

STA 6207 – Fall 2001 – Quiz 3

Print Name: _____ **SSN:** _____

A college swim team has a pool with 5 lanes (columns) and 5 swimmers (treatments). The coach would like to compare the mean times to swim 100 meters among her five swimmers under competitive conditions in a latin square design. She sets up 5 races (rows) among her swimmers, where each swimmer swims in lane 1 once, in lane 2 once, and so on. The response measured is y_{ij} , the amount of time for the swimmer in race i who is in lane j , after properly randomizing all swimmers, lanes, and races to the latin square. The partial design square is given below.

| | Lane | | | | |
|------|--------|-------|-------|--------|--------|
| Race | 1 | 2 | 3 | 4 | 5 |
| 1 | Amy | Erin | | Barb | Cathy |
| 2 | Barb | Amy | Cathy | Debbie | Erin |
| 3 | Erin | | Barb | Cathy | |
| 4 | Debbie | Cathy | Amy | Erin | Barb |
| 5 | | Barb | Erin | Amy | Debbie |

a) Complete the missing swimmers in the table.

b) The sample means for each swimmer (treatment), in seconds, are given below. Give the treatment sum of squares.

$$\bar{y}_A = 62.0 \quad \bar{y}_B = 59.0 \quad \bar{y}_C = 60.0 \quad \bar{y}_D = 58.0 \quad \bar{y}_E = 61.0$$

c) Complete the following ANOVA table.

| Source | df | SS | MS |
|-----------------|----|-------|----|
| Races (Rows) | | 32.5 | |
| Lanes (Cols) | | 120.0 | |
| Swimmers (Trts) | | | |
| Error | | | |
| Total | | 225.0 | |

d) Test whether there are differences among the true mean times among the five swimmers at the $\alpha = 0.05$ significance level. **Clearly state the null and alternative hypotheses, test statistic, rejection region, and conclusion.**

e) Regardless of your previous answer, suppose we are going to make pairwise comparisons among all pairs of swimmers. Based on Bonferroni's method with an overall (experimentwise) Type I error rate of 0.05, we will conclude that $\mu_i \neq \mu_j$ if $|\bar{y}_i - \bar{y}_j|$ exceeds what (minimum) value?

2) A study is conducted to compare the color intensity of the four varieties (treatments) of blond hair color manufactured by L'oreal. Six sets of identical twins (blocks) participate in the experiment in a Balanced Incomplete Block Design. Sets of twins are assigned at random to the block positions, and varieties are randomly assigned to treatment positions. Within blocks, the individuals are assigned at random to the varieties. After the color is applied, a measure of intensity is obtained. The overall mean $\bar{y}_{..} = 71.0$.

| Twins | Variety | | | |
|-------|---------|----|----|----|
| | A | B | C | D |
| 1 | – | 64 | – | 76 |
| 2 | 84 | – | – | 80 |
| 3 | 92 | 72 | – | – |
| 4 | 72 | – | 60 | – |
| 5 | – | 68 | 64 | – |
| 6 | – | – | 56 | 64 |

a) Give t, r, b, k, λ :

b) Write out the statistical model (don't describe components, but give ranges of subscripts).

c) Give least squares estimates (intra-block) for variety effects.

d) Give the ANOVA table for the intra-block analysis (including the F -statistic for variety effects and the critical F -value for an $\alpha = 0.05$ level test). Note: $SSTotal = 1220.0$.

e) Give the interblock estimate of τ_A , the effect of variety A.

3) A study was conducted to measure the effects of three drugs (treatments) on the clearance of Theophylline in humans. The rate that Theophylline cleared in 14 subjects (blocks) who simultaneously took: Placebo (Trt 1), Famotidine (Trt 2), and Cimetidine (Trt 3) were obtained (orders of treatments were randomly assigned in blocks). The following treatment means and Analysis of Variance were obtained:

$$\bar{y}_{1.} = 3.08 \quad \bar{y}_{2.} = 3.16 \quad \bar{y}_{3.} = 2.26$$

| Source | df | SS | MS | F_0 |
|-------------------|----|-------|------|-------|
| TRTS (Drugs) | 2 | 7.01 | 3.51 | 10.64 |
| BLOCKS (Subjects) | 13 | 71.81 | 5.52 | |
| ERROR | 26 | 8.60 | 0.33 | |
| Total | 41 | 87.42 | | |

a) The test for treatment effects is highly significant. Use Bonferroni's method to compare all pairs of treatment means simultaneously, with an experimentwise error rate of $\alpha = 0.05$.

b) Compute the relative efficiency estimate for having used the Randomized Block Design as opposed to the Completely Randomized Design (Don't worry about the correction for estimating σ^2).

c) How many subjects would be needed to be recruited to have an equally efficient estimate of treatment means as we have here, had this been conducted as a Completely Randomized Design?