Completely Randomized Design Problems

Q.1. An experiment is conducted to compare 3 equally spaced dryer temperatures on fabric shrinkage. The researcher samples 15 pieces of wool fabric (labeled specimen1-specimen15). He generates random numbers for each specimen, then assigns 5 to each treatment (the 5 specimens with the smallest random numbers are assigned to temperature 1, the 5 specimens with the largest random numbers receive temperature 3, and others receiving temperature 2).

spec1	spec2	spec3	spec4	spec5	spec6	spec7	spec8	spec9	spec10	spec11	spec12	spec13	spec14	spec15
0.541239	0.849694	0.460164	0.608456	0.202543	0.331311	0.186567	0.416428	0.442315	0.278932	0.699956	0.67784	0.197721	0.662758	0.799943

p.1.a. Specimens receiving Temp1 _________________ Temp2 _________________ Temp3 ______________________

p.1.b. The means and standard deviations for the amount of shrinkage for the three temperatures are given below. Compute the Treatment and Error Sums of Squares:

Temp	r	Mean	SD
1	5	6	3
2	5	14	2
3	5	16	3

SS_{TRT} =

SS_{ERR} =

p.1.c. Complete the following ANOVA table.

Source	df	Sum of Squares	Mean Square	F_0	F(.05)
Treatments					
Error					
Total					

p.1.d. Consider 2 Contrasts: $C_{Lin} = (-1)\mu_1 + (0)\mu_2 + (1)\mu_3$ and $C_{Quad} = (1)\mu_1 + (-2)\mu_2 + (1)\mu_3$

p.1.d.i. Show that these contrasts are orthogonal.

p.1.d.ii. Give the sums of squares for these contrasts and show they sum to SS_{TRT}

$SS(C_{Lin})$ =

$SS(C_{Quad})$ =

Q.2. An experimenter has g=8 methods of preparing steel rods from raw steel, and is interested in comparing their mean breaking strengths. She obtains 40 batches of steel, and randomly assigns them, so that batches are used for each method (that is, n=5). Before conducting the experiment, she envisions many potential comparisons (contrasts) among the treatments and decides she will use Scheffe's method to conduct all her tests concerning the contrasts (with experimentwise error rate of $\alpha_E = 0.05$). Suppose here Error Sum of Squares is SSE = 200. How large will a Contrast sum of squares need to be to conclude that the contrast among population means is not equal to 0 (reject the null hypothesis that the contrast is 0)?

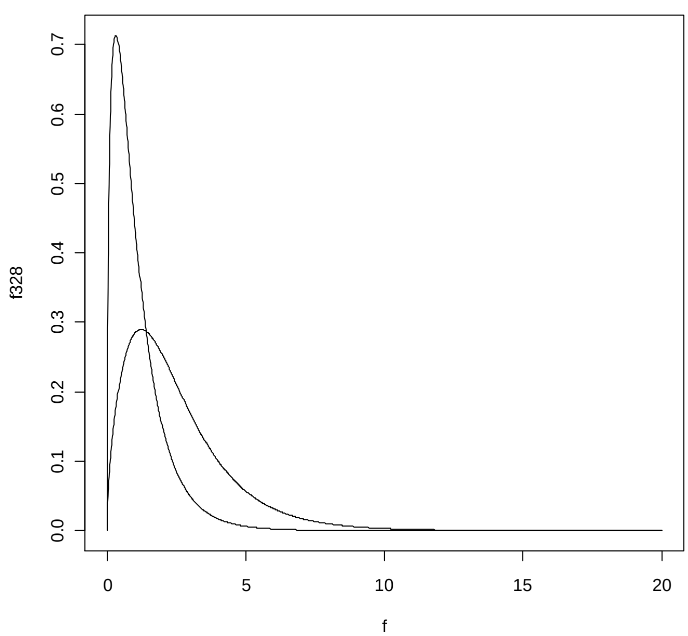
Q.3.. Compute Tukey's and Bonferroni's minimum significant differences (with experimentwise error rates of $\alpha_E = 0.05$) when the experiment consists of 5 treatments with, with 4 replicates per treatment and SSE = 400.

Q.4. We wish to conduct an experiment to compare t=4 treatments in a CRD. We would like the probability that we (correctly) reject the null hypothesis to be $1-\beta = 0.80$ when the test is conducted at $\alpha = 0.05$ and the α_i are -20,0,0,20 and $\sigma = 40$.

p.4.a. What is the non-centrality parameter in this setting when n=4, when n=8?

p.4.b. What is the Rejection Region for the test when n=4, when n=8?

p.4.c. Identify on the following plot, the rejection region and power of the F-test for the case n=8. The distribution to the "left" is the central F, the distribution to the "right" is the non-central F.



p.4.d. Does it appear that the power has reached .80 for n=8?

Q.6. For the CRD with fixed treatment effects: $y_{ij} = \mu_i + e_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i = 1,\ldots,g \quad j = 1,\ldots,n \quad \varepsilon_{ij} \sim NID\left(0,\sigma^2\right)$

p.6.a. Show that $\sum_{i=1}^{g}\sum_{j=1}^{n}\left(y_{ij} - \overline{y}_{i\bullet}\right)^2 = \sum_{i=1}^{g}\sum_{j=1}^{n} y_{ij}^2 - n\sum_{i=1}^{g}\overline{y}_{i\bullet}^2$ and $\sum_{i=1}^{g}\sum_{j=1}^{n}\left(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet}\right)^2 = n\sum_{i=1}^{g}\overline{y}_{i\bullet}^2 - ng\overline{y}_{\bullet\bullet}^2$

p.6.b. Derive E(MS_{ERR})

p.6.c. Derive E(MS_{TRT})

Q.7. A study was conducted to compare the effects of ad types (corporate social responsibility (CSR) and sex appeal (SA)) on people's attitude toward the firm. There were 4 conditions, with 100 people being assigned at random to each condition (there were a total of 400 people in the study). The 4 conditions were:

1: CSR ad only 2: CSR ad + SA1 ad 3: CSR ad + SA2 ad 4: CSR ad + SA3 ad

Condition (i)	Sample Size	Mean	SD
1	100	68	8
2	100	54	7
3	100	58	8
4	100	52	7

p.7.a. Complete the following ANOVA table (the overall mean is 58).

Source	df	SS	MS	F	F(0.05)
Treatments					
Error					
Total					

p.7.b. Consider the following 3 contrasts, fill in the table of coefficients, and show they are pairwise orthogonal:

C1 = Condition 1 vs Conditions 2,3,4 C2 = Condition 2 vs Conditions 3,4 C3 = Condition 3 vs Condition 4

	Trt1	Trt2	Trt3	Trt4
C1				
C2				
C3				

C1 & C2:

C1 & C3:

C2 & C3:

p.7.c. Compute the sum of squares for each contrast, and confirm they sum to SSTrts.

SSC1 = ______________ SSC2 = _______________ SSC3 = _____________ SSC1 + SSC2 + SSC3 = ____________________

Q.8. Consider the Completely Randomized Design with *gt* treatments and *n*replicates/treatment and the identity:

$$y_{ij} - \overline{y}_{\bullet\bullet} = \left(y_{ij} - \overline{y}_{i\bullet}\right) + \left(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet}\right) \qquad y_{ij} = \mu_i + \varepsilon_{ij} \quad \varepsilon_{ij} \sim NID\left(0,\sigma^2\right) \quad \overline{y}_{i\bullet} = \frac{1}{n}\sum_{j=1}^{n} y_{ij} \quad \overline{y}_{\bullet\bullet} = \frac{1}{gn}\sum_{i=1}^{g}\sum_{j=1}^{n} y_{ij} = \frac{1}{g}\sum_{i=1}^{g}\overline{y}_{i\bullet}$$

p.8.a. Show that $\sum_{i=1}^{g}\sum_{j=1}^{n}\left(y_{ij}-\bar{y}_{\bullet\bullet}\right)^2=\sum_{i=1}^{g}\sum_{j=1}^{n}\left(y_{ij}-\bar{y}_{i\bullet}\right)^2+g\sum_{i=1}^{g}\left(\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet}\right)^2$

p.8.b. Derive $E\left(y_{ij}-\bar{y}_{i\bullet}\right)$, $V\left(y_{ij}-\bar{y}_{i\bullet}\right)$ and $E\left[\left(y_{ij}-\bar{y}_{i\bullet}\right)^2\right]$

p.8.c. Derive $E\left(\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet}\right)$, $V\left(\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet}\right)$ and $E\left[\left(\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet}\right)^2\right]$

Q.9. A study is conducted to compare g = 4 treatments, based on n = 6 replicates per treatment.

p.9.a. Complete the following ANOVA table. Assume that the between treatments sum of squares accounts for 60% of the total variation in the sample data.

Source	df	SS	MS	F	F(0.05)
Treatments					
Error					
Total		10000			

p.9.b. Obtain the minimum significant differences when comparing all pairs of treatment means, for experiment-wise error rates of α_E = 0.05 based on the following methods:

p.9.b.i. Bonferroni's method:

p.9.b.ii. Scheffe's method:

p.9.b.iii. Tukey's method:

p.9.b.iv. Student-Newman-Keuls method (Note there will be multiple ranges):

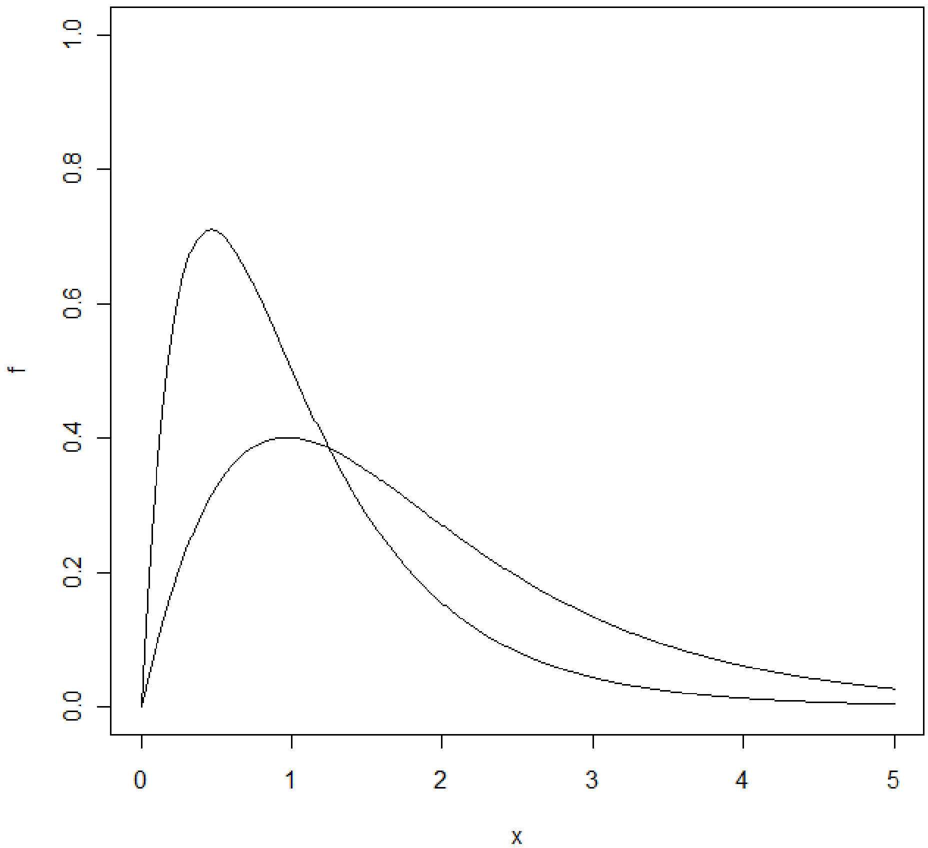
Q.10. A researcher wishes to compare g = 5 treatments. She is interested in determining the power of her test when the treatment effects are $\alpha_1 = -\sigma/2$, $\alpha_2 = \alpha_3 = \alpha_4 = 0$, $\tau_5 = \sigma/2$ and n = 6 replicates per treatment.

p.10.a. Compute $\lambda=2\Omega$, her non-centrality parameter.

p.10.b. On the following plot (where the "central" F-distribution has the higher mode),

p.10.b.i. identify her rejection region (be very specific on any numbers you are using).

p.10.b.ii. Shade in the area representing the power of her test.



p.10.c. Another researcher is reporting that he will reject his null hypothesis of no treatment effects if his F-statistic exceeds 5.143. How many treatments and how many replicates does he have in his study?

g = ______________ n = ____________

Q.11. Two statistical programs are fitting a 1-Way Analysis of Variance, based on the treatment effects model:

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i = 1,..,4 \quad j = 1,...,n \quad \text{Program A: } \alpha_4 = 0 \quad \text{Program B: } \sum_{i=1}^{4} \alpha_i = 0$$

The least squares estimates of the model parameters: μ, τ_1, τ_2, τ_3 are given below:

Parameter	Program A	Program B
μ	60	55
α1	-10	-5
α2	10	15
α3	20	-15

p.11.a. Compute the least squares estimates of the following estimable parameters, based on each program (Show all numbers used in calculations):

p.11.a.i.: $\mu + \alpha_1$ Program A ______________________________ Program B ________________________________

p.11.a.ii.: $\mu + \alpha_4$ Program A ______________________________ Program B ________________________________

p.11.a.iii.: $\alpha_2 - \alpha_1$ Program A ______________________________ Program B ________________________________

p.11.a.iv.: $\alpha_4 - \alpha_1$ Program A ______________________________ Program B ________________________________

p.11.b. For this experiment, what is the largest the error sum of squares could be, for us to reject H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$, if n = 5?

Q.12. For a one-way fixed effects analysis with t=3 treatments and unequal sample sizes (n_1, n_2, n_3), derive the ordinary least squares estimators for the treatment effects model. Show all work.

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i = 1,2,3 \quad j = 1,\ldots,n_i \quad \sum_{i=1}^{3} \alpha_i = 0$$

p.12.a. Derive the Ordinary Least Squares Estimator of μ:

p.12.b. Derive the Ordinary Least Squares Estimator of α_1:

p.12.c. Obtain $E\left\{Y_{ij}^2\right\}, \quad E\left\{\overline{Y}_{i\bullet}^2\right\}, \quad E\left\{\overline{Y}_{\bullet\bullet}^2\right\}$

Q.13. An experiment was conducted to compare preferred distances (cm) among subjects randomly assigned to watch HDTVs of g=3 sizes (32", 37", 42"). There were n = 150 subjects per size. The sample means for the 3 distances were 26.7 (32"), 29.8 (37"), and 33.8 (42"). The sums of squares are: SSTrts = 3801 and SSErr = 1710641, respectively.

p.13.a. Consider two contrasts among the 3 sizes: Linear: $C_1 = -\mu_{32} + \mu_{42}$ Quadratic: $C_2 = \mu_{32} - 2\mu_{37} + \mu_{42}$

Show that these are orthogonal contrasts.

p.13.b. Obtain the estimate for each contrast.

p.13.c. Compute the Sum of Squares for each contrast, and show they sum to SSTrts.

SSC_1 = ________________________________ SSC_2 = ___________________________ SSC_1+SSC_2 = _________________

p.13.d. For each contrast, test $H_0: C = 0$ vs: $H_A: C \neq 0$ by the F-test

Test $Stat_1$: ___________________________ Test $Stat_2$: ______________________ Rejection Region: ________________

Q.14. A study was conducted to compare 3 treatments for patients with HIV. The response was a measure of change in CD4+ counts. There were n = 90 patients in each of the 3 treatment conditions. The means and MSE are given below, note that positive responses implying increasing counts (good) and negative responses imply decreasing counts (bad).

$\overline{Y}_1 = 12.2 \quad \overline{Y}_2 = 5.1 \quad \overline{Y}_3 = -0.3 \quad MSE = 400$

Compute simultaneous 95% CIs for all pairs of treatments, based on Tukey's, Bonferroni's, and Scheffe's Methods:

Tukey: $\mu_1-\mu_2$: ______________________ $\mu_1-\mu_3$: ______________________ $\mu_2-\mu_3$: ______________________

Bonferroni: $\mu_1-\mu_2$: ______________________ $\mu_1-\mu_3$: _________________________ $\mu_2-\mu_3$: _____________________

Scheffe: $\mu_1-\mu_2$: ______________________ $\mu_1-\mu_3$: ______________________ $\mu_2-\mu_3$: ______________________

Q.15. Derive E{MS_{Trts}} for the balanced 1-Way ANOVA with t treatments and r replicates per treatment by completing the following parts. **SHOW ALL WORK to Receive any Credit!!!!!!!!!**

p.15.a. Show: $SS_{\text{Trts}} = \sum_{i=1}^{g}\sum_{j=1}^{n}\left(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet}\right)^2 = n\sum_{i=1}^{g}\left(\overline{y}_{i\bullet}\right)^2 - gn\left(\overline{y}_{\bullet\bullet}\right)^2$

p.15.b. Derive $E\left\{\overline{y}_{i\bullet}\right\}, \quad V\left\{\overline{y}_{i\bullet}\right\}, \quad E\left\{\left(\overline{y}_{i\bullet}\right)^2\right\}, \quad E\left\{\overline{y}_{\bullet\bullet}\right\}, \quad V\left\{\overline{y}_{\bullet\bullet}\right\}, \quad E\left\{\left(\overline{y}_{\bullet\bullet}\right)^2\right\}$

p.15.c. Use results from p.1.a. and p.1.b. to obtain: $E\left\{SS_{\text{TRTS}}\right\}$ and $E\left\{MS_{\text{TRTS}}\right\}$

Q.15.d. For which scenario of the parameter values, will the power of the F-test be highest (assume g = 3, $n_1 = n_2 = n_3 = n$):

Scenario 1: $\mu_1 = 80$, $\mu_2 = 100$, $\mu_3 = 120$, $\sigma = 10$ or **Scenario 2:** $\mu_1 = 90$, $\mu_2 = 100$, $\mu_3 = 110$, $\sigma = 5$

Compute E{MS_{Trts}} for each scenario (as a function of r).

Q.16. A published report, based on a **balanced 1-Way ANOVA** reports means (SDs) for the three treatments as:

Trt 1: 70 (8) Trt 2: 75 (6) Trt 3: 80 (10) Tragically, the authors fail to give the treatment sample sizes.

p.16.a The Treatment degrees of freedom is the same for each sample size. The error degrees of freedom are:

df_{TRTS} = ___________ $n_i = 2$: df_{ERR} = ___________ $n_i = 6$: df_{ERR} = ____________ $n_i = 10$: df_{ERR} = ____________

p.16.b. Complete the following table, given arbitrary levels of the number of replicates per treatment:

n	SSTrt	SSErr	MSTrt	MSErr	F_obs	F(0.05)
2						
6						
10						

p.16.c. The smallest r_i, so that these means are significantly different is:

i) $n_i <= 2$ ii) $2 < n_i <= 6$ iii) $6 < n_i <= 10$ iv) $n_i > 10$

Q.17. A study is conducted to compare 4 menus in terms of numbers of calories ordered by restaurant customers (in 100s of calories). The treatments (menus) are (consider them increasing in order of information provided):

1) No Calories Reported   2) Calories Reported   3) Rank-Ordered Calories   4) Color-Coded Calories

The sample sizes are all based on samples of r = 20 customers per menu. The sample means and estimated variance are:
$\overline{Y}_{1\bullet} = 17.6 \quad \overline{Y}_{2\bullet} = 16.8 \quad \overline{Y}_{3\bullet} = 16.0 \quad \overline{Y}_{4\bullet} = 14.4 \quad \overline{Y}_{\bullet\bullet} = 16.2 \quad s^2 = \text{MSErr} = 196.0$.

p.3.a. Complete the following table to test $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

ANOVA	df	SS	MS	F_obs	F(.05)	Reject H0?
Source						Yes / No
Trts (Menus)				#N/A	#N/A	#N/A
Error				#N/A	#N/A	#N/A
Total			#N/A	#N/A	#N/A	#N/A

p.17.b.: Give 3 orthogonal contrasts, and their estimates and estimated standard errors

Contrast 1: Menu 1 vs Menus {2, 3, 4}   Contrast 2: Menu 2 vs Menus {3, 4}   Contrast 3: Menu 3 vs Menu 4

Contrast 1: $C_1 =$ ____________________ $c_1 =$ ____________________ $SE[c_1] =$ ____________________

Contrast 2: $C_2 =$ ____________________ $c_2 =$ ____________________ $SE[c_2] =$ ____________________

Contrast 3: $C_3 =$ ____________________ $c_3 =$ ____________________ $SE[c_3] =$ ____________________

p.17.c. Obtain Simultaneous 95% CIs for each (population) contrast, based on Bonferroni's method

Contrast 1:

Contrast 2:

Contrast 3:

Q.18. Players in the English Premier Soccer League are classified as one of 4 positions (Defender, Forward, Goalie, or Midfielder). Random samples of 8 players were selected from each position for players during the 2014 season. The players' Body Mass Indices were measured. The within position (Error) sum of squares is SSE = 68. The means for the 4 positions are: 22.77 (D), 22.19 (F), 24.11 (G), and 23.16 (M).

p.18.a. Compute the standard error of the difference between 2 means: $s\left\{\overline{Y}_{i\bullet} - \overline{Y}_{i'\bullet}\right\}$

$s\left\{\overline{Y}_{i\bullet} - \overline{Y}_{i'\bullet}\right\} =$ ___________________________

p.18.b. Compute Tukey's Honest Significant Difference for simultaneously comparing all pairs of positions, with a family-wise error rate of 0.05. Identify significant differences (if any) among all pairs of means.

Tukey's HSD = _________________________________

p.18.c. Compute Bonferroni's Minimum Significant Difference for simultaneously comparing all pairs of positions, with a family-wise error rate of 0.05. Identify significant differences (if any) among all pairs of means.

Bonferroni's MSD = __________________________

p.18.d. Compute Scheffe's Minimum Significant Difference for simultaneously comparing all pairs of positions, with a family-wise error rate of 0.05. Identify significant differences (if any) among all pairs of means.

Scheffe's MSD = ________________________

Q.19. Consider the following 2 models:

Model 1: $y_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,...,g;\ j = 1,...,n_i$ Model 2: $y_{ij} = \mu + \varepsilon_{ij} \quad i = 1,...,g;\ j = 1,...,n_i$

p.19.a. Derive the least squares estimator of μ_k for Model 1. SHOW ALL WORK!!!!!

p.19.b. Derive the least squares estimator of μ for Model 2. SHOW ALL WORK!!!!!

p.19.c. Three brands of cellphones are to be compared in terms of sound quality. Samples of $n_1 = 2$, $n_2 = 3$, $n_3 = 2$ are obtained, with the following sound quality scores: Brand 1: 8, 9; Brand 2: 8, 7, 6; Brand 3: 5, 6.

Brand1	Brand2	Brand3
8	8	5
9	7	6
	6	

Obtain the Error Sum of Squares and Degrees of Freedom for each model:

Model 2: $SSE_2 =$ $df_{E2} =$

Model 1: $SSE_1 =$ $df_{E1} =$

p.19.d. Conduct the General Linear Test to test $H_0: \mu_1 = \mu_2 = \mu_3$

Test Statistic: ____ Rejection Region: ___________________

Q.20. For the balanced 1-Way Analysis of Variance model, complete the following parts.

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i = 1,...,g;\ j = 1,...,n \quad \varepsilon_{ij} \sim NID\left(0, \sigma^2\right)$$

p.20.a. Show that: $SS_{\text{Err}} = \sum_{i=1}^{g}\sum_{j=1}^{n} Y_{ij}^2 - n\sum_{i=1}^{g} \overline{Y}_{i\bullet}^2$ and $SS_{\text{Trts}} = n\sum_{i=1}^{g} \overline{Y}_{i\bullet}^2 - N\overline{Y}_{\bullet\bullet}^2$

p.20.b. Derive $E\{SS_{\text{Err}}\}$ and $E\{SS_{\text{Trts}}\}$

Q.21. A study compared the antioxidant activity for 4 varieties of green tea. Five replicates were obtained from each tea variety and the total phenolic content was measured. The means and standard deviations are given below.

Variety	1	2	3	4
Mean	160	170	140	190
SD	15	18	12	15

p.21.a. Complete the following Analysis of Variance table.

Source	df	Sum of Squares	Mean Square	F_obs	F(.95)
Treatments					
Error					
Total					

p.22.b. Use Tukey's Method to compare all pairs of variety means.

p.22.c. Compute Bonferroni's and Scheffe's minimum significant differences for comparing all pairs of treatments.

Q.23. A 1-Way ANOVA is to be fit with g = 3 treatments and sample sizes $n_1 = 2$, $n_2 = 4$, $n_3 = 3$

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,2,3;\ j = 1,...,n_i \qquad \underset{N\times 1}{\mathbf{Y}} = \underset{N\times 3}{\mathbf{X}}\underset{3\times 1}{\boldsymbol{\beta}} + \underset{N\times 1}{\boldsymbol{\varepsilon}}$$

Give the form of the **X** matrix, **X'X** matrix and $\boldsymbol{\beta}$ vector for each of the following parameterizations.

p.23.a. $\mu^* = 0 \quad \alpha_i^* = \mu_i \quad i = 1,2,3$

p.23.b. $\alpha_1^* = 0 \quad \mu^* = \mu_1 \quad \alpha_i^* = \mu_i - \mu_1 \quad i = 1,2,3$

p.23.c. $\mu^* = \mu \quad \sum_{i=1}^{3} \alpha_i^* = 0$

Q.24. A study was conducted to compare the effects of 4 evenly spaced doses of a drug on a measured response. There were 6 replicates per dose, sample mean responses were 10, 18, 22, and 26 respectively and $SS_{Err} = 500$.

p.24.a. Compute the Treatment sum of Squares, SS_{Trts}.

p.24.b. The goal was to partition the Treatment sum of squares into Linear, Quadratic, and Cubic components.

i	Linear	Quadratic	Cubic
1	-3	1	-1
2	-1	-1	3
3	1	-1	-3
4	3	1	1

Making use of the pairwise orthogonal contrasts, compute SS_{Lin}, SS_{Quad}, SS_{Cubic} and show they sum to SS_{Trts}

p.24.c. Compute the F-statistics and the critical F value (do not adjust for multiple tests) for testing these 3 (population) contrasts are 0.

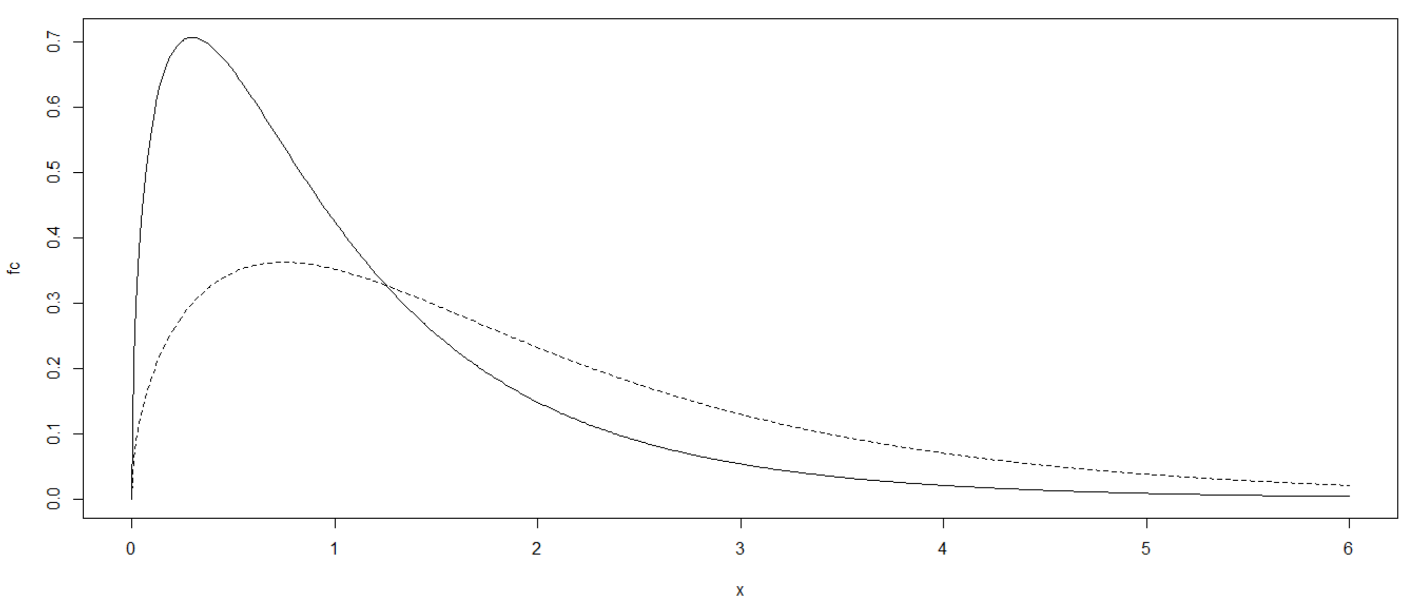
F_{Lin} = ________________ F_{Quad} = ________________ F_{Cubic} = ________________ $F_{.95,df1,df2}$ = ________________

Q.25. Consider a model with g = 4 treatments, with sample sizes $n_1 = n_2 = n_3 = n_4 = 5$.

p.25.a. Give the rejection region for testing $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

p.25.b. Give the non-centrality parameter for the F-statistic for testing $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ when $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 40$, $\mu_4 = 50$ and $\sigma = 20$. Give 2Ω

p.25.c. On the following graph, identify the distributions of the F-statistic under H_0 and under the parameter values in p.7.b. Sketch the power of the F-test under p.7.b.



Q.26. A published report, based on a balanced 1-Way ANOVA reports means (SDs) for the three treatments as:

Trt 1: 70 (8) Trt 2: 75 (6) Trt 3: 80 (10)

Unfortunately, the authors fail to give the sample sizes.

p.26.a. Complete the following table, given arbitrary levels of the number of replicates per treatment:

r	SSTrt	SSErr	MSTrt	MSErr	F_obs	F(.05)
2						
6						
10						

p.26.b. The smallest n, so that these means are significantly different is:

i) n <= 2 ii) 2 < n <= 6 iii) 6 < n <= 10 iv) n > 10

Q.27. An experiment with g = 3 treatments, $n_1 = n_2 = n_3 = 5$ and $\overline{y}_{1\bullet} = 20, s_1 = 3 \quad \overline{y}_{2\bullet} = 30, s_2 = 9 \quad \overline{y}_{3\bullet} = 10, s_3 = 1$

Estimated Weighted Least Squares with the weight of Y_{ij} being the inverse of its group's sample variance.

We plan to test H_0: $\mu_1 = \mu_2 = ... = \mu_g = \mu$ H_A: Not all μ_i are equal.

Under H_0 (the Reduced Model), the weighted Error sum of squares is $Q_R^W = \sum_{i=1}^{3}\sum_{j=1}^{5} w_{ij}\left(Y_{ij} - \mu\right)^2$ derive the weighted least squares estimator for μ and give its estimate based on this data.

Q.28. An experiment is conducted as a Completely Randomized Design to compare the durability of 5 green fabric dyes, with respect to washing. A sample of 30 plain white t-shirts was obtained, and randomized so that 6 received each dye (with each shirt receiving exactly one dye). A measure of the color brightness of the shirts after 10 wash/dry cycles is obtained (with higher scores representing brighter color). The error sum of squares is reported to be *SSE* = 2000. The mean scores for the 5 dyes are: $\overline{Y}_{1\bullet} = 30 \quad \overline{Y}_{2\bullet} = 25 \quad \overline{Y}_{3\bullet} = 40 \quad \overline{Y}_{4\bullet} = 35 \quad \overline{Y}_{5\bullet} = 20$

p.28.a. Compute Tukey's HSD, and determine which means are significantly different with an experiment-wise (overall) error rate of $\alpha_E = 0.05$, by joining lines to means that are not significantly different.

D5 D2 D1 D4 D3

p.28.b. Compute the Bonferroni MSD, and determine which means are significantly different with an experiment-wise (overall) error rate of $\alpha_E = 0.05$, by joining lines to means that are not significantly different.

D5 D2 D1 D4 D3

Q.29. Geographers are interested in the effect of map extent (Percentage of a map showing up on computer screen at one time) on response time for completing a map reading task. They sampled 80 individuals, and randomly assigned them to 4 extents (full, 40%, 25%, 10%), such that 20 individuals received each extent. Note that this a fixed effects model. $y_{ij} = \mu_i + \varepsilon_{ij}$ $\varepsilon_{ij} \sim NID(0,\sigma^2)$.

Treatment	r	Mean	SD
Full Extent	20	34	10
40% Extent	20	35	8
25% Extent	20	38	8
10% Extent	20	45	13

p.29.a. Complete the following ANOVA table

ANOVA					
Source	df	SS	MS	F	F(.05)
Treatments					
Error					
Total					

p.29.b. Give a contrast that compares the average of the last 3 treatments (less than full extent) with the full extent.

p.29.c. Give a 95% Confidence Interval for the contrast.

p.29.d. Use the F-test to test $H_0: C = 0$ vs $H_A: C \neq 0$

Test Statistic: ________________________________ Rejection Region: _____________________________

Q.30. For the balanced completely randomized design with g treatments, and n units per treatment, consider the following (cell means) model:

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i=1,\ldots,g;\ j=1,\ldots,n \quad \sum_{i=1}^{g} \alpha_i = 0 \quad \varepsilon_{ij} \sim NID\left(0,\sigma^2\right)$$

p.30.a. Derive the least squares estimate of μ: $\hat{\mu}$ SHOW ALL WORK

p.30.b. Derive the least squares estimate of α_k: $\hat{\alpha}_k$ SHOW ALL WORK

p.30.c. Derive: $E\left\{\hat{\mu}\right\},\ V\left\{\hat{\mu}\right\},\ E\left\{\hat{\alpha}_k\right\},\ V\left\{\hat{\alpha}_k\right\},\ COV\left\{\hat{\mu},\hat{\alpha}_k\right\},\ COV\left\{\hat{\alpha}_k,\hat{\alpha}_{k'}\right\}\ \left(k \neq k'\right)$ SHOW ALL WORK

p.30.d. If you had point estimates for all parameters in the model, how would you set up a 95% Confidence Interval for $\alpha_k - \alpha_{k'}$ (as a function of n and g)

Q.31. An experiment is conducted with 3 diets, and 4 replicates per diet. Weight loss (Y) is observed from pre-diet at 6 months. Positive values of Y imply the subject has lost weight since the trial began. You are given the following data. Compute the Analysis of Variance, including testing $H_0: \mu_1 = \mu_2 = \mu_3$

j	Diet1	Diet2	Diet3
1	12	4	-2
2	10	6	0
3	6	6	3
4	8	8	-1

ANOVA						
SOURCE	df	SS	MS	F	F(.05)	P-value
						< or > .05
				#N/A	#N/A	#N/A
Total			#N/A	#N/A	#N/A	#N/A

Q.32. In an experiment to compare g = 4 cold medications, brands 1 and 2 make use of one chemical, and brands 3 and 4 make use of a second chemical. A researcher is interested in whether there are differences among the effects of the 4 cold medications. In particular, she would like to compare medications within each chemical type, as well as compare the 2 chemical types. The response is a measure of the brand's ability to keep a cold virus from occurring in the subject.

p.32.a. Give 3 orthogonal contrasts (sets of weights) that will answer her research questions. Show that they are orthogonal.

	Contrast1	Contrast2	Contrast3
w1			
w2			
w3			
w4			

p.32.b. The sample means and standard deviations for each treatment are given below, based on samples of n = 8 subjects per treatment. Compute SS_{TRT} and MS_{Err}

	Mean	SD
Brand1	12	8
Brand2	16	6
Brand3	22	6
Brand4	30	8

SS_{Trt} = ____________________________________ MS_{Err} = ______________________________

p.32.c. Compute each of the 3 estimated contrasts, their sums of squares and their corresponding F-statistics and critical F-values

Contrast	Estimate	SumSq	F	F(.05)
1				
2				
3				

p.32.d. Show that the orthogonal sums of squares add to the Treatment sum of squares.

Q.33. An experiment was conducted on the phenobarbital sleep time of rats when they had been exposed to various fragrances in a Completely Randomized Design. There were 5 fragrances, including a control condition, and there were 10 rats per treatment. MS_{Err} = 16.0. The means are given below.

Trt	Mean
Control	68
Lemon	54
Jasmine	63
Rose	79
Sandalwood	77

p.33.a. Compute Tukey's Honest Significant difference. Draw lines below the treatment labels, joining pairs of treatments that are not significantly different.

Lemon Jasmine Control Sandalwood Rose

p.33.b. Compute Bonferroni's minimum Significant difference. Draw lines below the treatment labels, joining pairs of treatments that are not significantly different.

Lemon Jasmine Control Sandalwood Rose

p.33.c. Compute Scheffe's minimum Significant difference. Draw lines below the treatment labels, joining pairs of treatments that are not significantly different.

Lemon Jasmine Control Sandalwood Rose

Q.34. A one-way ANOVA is run as cell means model in matrix form $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ You are given the following matrix and vector:

X'X					X'Y
6	0	0	0		48
0	3	0	0		24
0	0	4	0		20
0	0	0	7		28

p.34.a. Complete the following parts:

g = ____________ n^s = ______________________________ $\overline{Y}_{\bullet\bullet}$ = ____________________

$$\hat{\beta} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad V\left\{\hat{\boldsymbol{\beta}}\right\} = \sigma^2 \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Q.35. You would like to obtain the uncorrected sum of squares (USS) for a sample, based on published summary statistics: mean, standard deviation, and sample size. Given their published summary, how would you compute it?

$$USS = \sum_{i=1}^{g}\sum_{j=1}^{n_i} Y_{ij}^2 \qquad \overline{Y}_{i\bullet} = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij} \qquad s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}\left(Y_{ij} - \overline{Y}_{i\bullet}\right)^2$$

Q.36. A researcher wants to run a 1-Way ANOVA with 3 treatments with $\alpha = 0.05$ level test of $H_0: \mu_1 = \mu_2 = \mu_3$

An important difference in treatment means he would like to detect is: $\frac{1}{\sigma^2}\sum_{i=1}^{3}\alpha_i^2 = 0.18$

p.36.a. He considers balanced designs with n = 5, 10, 15. For each n, give the critical value for the F-test and the non-centrality parameter (2Ω in the notes from Wednesday's lecture).

n	F(.05)	2Ω
5		
10		
15		

p.36.b. This plot represents the central and non-central F distributions when n = 10. Sketch the area that represents the power of the test for this set of treatment effects.