and alternative hypotheses; use a two-sided test.

(iii) State the assumptions for the standard ("pooled") two-sample t-test in this context.

(iv) Compute the test statistic. Show your work clearly. State the null distribution of the test statistic.

(v) Find the P-value; explain briefly how you found it. (You may use a table of the t distribution, or R.)

(vi) State your conclusion: Do you reject or fail to reject H_0? What does this result tell you about the comparison between breast-feeding and formula?

Solution

Population 1 is the values of hemoglobin levels (Y) among all breast-fed babies. We assume the distribution of Y is $\mathcal{N}(\mu_Y, \sigma^2)$. Population 2 is the values of hemoglobin levels (Z) among all babies, if fed the standard baby formula without iron supplement. We assume the distribution of Z is $\mathcal{N}(\mu_Z, \sigma^2)$. We have assumed the variances of the two populations are equal. We also will act as if we have random samples from each of the two populations, although this assumption is not likely to hold.

(i) The parameter of interest is the difference between the two population means: $\mu_Y - \mu_Z$. Note: It is up to the student whether to use $\mu_Y - \mu_Z$, or $\mu_Z - \mu_Y$.

(ii) The hypotheses: $H_0 : \mu_Y - \mu_Z = 0$ vs. $H_a : \mu_Y - \mu_Z \neq 0$.

(iii) The assumptions are stated in the introductory paragraph. Or, you can say: The assumptions are that the Ys and the Zs are all independent and normally distributed with the same variance σ^2.

(iv) The test statistic: First compute the difference of the two sample means, $\bar{Y} - \bar{Z} = 13.3 - 12.4 = .9$. Next compute the pooled estimate of variance,

$$\hat{\sigma}^2 = \frac{(n_Y - 1)S_Y^2 + (n_Z - 1)S_Z^2}{(n_Y + n_Z - 2)} = \frac{22(1.7^2) + 18(1.8^2)}{23 + 19 - 2} = \frac{121.9}{40} = 3.0475$$

Now we can compute the test statistic:

$$t_{\text{obs}} = \frac{\bar{Y} - \bar{Z}}{\sqrt{\left(\frac{1}{n_Y} + \frac{1}{n_Z}\right)\frac{(n_Y-1)S_Y^2+(n_Z-1)S_Z^2}{(n_Y+n_Z-2)}}}$$
$$= \frac{.9}{\sqrt{(\frac{1}{23} + \frac{1}{19}) \times 3.047}} = \frac{.9}{.5412} = 1.663$$

Under the assumptions stated above, the null distribution of the test statistic is t with $n_Y + n_Z - 2 = 40$ d.f.

(v) The P-value is found as $P = P(|t_{40}| > 1.663)$. From the t-table row for 40 d.f. we find the observed test statistic is between the .9 and .95 quantiles ($1.303 < 1.663 < 1.684$), so $.1 < P < .2$ for a two-sided test.

In R we get the exact P-value

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> P <- 2*(1-pt(1.663,df=40)); P
[1] 0.10413
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(vi) Since .1 is the largest conventional Type I error probability, we fail to reject H_0 at all the usual significance levels. We cannot conclude that there is difference between the population mean hemoglobin levels for breast-fed babies versus formula-fed babies. We do note that for these particular infants, the observed difference $\bar{Y} - \bar{Z} = .9$g/dl is a large practical difference, such that these breast-fed babies are healthier on average than the formula-fed babies in terms of hemoglobin. It is possible that with larger sample sizes, the observed difference would be statistically significant as well.

The main limitation of the study is that it is observational. There are many possible confounding variables that might explain the observed difference, other than source of food.