STA 6208 – Spring 2017 – Exam 2 PRINT Name ________________________

Note: Conduct all tests at at α = 0.05 significance level

Q.1. An experiment was conducted to compare a = 3 theories for the apparent modulus of elasticity (Y) of b = 3 apple varieties. The 3 theories were: Hooke's, Hertz's, and Boussineq's; the 3 apple varieties were: Golden Delicious, Red Delicious, and Granny Smith. The researchers determined the elasticity for n = 15 based on each combination of theory and variety. For the purposes of this experiment, each factor is fixed.

Model: $Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim NID(0,\sigma^2) \quad \sum_{i=1}^{a}\alpha_i = \sum_{j=1}^{b}\beta_j = \sum_{i=1}^{a}(\alpha\beta)_{ij} = \sum_{j=1}^{b}(\alpha\beta)_{ij} = 0$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(Y_{ijk} - \overline{Y}_{ij\bullet}\right)^2 = 17.095 \qquad \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(Y_{ijk} - \overline{Y}_{\bullet\bullet\bullet}\right)^2 = 113.119$$

Cell Means	GoldenDelicious	RedDelicious	GrannySmith	Row Mean
Hooke	2.68	3.46	4.23	3.457
Hertz	2.44	3.06	3.84	3.113
Boussinesq	1.53	1.89	2.36	1.927
Column Mean	2.217	2.803	3.477	2.832

Complete the following Analysis of Variance Table, and test for interaction effects and main effects.

Source	df	SS	MS	F	F(.95)	P-value
Theory						>0.05 or <0.05
Variety						>0.05 or <0.05
Theory*Variety						>0.05 or <0.05
Error				#N/A	#N/A	#N/A
Total			#N/A	#N/A	#N/A	#N/A

Q.2. For the 1-Way Random Effects model, derive the Expected Mean Squares for Treatments and Error by completing the following steps. $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad \alpha_i \sim NID\left(0, \sigma_\alpha^2\right) \quad \varepsilon_{ij} \sim NID\left(0, \sigma^2\right) \quad \{\alpha\} \perp \{\varepsilon\} \quad i = 1,...,g;\ j = 1,...,n$

p.2.a. Derive: $E\left\{Y_{ij}\right\}, \quad V\left\{Y_{ij}\right\}, \quad E\left\{Y_{i\bullet}\right\}, \quad V\left\{Y_{i\bullet}\right\}, \quad E\left\{Y_{\bullet\bullet}\right\}, \quad V\left\{Y_{\bullet\bullet}\right\}$ SHOW ALL WORK.

p.2.b. Making use of p.2.a., derive: $E\left\{\sum_{i=1}^{g}\sum_{j=1}^{n}Y_{ij}^{2}\right\}, \quad E\left\{n\sum_{i=1}^{g}\overline{Y}_{i\bullet}^{2}\right\}, \quad E\left\{ng\overline{Y}_{\bullet\bullet}^{2}\right\}$

p.2.c. Making use of p.2.b., derive: $E\left\{MS_{\text{TRT}}\right\}$ and $E\left\{MS_{\text{ERR}}\right\}$

Q.3. Consider a 2-factor, fixed effects, interaction model with $a = b = n = 2$.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \varepsilon_{ijk} \sim NID(0, \sigma^2) \quad \sum_{i=1}^{2}\alpha_i = \sum_{j=1}^{2}\beta_j = \sum_{i=1}^{2}(\alpha\beta)_{ij} = \sum_{j=1}^{2}(\alpha\beta)_{ij} = 0$$

p.3.a. Given the following data, obtain MS_{Err}.

Y111	Y112	Y121	Y122	Y211	Y212	Y221	Y222
22	18	31	29	8	12	37	43

SS_{Err} = ______________________________ MS_{Err} = ________________________

p.3.b. Use the matrix form of the model to obtain: the least squares estimate of the parameter vector $\boldsymbol{\beta}^*$, its estimated variance-covariance matrix, standard errors, and t-tests for all of the parameters.

$$\boldsymbol{\beta}^* = \begin{bmatrix} \mu \\ \alpha_1 \\ \beta_1 \\ (\alpha\beta)_{11} \end{bmatrix}$$

$\mathbf{X} =$

$\mathbf{X'X} =$

$(\mathbf{X'X})^{-1} =$

$\mathbf{X'Y} =$

$\hat{\boldsymbol{\beta}}^* =$

$\hat{V}\left\{\hat{\boldsymbol{\beta}}^*\right\} =$

Continued Below

Parameter	Estimate	Std Error	t	t(.975)
μ				
α1				
β1				
(αβ)11				

Q.4. A 1-Way Random Effects model is fit, comparing readability scores among a random sample of g=6 Business/Economics columnists. A sample of n=3 essays were selected from each columnist and their readability was assessed based on the Flesch-Kincaid scale.

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad \alpha_i \sim N\left(0, \sigma_\alpha^2\right) \quad \varepsilon_{ij} \sim NID\left(0, \sigma^2\right) \quad \{\alpha\} \perp \{\varepsilon\} \quad \sum_{i=1}^{6}\sum_{j=1}^{3}\left(Y_{ij} - \overline{Y}_{i\bullet}\right)^2 = 35.4 \qquad \sum_{i=1}^{6}\sum_{j=1}^{3}\left(Y_{ij} - \overline{Y}_{\bullet\bullet}\right)^2 = 90.1$$

p.4.a. Test $H_0 : \sigma_\alpha^2 = 0 \quad H_A : \sigma_\alpha^2 > 0$

Test Statistic: ________________________________ Rejection Region: __________________________________

p.4.b. Obtain a point estimate and an approximate 95% Confidence Interval for σ_α^2 (based on Satterthwaite's Approx.)

Point Estimate: __________________ Approximate 95% Confidence Interval: _______________________________

Q.5. A 2-Way fixed effects model is fit, with factor A at 4 levels, and factor B at 3 levels. There are 3 replicates for each combinations of factors A and B. The error sum of squares is $SS_{Err} = 720$.

p.5.a. Suppose that the interaction is not significant. What will be Tukey's and Bonferroni's minimum significant differences be for comparing the means for factor A (averaged across levels of factor B).

Tukey's HSD: ________________________________ Bonferroni's MSD: ____________________________

p.5.b. Suppose that the interaction is significant. What will be Tukey's and Bonferroni's minimum significant differences be for comparing the means for factor A (within a particular level of factor B).

Tukey's HSD: ________________________________ Bonferroni's MSD: ____________________________

p.5.c. How large would the interaction sum of squares, SS_{AB} need to be to reject H_0: No interaction between factors A&B?

Q.6. A 2-Way Random Effects model is fit, where a sample of a = 8 products were measured by a sample of b = 6 machinists, with n = 3 replicates per machinist per product. The model fit is as follows (independent random effects):

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \quad \alpha_i \sim NID\left(0,\sigma_\alpha^2\right) \quad \beta_j \sim NID\left(0,\sigma_\beta^2\right) \quad (\alpha\beta)_{ij} \sim NID\left(0,\sigma_\beta^2\right) \quad \varepsilon_{ijk} \sim NID\left(0,\sigma^2\right)$$

You are given the following sums of squares: $SS_A = 420 \quad SS_B = 350 \quad SS_{AB} = 140 \quad SS_{\text{Err}} = 210$

Give the test statistic and rejection region for the following 3 tests. Note for test 1, your rejection region will be symbolic, give the specific numerator and denominator degrees of freedom. Also give unbiased (ANOVA) estimates of each variance component.

1) $H_0^{AB}: \sigma_{\alpha\beta}^2 = 0 \quad H_A^{AB}: \sigma_{\alpha\beta}^2 > 0$ 2) $H_0^{A}: \sigma_{\alpha}^2 = 0 \quad H_A^{A}: \sigma_{\alpha}^2 > 0$ 3) $H_0^{B}: \sigma_{\beta}^2 = 0 \quad H_A^{B}: \sigma_{\beta}^2 > 0$

1: Test Stat: ______________________________ Rejection Region: ___________________ Estimate: __________________

2: Test Stat: ______________________________ Rejection Region: ___________________ Estimate: __________________

3: Test Stat: ______________________________ Rejection Region: ___________________ Estimate: __________________

Q.7. In a population of stock market analysts, the mean rate of return is 2%. Approximately 95% of all analysts have a personal mean return within 5% of the overall mean. Among individual stocks selected by a particular analyst, approximately 95% have returns within 7% of his/her mean.

Compute: $\sigma_{\alpha}^{2} =$ ____________ $\sigma^{2} =$ _____________ $V\{Y\} =$ ______________ $\rho_{I} =$ ___________

Q.8. An experiment with g = 3 treatments, $n_1 = n_2 = n_3 = 5$ and $\bar{y}_{1\bullet} = 20,\ s_1 = 3 \quad \bar{y}_{2\bullet} = 30,\ s_2 = 9 \quad \bar{y}_{3\bullet} = 10,\ s_3 = 1$
Estimated Weighted Least Squares with the weight of Y_{ij} being the inverse of its group's sample variance.
We plan to test H_0: $\mu_1 = \mu_2 = \ldots = \mu_g = \mu$ H_A: Not all μ_i are equal.

Under H_0 (the Reduced Model), the weighted Error sum of squares is $Q_R^W = \sum_{i=1}^{3}\sum_{j=1}^{5} w_{ij}\left(Y_{ij} - \mu\right)^2$ derive the weighted least squares estimator for μ and give its estimate based on this data.