Project 5 - Design Problems / RBD / Latin Square

P.1. The Professional Bowling Association (PBA) has 15 traditional tournaments per year (as well as several others with different formats). The tournaments are played at 15 different bowling centers. There are many PBA bowlers, however, not all bowlers play at all tournaments. Tournaments involve 2 preliminary rounds, each made up of 7 games each (games involve 10 frames with a perfect score being 300). After these rounds, the leading bowlers move on in the tournament. There are 5 oiling patterns (Chameleon, Cheetah, Scorpion, Shark, and Viper), each is used at 3 of the tournaments. Set up this model/design, with respect to factors, and effects and variances to be estimated. Note that we consider only a random sample of 20 bowlers who competed in all tournaments. Each observation is a 7 game total for a bowler at a tourney in a 7-game preliminary round. Obtain the Expected Mean Squares and F-tests for all model parameters (fixed effects and variance components). Compare all pairs of pattern means.

Q.2. An experiment was conducted to determine whether Pepcid or Tagamet interact with theophylline in patients. A sample of 14 patients were selected, and each received theophylline with: (Placebo, Tagamet, and Pepcid). The response was the clearance rate of theophylline for the patients under each condition.

- Plot y vs subject by drug (Placebo, Tagamet, Pepcid)
- Obtain the Analysis of Variance and test for differences in means among the 3 conditions.
- Use Tukey's and Bonferroni's methods to compare all pairs of treatments.
- Obtain the relative efficiency for using patients as a blocking factor
- Dataset: theoph.dat
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- Source: K. Bachmann, T.J. Sullivan, J.H. Reese, et al. (1995).
- "Controlled Study of the Putative Interaction Between Famotidine
- and Theophylline in Patients with Chronic Obstructive Pulmonary
- Disorder," Journal of Clinical Pharmacology, 35:529-535.
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- Description: Randomized Block design in 14 patients with chronic
- obstructive pulmonary disease. Each subject received theophylline
- along with (famotidine(pepcid), cimetidine(tagamet), placebo)
- and theophylline clearance (liters/hour) was measured.
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- Variables/Columns
- Subject Number 8
- Interacting agent 16 /* 1=Placebo, 2=Pepcid, 3=Tagamet */
- Theophylline clearance (ltr/hr) 21-24

Q.3. For the following latin square design:

- Obtain the Analysis of Variance and test for differences in means among the 6 Container Types.

- Use Tukey's and Bonferroni's methods to compare all pairs of treatments.
- Obtain the Relative efficiencies for each blocking factor

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Dataset: juice1.dat

Source: S. Banks (1965), "Latin Square Experiments," Journal of Advertising
Research, Vol5, #3, pp. 37-46

Description: Results of an experiment based on a latin square design
relating monthly juice sales (gallons) to container (3 jugs, 3 dispensers)
over 6 months in 6 stores in Washington, DC from Dec.1949 through May 1950.

Variables/Columns
Month    8  /* 1=12/1949, ..., 6=5/1950  */
Container   16  /* 1=Jug1, 2=Jug2, 3=Jug3, 4=Dispenser1, 5=Disp2, 6=Disp3 */
Store      24
Juice Sales (Gallons)  27-32
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Q.4. A tire company with 4 factories has 3 teams of workers at each factory (the teams at Factory A are not the same as those at Factory B, and so on). The company is interested in comparing output among factories, as well as among teams within factories. The experiment is conducted as measuring the output of each team on 3 occasions.

p.4.a. Write out the appropriate statistical model, stating all elements (parameters and random variables) and ranges of subscripts.

p.4.b. The following table gives means (SDs) for all teams for all factories. Give the ANOVA table, and test statistics and critical values.

Factory\Team	**1**	**2**	**3**
A	**100 (10)**	**90 (8)**	**110 (6)**
B	**65 (8)**	**70 (6)**	**75 (10)**
C	**110 (8)**	**120 (6)**	**130 (10)**
D	**95 (6)**	**125 (8)**	**110 (10)**

Source	**df**	**SS**	**MS**	**F_0**	**F(.05)**

p.4.c. Use Bonferroni's and Tukey's methods to obtain simultaneous 95% Confidence Intervals among Factory Means.

Q.5. A 2-Factor (nested, fixed effects ANOVA) is fit, with factor A at 4 levels, and factor B at 3 levels (within each level of A). There were 6 replicates for each combination of factor levels. Compute Bonferroni's and Tukey's minimum significant differences for comparing all pairs of means (Factor A) with an experiment-wise error rate of 0.05. Note: This is the same as the margin of error for the point estimates (half-width of simultaneous CI's). Give your results as functions of only MSE. The model is:

$$y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + e_{k(ij)} \quad i = 1,2,3,4 \quad j = 1,2,3 \quad k = 1,\ldots,6$$

$$\sum_{i=1}^{4} \alpha_i = 0 \quad \sum_{j=1}^{3} \beta_{j(i)} = 0 \; \forall \, i \quad e_{k(ij)} \sim NID\left(0, \sigma^2\right)$$

Bonferroni ____ Tukey ____

Q.6 Researchers conducted an experiment measuring acoustic metric values in *a* = 3 habitats (1=Cliff, 2=Mud, 3=Gravel). Nested within each habitat, there were *b* = 3 random patches. Replicates representing *r* = 5 sites within each patch were obtained. The habitats are considered to be fixed levels, while patches within habitats are considered to be random. The response measured was snap amplitude. The model is given below, numbers in the ANOVA are rounded.

$$y_{ijk} = \mu + \alpha_i + b_{j(i)} + e_{k(ij)} \quad i = 1,\ldots,a \quad j = 1,\ldots,b \quad k = 1,\ldots,r$$

$$\sum_{i=1}^{a} \alpha_i = 0 \quad b_{j(i)} \sim NID\left(0, \sigma_P^2\right) \quad e_{k(ij)} \sim NID\left(0, \sigma^2\right) \qquad \{b\} \perp \{e\}$$

p.6.a. Complete the following ANOVA Table and give the expected mean square for each row. Note that for the F-Stats, you are testing: $H_0^H : \alpha_1 = \ldots = \alpha_a = 0 \quad H_A^H : \text{ Not all } \alpha_i = 0 \qquad H_0^P : \sigma_P^2 = 0 \quad H_A^J : \sigma_P^2 > 0$

Source	df	SS	MS	F-Stat	F(.05)	E(MS)
Habitat		400.0				
Patch(Hab)		300.0				
Error						
Total		1100.0				

p.6.b Compute Tukey's and Bonferroni's minimum significant difference for comparing pairs of habitat means.

Tukey's HSD: _____________________________________ Bonferroni's MSD: ____________________________

p.6.c Obtain point estimate for σ_P^2 and σ^2

Q.7. Jack and Jill wish to compare the effects of 3 interior design presentation methods (dp1, dp2, dp3) on ease of visualizing the task (y = 1 (very difficult) to 7 (very easy)). The sample consisted of 32 participants who had recently remodeled or built a home. Each subject was exposed to each design presentation method, in random order.

The means for the 3 design presentation method are:

$$\overline{y}_{1\bullet} = \frac{188}{32} = 5.875 \quad \overline{y}_{2\bullet} = \frac{218}{32} = 6.813 \quad \overline{y}_{3\bullet} = \frac{199}{32} = 6.219 \quad \overline{y}_{\bullet\bullet} = \frac{605}{96} = 6.302$$

$$SSTR = b\sum_{i=1}^{t}\left(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet}\right)^2 = 14.4 \quad SSBL = t\sum_{j=1}^{b}\left(\overline{y}_{\bullet j} - \overline{y}_{\bullet\bullet}\right)^2 = 70.5 \quad TSS = \sum_{i=1}^{t}\sum_{j=1}^{b}\left(\overline{y}_{ij} - \overline{y}_{\bullet\bullet}\right)^2 = 122.5$$

p.7.a. Jack conducts the analysis as a Completely Randomized Design (independent samples), Give Jack's test statistic for testing H_0: No design presentation effects:

p.7.a.i. Test Statistic:

p.7.a.ii. Reject H_0 if Jack's test statistic falls in the range ____

p.7.b. Jill conducts the analysis as a Randomized Block Design, treating participants as blocks,

Give Jill's test for testing H_0: No design presentation effects:

p.7.b.i. Test Statistic:

p.7.b.ii. Reject H_0 if Jill's test statistic falls in the range ____

p.7.c. Obtain Jack's and Jill's minimum significant differences based on Tukey's method for comparing all pairs of design presentation effects:

Jack's W_{ij} = _______________________________ Jill's W_{ij} = _________________________________

Q.8. A randomized complete block design is conducted with 3 (fixed) treatment in 3 (random) blocks.

$$y_{ij} = \mu + \tau_i + b_j + e_{ij} \quad i, j = 1,2,3 \quad b_j \sim NID\left(0, \sigma_b^2\right) \quad e_{ij} \sim NID\left(0, \sigma^2\right) \quad \left\{b_j\right\} \perp \left\{e_{ij}\right\}$$

p.8.a. $E(y_{ij})$ = p.8.b. $V(y_{ij})$ = p.8.c. $COV(y_{ij}, y_{i'j})$ = p.8.d. $COV(y_{ij}, y_{ij'})$ =

p.8.e. **Derive** $V\left(\overline{y}_{i\bullet}\right), \quad COV\left(\overline{y}_{i\bullet}, \overline{y}_{i'\bullet}\right), \quad V\left(\overline{y}_{i\bullet} - \overline{y}_{i'\bullet}\right)$ **(SHOW ALL WORK)**

Q.9. A latin-square design is used to compare 6 brands of energy enhancing drinks on alertness, measured by the amount of items completed in a fixed amount of time on skills tests. There are 6 different skills tests, and 6 participants, such that each energy drink is observed once on each skills test and once for each participant. The drink means and partial ANOVA table are given below.

p.9.a. Complete the tables.

	Drink1	Drink2	Drink3	Drink4	Drink5	Drink6	Overall
Mean	30	50	36	42	44	38	40
SS							

Source	df	SS	MS	F	F(.05)
Energy Drink					
Skills Test		460			
Participant		2000			
Error					
Total		4600			

p.9.b. Compute Tukey's HSD and Bonferroni's MSD for comparing all pairs of drinks.

Tukey's HSD: ______________________________ Bonferroni's MSD __________________________________

Q.10. A Balanced Incomplete Block Design is conducted, to compare 7 (fixed) machines in terms of output. The supplier provides (random) batches (blocks) of components that only have 4 components.

$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad i = 1,...,t;\ j = 1,...,b$ Note: not all pairs (i, j)

p.10.a. How many batches will be needed so that each machine produces 4 items? Fill in the following numbers, where t = #trts, k = block size, r = # reps/trt, b = # blocks, λ = # blocks each pair of treatments appears in together.

$t =$ ____________ $k =$ ____________ $r =$ ____________ $b =$ ____________ $\lambda =$ ____________

p.10.b. The error sum of squares for a model that contains only blocks as a factor is 200. The error sum of squares for a model that contains both blocks and treatments as factors is 120. Test for treatment effects (adjusted for blocks). H_0: $\tau_1 = ... = \tau_\tau = 0$.

Test Statistic: ____________________________ Rejection Region: _____________________________

p.10.c. We wish to obtain simultaneous 95% Confidence Intervals for all pairs of treatment differences based on all the intra-block analysis, where: $\hat{V}\left\{\hat{\tau}_i - \hat{\tau}_{i'}\right\} = \frac{2kMS_{\text{ERR}}}{\lambda t}$. Give the form of the simultaneous 95% CI's:

$$\left(\hat{\tau}_i - \hat{\tau}_{i'}\right) \pm$$