Split Plot and Repeated Measures Designs

Part A. Split Plot Designs

QA.1. An experiment is to be conducted to compare 4 cooking temperatures and 3 mixtures of alloys on strength measurements of steel rods. The cooking period is 2 hours, so that only 4 cooking periods can be conducted on a business day (the temperatures are randomly assigned to periods). The experimenter decides she will assign the 3 mixtures at random to the 3 positions in the oven (experience implies there are no position effects), separately for the 4 runs on a given day. She repeats the experiment over 3 days (randomizing separately on each day). Her assistant provides her the following sequences of random numbers for temperature and mixtures. Give the assignment of treatments to experimental positions (in each cell, enter Ti/Mj where i=Temperature level, j=Mixture level).

Temp	1	2	3	4	1	2	3	4	1	2	3	4
Ran#	0.96	0.91	0.16	0.22	0.21	0.34	0.28	0.27	0.75	0.23	0.37	0.20
Mix	1	2	3	1	2	3	1	2	3	1	2	3
Ran#	0.10	0.54	0.58	0.59	0.70	0.79	0.26	0.74	0.72	0.83	0.17	0.56
Mix	1	2	3	1	2	3	1	2	3	1	2	3
Ran#	0.29	0.64	0.43	0.76	0.65	0.56	0.97	0.28	0.06	0.83	0.76	0.64
Mix	1	2	3	1	2	3	1	2	3	1	2	3
Ran#	0.91	0.55	0.83	0.43	0.69	0.19	0.98	0.79	0.36	0.26	0.78	0.11

	Day1	Day1	Day1	Day2	Day2	Day2	Day3	Day3	Day3
	Pos1	Pos2	Pos3	Pos1	Pos2	Pos3	Pos1	Pos2	Pos3
Per1									
Per2									
Per3									
Per4									

QA.2. A split-plot experiment is conducted to compare 5 cooking conditions (combinations of temperature/time) and 8 recipes for quality of taste of cupcakes. Because of the logistics of the experiment, each of the 5 cooking conditions can be conducted once per day (in random order). The recipes are randomly assigned to the slots in the oven (each recipe is observed once in each cooking condition). The experiment is conducted on 3 different days (blocks). Give the Analysis of Variance (sources and degrees of freedom and critical F-values), assuming no interaction between blocks and subplot units. The response is an average taste rating among a panel of judges.

Source	Label	df	Error df	F(.05)
Whole Plot Factor				
Blocks			#N/A	#N/A
Error1			#N/A	#N/A
Sub Plot Factor				
WP*SP Interaction				
Error2			#N/A	#N/A
Total			#N/A	#N/A

QA.3. A study was conducted to measure Irrigation and Nitrogen effects on Sweet Corn Row Numbers. Plots were 115 by 63-feet, arranged in a split-plot design, with four irrigation treatments as whole plots and 15 nitrogen applications randomly allocated on 23 by 21-foot subplots in each irrigation treatment. There were 3 replications of the experiment (blocks).

pA.3.a. Complete the following ANOVA Table.

Source	df	SS	MS	F	F(.05)
Rep		5.94			
Irrigation		2.49			
RepxIrrigation		3.54			
Nitrogen		80.92			
IrrigationxNitrogen		14.28			
Error		29.12			
Total		136.29			

pA.3.b. Compute Bonferroni's MSD for comparing all pairs of Irrigation treatments.

pA.3.c. Compute Tukey's HSD for comparing all pairs of Nitrogen means.

QA.4. A study was conducted to compare 5 Nitrogen levels (0, 45, 90, 135, 180 kg/hectare) and Rice Straw (Absent/Present) on Rice Grain Yield (100s of kg/hectare). The experiment was conducted as a split-plot design, with whole plot factor being Nitrogen level, and sub-plot factor being Rice Straw. The experiment was conducted in 3 blocks (Years).

pA.4.a. Complete the following Analysis of Variance Table.

ANOVA					
Source	df	SS	MS	F	F(.05)
WP		5693.63			
Block		216.82			
WP*Block		60.18			
SP		110.82			
WP*SP		0.96			
Error					
Total		6098.94			

pA.4.b. Assuming no significant Nitrogen/Rice Straw Interaction, compute Bonferroni's Minimum significant Difference for comparing pairs of Nitrogen level effects. Show which levels are significantly different.

Nitrogen	Mean	Rice Straw	Mean
0	48.65	0	73.02
45	75.19	1	76.87
90	79.07		
135	85.85		
180	85.96		

Bonferoni MSD = ________________________ Nit_0 Nit_{45} Nit_{90} Nit_{135} Nit_{180}

pA.4.c. Compute a 95% Confidence Interval for the effect of Using Rice Straw, versus not using rice straw.

QA.5. A split-plot experiment was conducted, comparing 4 coagulant treatments of camel chymosin (HBCC, LBCC, HCC, LCC) and age (4 levels) on Y = strand thickness of melted mozzarella cheese. The experiment was conducted on 3 cheese-making days (blocks), with coagulant treatment as the whole-plot factor, and age as the sub-plot factor.

pA.5.a. Complete the ANOVA Table.

Source of Variation	df	SS	MS	F_obs	F(0.05)
WP Factor		59.8			
Block		4.8		#N/A	#N/A
WP*Block		3.36		#N/A	#N/A
SP Factor		68.7			
WP*SP		4.91			
Error2		14.28		#N/A	#N/A
Total			#N/A	#N/A	#N/A

pA.5.b. Assuming the coagulant treatment/age interaction is not significant, compute Tukey's W and compare all pairs of coagulant treatments. $\overline{Y}_{HBCC} = 3.42$ $\overline{Y}_{LBCC} = 4.06$ $\overline{Y}_{HCC} = 5.94$ $\overline{Y}_{LCC} = 5.92$

pA.5.c. Assuming the coagulant treatment/age interaction is not significant, compute Bonferroni's B and compare all pairs of age.

$\overline{Y}_{Age1} = 5.98 \quad \overline{Y}_{Age2} = 5.86 \quad \overline{Y}_{Age3} = 4.45 \quad \overline{Y}_{Age4} = 5.92$

QA.6. An experiment was conducted as a split-plot design in 4 blocks. Within blocks, 4 seed lots (Factor A) were assigned at random to the 4 whole plots. There were 4 seed protectants (Factor C) assigned at random to the 4 subplots within each whole plot. Assume factors A and C are fixed, and blocks are random. The design of the experiment may have looked like this for Block 1:

	WP1	WP1	WP1	WP1	WP2	WP2	WP2	WP2	WP3	WP3	WP3	WP3	WP4	WP4	WP4	WP4
	SP1	SP2	SP3	SP4	SP1	SP2	SP3	SP4	SP1	SP2	SP3	SP4	SP1	SP2	SP3	SP4
Block1	A4C2	A4C1	A4C3	A4C4	A1C1	A1C4	A1C3	A1C2	A2C2	A2C3	A2C4	A2C1	A3C4	A3C2	A3C3	A3C1

pA.6.a. Complete the following ANOVA table.

Source	df	SS	MS	F_obs	F(0.05)
Seed Lots (A)		2848			
Blocks		2843		#N/A	#N/A
Seed Lots x Blocks		618		#N/A	#N/A
Seed Protectants (B)		171			
Lots x Protectants (AB)		586			
Error		731		#N/A	#N/A
Total		7797	#N/A	#N/A	#N/A

pA.6.b. Compute Bonferroni's minimum significant difference for comparing all pairs of seed lot means.

pA.6.c. Compute Bonferroni's minimum significant difference for comparing all pairs of seed protectant means.

Part B. Repeated Measures Designs

QB.1. A repeated measures design is conducted to compare 3 treatments over 4 equally space time points. A random sample of 60 subjects are selected and randomized, so that 20 receive treatment A, 20 receive B, and 20 receive C.

pB.1.a. Complete the following ANOVA table.

ANOVA						
Source	**df**	**SS**	**MS**	**F**	**F(.05)**	**Sig Effect?**
Treatments		**4000**				
Subject(Trt)		**22800**		**#N/A**	**#N/A**	**#N/A**
Time		**1260**				
TrtxTime		**84**				
Error2				**#N/A**	**#N/A**	**#N/A**
Total		**30538**	**#N/A**			

pB.1.b. Assuming no significant Time by Treatment Interaction, compute Bonferroni's and Tukey's Minimum Significant Differences for comparing all pairs of treatment means.

pB.1.c. Assuming no significant Time by Treatment Interaction, compute Bonferroni's and Tukey's Minimum Significant Differences for comparing all pairs of time means.

QB.2. A Repeated measures design is used to test for effects among 3 treatments at 2 points in time. A sample of 30 subjects is selected, and assigned at random so that 10 subjects receive Trt A, 10 receive Trt B, and 10 receive Trt C.

The (univariate) model fit is: $y_{ijk} = \mu + \alpha_i + d_{k(i)} + \beta_j + (\alpha\beta)_{ij} + e_{ijk} \quad i = 1,2,3 \quad j = 1,2 \quad k = 1,\ldots,10$

The subject within treatment sum of squares is: $SS\left(\text{Subject(Trt)}\right) = 2\sum_{i=1}^{3}\sum_{k=1}^{10}\left(\overline{y}_{i\bullet k} - \overline{y}_{i\bullet\bullet}\right)^2 = 4000$

The table of treatment/time means is:

Trt\Time	1	2
1	50	70
2	40	60
3	60	80

pB.1.a. Complete the following ANOVA table with corrected total sum of squares = 20,000

Source	df	SS	MS	F	F(.05)
Treatments					
Subject(Trts)					
Time					
Trt*Time					
Error					
Total					

pB.1.b. Compute Tukey's HSD and Bonferroni's MSD for comparing pairs of treatment means

Tukey's W = ________________________________ Bonferroni's B = ________________________________

QB.3. A Repeated measures design was used to test for effects of an treatment to reduce fear of spiders. There were 2 treatments (eye movement desensitization (EMD) and control) at 2 points in time. A sample of 14 subjects was selected, and assigned at random so that 7 subjects received EMD, and 7 received Control. The response was distance that the subject moved spider cage to him/her on a conveyor.

The (univariate) model fit is: $y_{ijk} = \mu + \alpha_i + d_{k(i)} + \beta_j + (\alpha\beta)_{ij} + e_{ijk} \quad i = 1,2 \quad j = 1,2 \quad k = 1,\ldots,7$

The subject within treatment sum of squares is: $SS\left(\text{Subject(Trt)}\right) = 2\sum_{i=1}^{2}\sum_{k=1}^{7}\left(\overline{y}_{i\bullet k} - \overline{y}_{i\bullet\bullet}\right)^2 = 900$

The table of treatment/time means is:

Trt\Time	1	2
1	62	56
2	50	44

pB.3.a. Complete the following ANOVA table with corrected total sum of squares = 14,450

Source	df	SS	MS	F	F(.05)
Treatments					
Subject(Trts)					
Time					
Trt*Time					
Error					
Total					

pB.3.b. Compute a 95% Confidence Interval for the difference $\mu_{EMD} - \mu_{Control}$

QB.4. A study compared 3 foods on serum glucose levels. A sample of 12 subjects were selected, and randomized such that 4 people received Food A, 4 received Food B, and 4 received Food C. Each subject's glucose levels were observed at 3 time points after eating the meal (15, 30, and 45 minutes).

pB.4.a. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.05)	Significant?
Food		1020.67				
Subject(Food)		413.33				
Time		170.17				
Food*Time		869.67				
Error2		128.17				
Total		2602.00				

pB.4.b. The means for each food/time combination are given below. Use Bonferroni's method to compare all pairs of food, separately for each time (treat each time as a separate "family", when making adjustment).

Food\Time	1	2	3
1	19	35	31.5
2	22	20	11.5
3	26	27.25	35.75

Time 1: Food_1 Food_2 Food_3 Time 2: Food_2 Food_3 Food_1 Time 1: Food_2 Food_1 Food_3

QB.5. A study compared doses of a drug on female rats' activity levels in a maze. A sample of 91 rats were selected, and randomized such that 21 rats received the Control Dose, 25 received Low Dose, 24 received Medium Dose, and 21 received High Dose. Each rat's activity levels were observed at 4 time points after dosing (15, 30, 45, and 60 minutes).
Hint: There are a total of 21+25+24+21=91 rats in the study.

Complete the following ANOVA table. The dose and time means are given in the second table.

Source	df	SS	MS	F	F(0.05)	Significant?
Dose						
Rat(Dose)		176677		#N/A	#N/A	#N/A
Time						
Dose*Time		2161				
Error2				#N/A	#N/A	#N/A
Total		465319	#N/A	#N/A	#N/A	#N/A

Trt	n_dose	Time1	Time2	Time3	Time4	Mean
Control	21	165.9	136.3	121.4	103.4	131.8
LowDose	25	167.8	148.4	123.7	118.4	139.6
MidDose	24	168.3	140.3	117.2	109.8	133.9
HighDose	21	184.4	156.2	136.3	122.7	149.9
Sum/Mean	91	171.3	145.3	124.4	113.7	138.7

QB.6. A repeated measures design is conducted to compare 3 treatments over 3 time points, with 10 subjects per treatment. The model is:

$$Y_{ij} = \mu + \alpha_i + \beta_{j(i)} + \tau_k + (\alpha\tau)_{ik} + \varepsilon_{jk(i)} \quad i = 1,2,3,\ j = 1,\ldots,10;\ k = 1,2,3$$

$$\beta_{j(i)} \sim NID\left(0,\sigma_\beta^2\right) \quad \varepsilon_{jk(i)} \sim NID\left(0,\sigma^2\right) \quad \{\beta\} \perp \{\varepsilon\} \quad \sum_i \alpha_i = \sum_k \tau_k = \sum_i (\alpha\tau)_{ik} = \sum_k (\alpha\tau)_{ik} = 0$$

pB.6.a. Under this model, give the within subject variance-covariance matrix.

pB.6.b. The treatment means are: $\overline{y}_{1\bullet\bullet} = 90.6 \quad \overline{y}_{2\bullet\bullet} = 95.3 \quad \overline{y}_{3\bullet\bullet} = 108.7$. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.95)
Trts					
Subj(Trt)		11360		#N/A	#N/A
Time		727			
TrtxTime		279			
Error=Time*S(Trt)				#N/A	#N/A
Total		18473		#N/A	#N/A

pB.6.c. Assuming there is a significant interaction, based on Bonferroni's method, test for significant differences among all pairs of treatments at the 3rd time point

$$\overline{Y}_{1\bullet 3} = 102.6 \quad \overline{Y}_{2\bullet 3} = 108.1 \quad \overline{Y}_{3\bullet 3} = 102.6 \quad \hat{V}\left\{\overline{Y}_{i\bullet k} - \overline{Y}_{i'\bullet k}\right\} = \frac{2\left(MS_{B(A)} + (t-1)MS_{\text{ERR2}}\right)}{nt}$$

pB.6.d. Give the ANOVA estimates for σ^2 and σ_{β}^2