STA 6208 – Spring 2002 – Exam 4

Print Name: **SSN:**

1) A model is fit relating therapeutic response (Y) to dose (X) given to the patient. The model fit is:

$$Y = \frac{\beta_1 X}{X + \beta_2} + \varepsilon$$

A nonlinear least squares program obtains the estimates: $\hat{\beta}_1 = 5.0$ and $\hat{\beta}_2 = 500$.

a) Give the fitted values at doses of 0, 250, 500, 1000, 2000, ∞.

b) Sketch the function, and identify approximately the range of doses that correspond to providing 20% to 80% of maximal effect (this is sometimes referred to as the therapeutic window).

2) A marketing firm wishes to compare 3 marketing strategies for a new product. They sample 12 test cities of similar sizes, and randomly assign 4 cities to each strategy. Further, they have information regarding sales in the past year of a similar product (X) in each city (in the past they used only one strategy). They observe Y, the sales of the new product in the first six weeks on the market in each city. They fit the model (i represents strategy, j represents city/replicate):

$$Y_{ij} = \mu_i + \beta(X_{ij} - \overline{X}_{..}) + \varepsilon_{ij} \qquad \varepsilon \sim NID(0, \sigma^2)$$

where $\mu_i = \mu + \tau_i$ is the true mean for strategy i (this makes this a full rank model).

a) Give the expected value of $\overline{Y}_{i.}$, the unadjusted sample mean for the i^th strategy.

b) Suppose $\hat{\boldsymbol{\beta}}$ is the least squares estimate of $\boldsymbol{\beta}$ in the matrix version of this problem. What matrix would you multiply by $\hat{\boldsymbol{\beta}}$ to obtain the vector of adjusted means? What would be the variance covariance matrix of the vector of adjusted means?

3) A very poorly designed experiment results in the following sample summary statistics for an experiment to fit a simple linear regression relating a response (Y) to an exposure (X). The replicate sizes and sample means are given for each level of X.

X	reps	$\overline{Y}$
0	1	10
2	4	14
4	9	16
6	9	20
8	4	22

a) Give the weighted least squares estimator.

b) Give its estimated variance-covariance matrix (up to the constant $MS(Residual)$).

4) In a multiple regression model with $p = 2$ predictors, for what values of the correlation of coefficient between X_1 and X_2 would multicollinearity be considered a serious problem?