STA 6166 – Exam 1 – Spring 2016 **PRINT** Name ANSWER KEY

(123)

Unless stated otherwise, conduct all tests at $\alpha = 0.05$ significance level.

True/False, Multiple Choice, and Fill in the Blanks Questions

3 Q.1. Two researchers analyze the same set of observations from 2 samples of equal sample sizes ($n_1 = n_2 = n$). One researcher uses the independent sample t-test, based on equal variances. The other researcher uses the independent sample t-test, based on unequal variances. Choose the correct answer:

- Their test statistics will be the same, their degrees of freedom will be the same.
- Their test statistics will be the same, their degrees of freedom will be different.
- Their test statistics will be different, their degrees of freedom will be the same.
- Their test statistics will be different, their degrees of freedom will be different.

4 Q.2. A scientist wants to compare the effects of 3 treatments on behavior in mice. The treatments are: 1) Placebo, 2) Drug A, 3) Drug B. The experiment is balanced. The researcher is interested in 2 specific contrasts: Contrast 1: Placebo (μ_1) versus Average of Drug A (μ_2) and Drug B (μ_3), Contrast 2: Drug A versus B. Give the two contrasts (note there are many ways of writing these, but they share a specific pattern):

$l_1 = \underline{\;2\;}\,\mu_1 \underline{\;-1\;}\,\mu_2 \underline{\;-1\;}\,\mu_3 \qquad l_2 = \underline{\;0\;}\,\mu_1 \underline{\;1\;}\,\mu_2 \underline{\;-1\;}\,\mu_3$

3 Q.3. A study is conducted to compare 2 methods of teaching foreign language to children (independent samples). One analyst uses a 2-sided test of $H_0: \mu_1 - \mu_2 = 0$ versus $H_A: \mu_1 - \mu_2 \neq 0$ based on the independent sample t-test, assuming equal variances. The other analyst uses a 1-Way Analysis of Variance F-test to test $H_0: \mu_1 = \mu_2$ versus $H_A: \mu_1 \neq \mu_2$. They use the same computing software, so there are no issues due to rounding. Choose the correct answer:

- The p-value from the t-test will always be higher than the p-value from the F-test
- The p-value from the t-test will always be lower than the p-value from the F-test
- The p-value from the t-test will always be the same as the p-value from the F-test
- None of the above

3 Q.4. For a balanced 1-Way ANOVA, with $t > 2$ groups, when making all pairwise comparisons, Tukey's W will always be smaller than Bonferroni's B.

TRUE or **FALSE**

13

Problems

Q.5. Among 2 large populations (Males and Females) who completed the Rock and Roll marathon in 2015, the **population** means and standard deviations of velocities (miles per hour) were:

$\mu_M = 6.34$ $\sigma_M = 1.06$ $\mu_F = 5.84$ $\sigma_F = 0.83$

Suppose you simultaneously took many random samples of size $n_M = n_F = 20$ from each population, and for each pair of random samples, you computed $\overline{Y}_M - \overline{Y}_F$ and saved each difference.

p.5.a. What would you expect the mean of the $\overline{Y}_M - \overline{Y}_F$ values to be.

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$$\mu_M - \mu_F = 6.34 - 5.84 = 0.50$$

p.5.b. What would you expect the standard deviation of the $\overline{Y}_M - \overline{Y}_F$ values to be.

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$$S_{\bar{Y}_M - \bar{Y}_F} = \sqrt{\frac{1.06^2}{20} + \frac{0.83^2}{20}} = \sqrt{\frac{1.8125}{20}} = \sqrt{.090625} = 0.3010$$

p.5.c. Between what 2 bounds would you expect 95% of the $\overline{Y}_M - \overline{Y}_F$ values to lie between?

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$$0.50 \pm 1.96(.3010) = .50 \pm 0.5900$$

$$= (-0.09, 1.09)$$

Q.6. Two models of video cameras are being compared for detecting animals in a wildlife setting. The cameras will film the same locations in fixed time intervals in a paired difference experiment. The parameter μ_D is the population mean difference across all possible locations in the fixed time intervals. From a pilot study, it is believed $\sigma_d = 5$. How many samples will be needed if we wish for the margin of error in estimating μ_D within $E = 0.5$ with 95% Confidence?

8

$$E = \frac{z_{\alpha/2}\,\sigma_d}{\sqrt{n}} \Rightarrow n = \left(\frac{z_{\alpha/2}\,\sigma_d}{E}\right)^2 = \left(\frac{1.96(5)}{.5}\right)^2$$

$$= 19.6^2 = 384.16 \approx 385$$

23

Q.6. An experiment is conducted to compare breaking strengths of 2 types of fibers. The means, standard deviations, and sample sizes of random samples from each fiber type are: $\overline{y}_{1\bullet} = 50 \quad s_1 = 12 \quad n_1 = 10 \quad \overline{y}_{2\bullet} = 45 \quad s_2 = 8 \quad n_2 = 10$

p.6.a. Test H_0: $\sigma_1^2 = \sigma_2^2$ versus H_A: $\sigma_1^2 \neq \sigma_2^2$ at $\alpha = 0.10$ significance level

$$F_{obs} = \frac{S_1^2}{S_2^2} = \frac{12^2}{8^2} = \frac{144}{64} = 2.25$$

$F_{.05,9,9} = 3.179$

$\Rightarrow F_{.95,9,9} = \frac{1}{3.179} = 0.315$

6

2 2

Test Statistic: 2.25 Reject H_0 if Test Statistic < 0.315 or > 3.179

p.6.b. Regardless of your previous answer, assume $\sigma_1^2 = \sigma_2^2$, Test H_0: $\mu_1 - \mu_2 = 0$ versus H_A: $\mu_1 - \mu_2 \neq 0$

$$\sqrt{\frac{(10-1)12^2 + (10-1)8^2}{10+10-2}} = 13.27$$

$$t_{obs} = \frac{50-45}{13.27\sqrt{\frac{1}{10}+\frac{1}{10}}} = \frac{5}{5.935} = 0.843$$ 8

3

Test Statistic: 0.843 Reject H_0 if Test Statistic < −2.101 or > 2.101

Q.7. A 1-Way ANOVA is conducted, comparing clarity of t = 3 methods if meniscal repair. A sample of N=18 subjects was obtained and assigned at random such that $n_1 = 6$ received method 1, $n_2 = 6$ received method 2, and $n_3 = 6$ received method 3. The response was Y = Displacement (mm). Complete the following table to test:

$H_0: \mu_1 = \mu_2 = \mu_3$ vs H_A: Not all μ_i are equal

Source	df 4/4	SS 5	MS 3/3	F_obs 3	F(.05) 3
Treatment(Between)	3-1=2	105	52.50	3.768	3.682
Error(Within)	18-3=15	209	13.93	#N/A	#N/A
Total	18-1=17	314	#N/A	#N/A	#N/A

2 2

Do we reject the null hypothesis? Yes or No Is the P-value < 0.05 or > 0.05

48

Q.8. A study is conducted to compare lifetimes of 2 brands of light bulbs. Random samples of $n_1 = n_2 = 20$ are obtained from each manufacturer. Due to the highly skewed distributions of lifetimes, the large-sample Wilcoxon rank-sum test was used. The investigators tested each bulb, measuring its lifetime, and ranked all of the N = 40 bulbs. The rank sum for the two brands are $T_1 = 460$ and $T_2 = 360$. Complete the following parts to test $H_0: M_1 = M_2$ $H_A: M_1 \neq M_2$

p.8.a. Compute μ_T: $\frac{n_1(N+1)}{2} = \frac{20(40+1)}{2} = 410$ 8

p.8.b. Compute σ_T: $\sqrt{\frac{n_1 n_2 (N+1)}{12}} = \sqrt{\frac{20(20)(41)}{12}} = 36.97$ 8

p.8.c. Test Statistic: $z_{obs} = \frac{460-410}{36.97} = \frac{50}{36.97} = 1.353$ 5

p.8.d. Rejection Region: $|z_{obs}| \geq 1.96$ 2 p.8.e. Reject H_0? **Yes** or **No** 2

Q.9. An experiment was conducted to compare wine color intensity (Y) among t = 6 types of wine barrels. There 9 replicates for each of the wine barrel types. The Mean Square Error (MSW) was 1.04.

p.9.a. Compute Tukey's Honest Significant Difference for Comparing all pairs of wine barrel types

$df_{Error} = 6(9-1) = 48$ $q(.05, 6, 48) \approx 4.198$ 4 $\sqrt{\frac{MSE}{n}} = \sqrt{\frac{1.04}{9}} = .340$ 4

Conclude Conclude $\mu_i \neq \mu_j$ if $|\bar{y}_i - \bar{y}_j| \geq W_{ij} =$ $4.198(.340) = 1.427$ 2

p.9.a. Compute Bonferroni's Minimum Significant Difference for Comparing all pairs of wine barrel types $C = \frac{6(5)}{2} = 15$

$t(.025/15, 48) \approx 3.091$ 4 $\sqrt{MSE\left(\frac{2}{n}\right)} = \sqrt{1.04\left(\frac{2}{9}\right)} = 0.481$ 4

Conclude Conclude $\mu_i \neq \mu_j$ if $|\bar{y}_i - \bar{y}_j| \geq B_{ij} =$ $3.091(.481) = 1.486$ 2

39