Chapter 12 - Fractional Factorial Experiments

2^n Fractional Factorial Designs in Fewer than $N = 2^n$ observations.

- Number of treatments required > resources.
- Info needed only on main effects and lower order interactions.
- Screening experiments for many factors.
- Belief that only a few effects are important.
- Factor Sparsity Hypothesis: Few effects are large/important

12.2 One-Half Fraction of 2^n Factorial

Defining Contrasts - Place observations into 2 groups via +/- conventions on the "sacrificed" contrast.

Similar principle here. Use only those observations that have +/- coefficients for the defining contrast.

e.g. $2^3 = 8$ observations in 4 observations w/ ABC as defining contrast.

TRT	I	A	B	C	AB	AC	BC	ABC*	y
(1)	+	−	−	−	+	+	+	−	32
a	+	+	−	−	−	−	+	+	35 ✓
b	+	−	+	−	−	+	−	+	28 ✓
ab	+	+	+	−	+	−	−	−	31
c	+	−	−	+	+	−	−	+	48 ✓
ac	+	+	−	+	−	+	−	−	39
bc	+	−	+	+	−	−	+	−	28
abc	+	+	+	+	+	+	+	+	29 ✓

$\frac{1}{2}$ Fractions based on ABC as Defining contrast: (1), ab, ac, bc

Use the group with "+" or $\boxed{a, b, c, abc}$

Problems: ① No estimate of 3-factor Interaction
② 2-factor Interactions Confounded w/ main effects. (aliased)

$l_A = a - b - c + abc = l_{BC}$
$l_B = -a + b - c + abc = l_{AC}$
$l_C = -a - b + c + abc =$

=> $a - b - c + abc$ ESTIMATES $A + BC$, Simililar for other contrasts.

Two effects estimated by same contrast ≡ <u>ALIASES</u>

<u>ALIAS RELATIONSHIPS</u>: $A = BC$, $B = AC$, $C = AB$, $I = ABC$
(same coeffs as mean)

$I = ABC$ ≡ <u>DEFINING RELATION</u>

COULD ALSO HAVE USED THE NEGATIVE ABC VALUES $l_A = -l_{BC}$
=> $I = -ABC$, $A = -BC$, $B = -AC$, $C = -AB$

The alias for any trt effect can be determined from the <u>generalized interaction</u> w/ the <u>defining Contrast</u>:

USING + Coeffs of ABC: $A \times ABC = A^2BC = BC$ ...
" – " " " $A \times (-ABC) = -A^2BC = -BC$ ...

- When no interactions exist, can use either half-replicate.

$I = ABC$ => Principal Fraction $I = -ABC$ => Complementary Fraction

If both half fractions are run, this is 2^3 in incomplete blocks w/ ABC Confounded in BLOCKS.

<u>Constructing Half Replicates of 2^{n-1} Designs</u>

① Use highest order interaction as design generator.
② Write the + and – Coefficients in standard order for factors in 2^{m}
③ Identify coeffs for the n^{th} factor by equating coeffs to highest order interaction in 2^{n-1} factorial

→

EXAMPLE: $2^4 = 16$ in 8 observations

① I = ABCD ⇒ + coefficients w/ (1), ab, ac, bc, ad, bd, cd, abcd

② Get +/− Coefficients for A, B, C for these observations

TRT	A	B	C	D=ABC
(1)	−	−	−	−
ab	+	+	−	−
ac	+	−	+	−
bc	−	+	+	−
ad	+	−	−	+
bd	−	+	−	+
cd	−	−	+	+
abcd	+	+	+	+

③ Get coeffs for D by multiplying A, B, C coeffs

- Always use highest order interaction as design generator for "optimal aliases".
- Randomly order treatment runs or assign @ random to exp'tl unit.

DESIGN RESOLUTION WRT ALIASES

Resolution III – No main effects aliased w/ one another, but main effects ~~are~~ are aliased w/ 2-factor interactions and 2-factor interactions are aliased w/ one another.

Resolution IV – No main effect is confounded w/ a main effect or 2-factor interaction, but 2-factor interactions are confounded w/ one another.

Resolution V – No main effect or 2-factor interaction is confounded w/ a main effect or 2-factor interaction, but 2 factor interactions are confounded w/ 3-factor interactions.

Notation: 2^{5-1}_{V} ⇒ 2^{5-1} fractional factorial w/ Resolution V.

Analysis of Half Replicate 2^{n-1} Designs

1. Set up table of contrasts (+/- coeffs for each aliased effect)
2. Get linear contrasts for each effect:

$$l_{AB...} = \sum k_i y_i \qquad k_i \equiv +/-1 \qquad y_i \equiv \text{observed data}$$

3. ESTIMATE EFFECTS:

$$AB... = \frac{2(l_{AB...})}{N} \qquad N = 2^{n-1} \equiv \text{Total \# of observations.}$$

4. OBTAIN SUMS OF SQUARES:

$$SS(AB...) = \frac{1}{2^{n-1}}(l_{AB...})^2$$

Determining Large Effects via Normal Probability (Pareto) Plot

1. Order estimated effects from smallest to largest
2. Get standard normal quantiles $\equiv Z\left(\frac{\text{rank} - 0.5}{2^{n-1}}\right)$
3. Plot std normal quantiles (y) vs estimated effects.
4. Estimated effects to the ~~right~~ NE and SW of straight line thru middle effects are important, and should be further studied via graphs (main effects and interaction plots).

Estimating the Experimental Error Variance

1. Fit model w/ those important effects from above
2. Obtain SSE by pooling all other sources & get $MSE = s^2$

- Standard Errors of Effects Estimates: $S.E.(\text{Effect estimate}) = \frac{4s^2}{N^2}$ $\quad N = 2^{n-1}$

Then t-test