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STA 6166 – Exam 2 – Fall 2016 PRINT Name ANSWER KEY

Conduct all tests at $\alpha = 0.05$ significance level

Q.1. Researchers were interested in the effect of a testosterone injection on men with erectile dysfunction. Scores for erectile function were taken at baseline (pre-treatment) and after 12 weeks of treatment on a sample of 20 men with erectile dysfunction. The appropriate analyses for normally distributed and non-normally distributed scores would be:

4

- Normal: Independent Sample t-test Non-Normal: Wilcoxon Rank-sum test
- Normal: Independent Sample t-test Non-Normal: Wilcoxon Signed-Rank test
- Normal: Paired t-test Non-Normal: Wilcoxon Rank-sum test
- Normal: Paired t-test Non-Normal: Wilcoxon Signed-Rank test

Q.2. In which case will the power of the test of H_0: $\mu = \mu_0$ versus H_A: $\mu > \mu_0$ for the specific alternative $\mu_A > \mu_0$ be highest?

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a) $n = 10, \sigma = 10, \mu_A - \mu_0 = 3$ b) $n = 20, \sigma = 5, \mu_A - \mu_0 = 3$ c) $n = 5, \sigma = 20, \mu_A - \mu_0 = 3$

Q.3. When conducting an independent sample t-test with and without equal variance, when $n_1 = n_2$, which of the following statements is correct?

4

a) The t-statistics are the same, the P-value is smaller for the unequal variance case
b) The t-statistics are the same, the P-value is smaller for the equal variance case
c) Neither the t-statistics or the ~~manufacturer~~ P-values will be the same

Q.4. A manufacturer of a new swimsuit for competitive swimmers would like to compare the velocities of swimmers in their swimsuit with their competitor's swimsuit. They plan a paired experiment, where a sample of swimmers will be obtained, and their velocity (cm/second) will be measured in each swimsuit. They believe the standard deviation of the within swimmer differences is 10 cm/sec. They are conducting a 2-sided test of H_0: $\mu_D = 0$ H_A: $\mu_D \neq 0$ with $\alpha = 0.05$. How many swimmers should they use if they want the power to be 0.90 to detect a difference of $\Delta = 2.0$ cm/sec?

$\sigma = 10$ $\alpha = .05$ $\alpha/2 = .025$ $z_{.025} = 1.96$ 3

$1-\beta = .90 \Rightarrow \beta = .10 \Rightarrow z_{.10} = 1.282$ 5

$\Delta = 2.0$ 4 2

$$n = \frac{\sigma^2 (z_{\alpha/2} + z_\beta)^2}{\Delta^2} = \frac{10^2 (1.96 + 1.282)^2}{2^2} = 262.76 \approx 263$$

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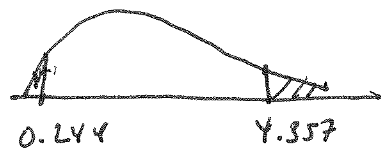
Q.5. A study compared the strength of silk spun from 2 species of New Zealand spiders. The species were: *S. Cavernicola* and *L. Katipo*. The summary statistics are given below for the response variable maximum load for samples from each species.

Species	S.C. (i=1)	L.K. (i=2)
n	10	9
Mean	5.869	2.917
Std dev	1.966	1.449

p.5.a. Test whether the population variances are equal. $H_0: \sigma_1^2 = \sigma_2^2 \quad H_A: \sigma_1^2 \neq \sigma_2^2$

$T.S.\ F_{obs} = \frac{s_1^2}{s_2^2} = \frac{1.966^2}{1.449^2} = 1.841$ (4)



RR: $F_{obs} \geq F_{.025,10-1,9-1} = 4.357$

$F_{obs} \leq F_{.975,9,8} = \frac{1}{F_{.025,8,9}} = \frac{1}{4.102} = 0.244$

Test Statistic 1.841 Rejection Region ≥ 4.357, ≤ 0.244 P-value > 0.05 or < 0.05 (3)

p.5.b. Using the equal variance case, test whether the population means are equal. $H_0: \mu_1 - \mu_2 = 0 \quad H_A: \mu_1 - \mu_2 \neq 0$

$$\sqrt{\frac{(10-1)(1.966^2)+(9-1)(1.449^2)}{10+9-2}} = 1.742$$

$\sqrt{\frac{1}{10}+\frac{1}{9}} = \sqrt{.21111} = 0.459$

$\bar{y}_1 - \bar{y}_2 = 5.869 - 2.917 = 2.952$

$SE\{\bar{y}_1 - \bar{y}_2\} = 1.742(0.459) = 0.800$ (6)

$t_{obs} = \frac{2.952}{.800} = 3.692$ (2)

$t_{.025,19-2} = 2.110$ (3)

Test Statistic $t_{obs} = 3.692$ Rejection Region $|t_{obs}| \geq 2.110$ (2) P-value > 0.05 or < 0.05 (3)

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Q.6. An experiment was conducted as a paired experiment to compare the effects of tumble drying and line drying of fabrics. The researchers created n = 24 matched pairs, where the conditions with respect to other factors were identical, and one fabric was tumble dried, the other clothesline dried. The response was length shrinkage of the fabric after drying.

p.6.a. The sample mean difference was 1.08 and the sample standard deviation was 0.71. Compute the 95% Confidence Interval for the population mean difference μ_D.

$\bar{d} = 1.08$ (3) $\quad \frac{s_d}{\sqrt{n}} = \frac{0.71}{\sqrt{24}} = 0.145$ (5) $\quad t_{.025, 24-1} = 2.069$ (3)

$1.08 \pm 2.069(0.145) = 1.08 \pm 0.30 = (0.78, 1.38)$ (3)

95% CI for μ_D $(0.78, 1.38)$ (3)

p.6.b. Based on your confidence interval, would the P-value for $H_0: \mu_D = 0$ vs $H_A: \mu_D \neq 0$ be: **> 0.05** or **< 0.05** ?

p.6.c. Conduct the large-sample Wilcoxon Signed-Rank Test of $H_0: M_1 = M_2$ vs $H_A: M_1 \neq M_2$. $T^- = 5.5$.

$E\{T\} = \frac{n(n+1)}{4} = \frac{24(25)}{4} = 150$ (4)

$\sigma_T = \sqrt{\frac{24(24+1)(2(24)+1)}{24}} = \sqrt{25(49)} = 5(7) = 35$ (4)

$z_{obs} = \frac{5.5 - 150}{35} = -4.129$ (3)

Test Statistic $z_{obs} = -4.129$ Rejection Region $|z_{obs}| \geq 1.96$ (3) P-value is **> 0.05** or **< 0.05** (3)

p.6.d. What is T^+ for this experiment?

$T^- + T^+ = \frac{24(25)}{2} = 300$ (4)

$T^+ = 300 - 5.5 = 294.5$

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Q.7. An engineer wishes to compare 2 brands of propane gas tank firms in terms of mean hours of cooking per tank at a fixed temperature. She believes the standard deviation for each brand is 30 hours based on a pilot study. She would like to estimate the population mean difference $\mu_1 - \mu_2$ within 3 hours with 95% confidence. How many tanks should she sample from each firm?

$\sigma = 30$ $\quad E = 3$ $\quad z_{\alpha/2} = 1.96$ 3

8

$$n = \frac{2 z_{\alpha/2}^2 \sigma^2}{E^2} = \frac{2(1.96^2)(900)}{9} = 768.32 \approx 769$$ 2

$n_1 = n_2 =$ 769

Q.8. Anita Wlodarczyk is considered the all-time greatest women's hammer thrower. A random sample of n = 12 of her competitive throws yields a sample standard deviation of s = 3.075 meters. Assuming her distribution of throws is normally distributed, compute a 95% Confidence Interval for her "true" standard deviation of throws. Hint: Compute a CI for the variance, then compute the CI for the standard deviation from that.

$$\frac{(n-1)s^2}{\chi_U^2}, \frac{(n-1)s^2}{\chi_L^2}$$

$\chi_U^2 = \chi^2_{.025, 12-1} = 21.920$ 3

$\chi_L^2 = \chi^2_{.975, 12-1} = 3.816$ 3

$s^2 = 3.075^2 = 9.456$ 3

$(n-1)s^2 = 11(9.456) = 104.012$

3

95% CI for σ^2: $\left[\frac{104.012}{21.920} = 4.745, \frac{104.021}{3.816} = 27.257\right]$

95% Confidence Interval for σ: $\left[\sqrt{4.745} = 2.178 \quad \sqrt{27.257} = 5.221\right]$

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~~20~~ 12 ~~45~~

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Q.1. Researchers were interested in the effect of a testosterone injection on men with erectile dysfunction. Scores for erectile function were taken at baseline (pre-treatment) and after 12 weeks of treatment on a sample of 20 men with erectile dysfunction. The appropriate analyses for normally distributed and non-normally distributed scores would be:

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- Normal: Independent Sample t-test Non-Normal: Wilcoxon Signed-Rank test
- Normal: Independent Sample t-test Non-Normal: Wilcoxon Rank-sum test
- Normal: Paired t-test Non-Normal: Wilcoxon Signed-Rank test
- Normal: Paired t-test Non-Normal: Wilcoxon Rank-sum test

Q.2. In which case will the power of the test of H_0: $\mu = \mu_0$ versus H_A: $\mu > \mu_0$ be highest for the specific alternative $\mu_A > \mu_0$?

4

a) $n = 15, \sigma = 20, \mu_A - \mu_0 = 5$ b) $n = 30, \sigma = 15, \mu_A - \mu_0 = 5$ c) $n = 20, \sigma = 30, \mu_A - \mu_0 = 5$

Q.3. When conducting an independent sample t-test with and without equal variance, when $n_1 = n_2$, which of the following statements is correct?

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a) The t-statistics are the same, the P-value is smaller for the unequal variance case
b) The t-statistics are the same, the P-value is smaller for the equal variance case
c) Neither the t-statistics or the ~~manufacturer~~ P-values will be the same

Q.4. A manufacturer of a new swimsuit for competitive swimmers would like to compare the velocities of swimmers in their swimsuit with their competitor's swimsuit. They plan a paired experiment, where a sample of swimmers will be obtained, and their velocity (cm/second) will be measured in each swimsuit. They believe the standard deviation of the within swimmer differences is 15 cm/sec. They are conducting a 2-sided test of H_0: $\mu_D = 0$ H_A: $\mu_D \neq 0$ with $\alpha = 0.05$. How many swimmers should they use if they want the power to be 0.80 to detect a difference of $\Delta = 1.0$ cm/sec?

$\sigma = 15$ $\alpha = .05$ $\frac{\alpha}{2} = .025$ $z_{.025} = 1.96$ (3)

$1-\beta = .80 \Rightarrow \beta = .20$ $z_{.20} = 0.842$ (3)

$\Delta = 1.0$ (4)

$$n = \frac{\sigma^2 (z_{\alpha/2} + z_\beta)^2}{\Delta^2} = \frac{15^2(1.96+0.842)^2}{1^2} = \cancel{235.06 \approx 236}$$ (2)

$1766.5 \approx 1767$

24

Q.5. A study compared the strength of silk spun from 2 species of New Zealand spiders. The species were: *P.antipodiana* and *L. katipo*. The summary statistics are given below for the response variable maximum load for samples from each species.

Species	P.A. (i=1)	L.K. (i=2)
n	8	10
Mean	1.564	2.391
Std Dev	1.083	1.158

p.5.a. Test whether the population variances are equal. $H_0:\sigma_1^2=\sigma_2^2 \quad H_A:\sigma_1^2\neq\sigma_2^2$

$$T.S.\ F_{obs}=\frac{S_1^2}{S_2^2}=\frac{1.083^2}{1.158^2}=0.875$$ (4)

(4)

$$RR:\ F_{obs}\geq F_{.025,8-1,10-1}=4.197$$

$$F_{obs}\leq F_{.975,7,9}=\frac{1}{F_{.025,9,7}}=\frac{1}{4.823}=0.207$$

(3)

(2) ≥ 4.197

Test Statistic 0.875 Rejection Region ≤ 0.207 P-value >0.05 or <0.05

p.5.b. Using the equal variance case, test whether the population means are equal. $H_0:\mu_1-\mu_2=0 \quad H_A:\mu_1-\mu_2\neq 0$

$$\sqrt{\frac{(8-1)(1.083^2)+(10-1)(1.158^2)}{8+10-2}}=1.126 \qquad \sqrt{\frac{1}{8}+\frac{1}{10}}=\sqrt{.225}=0.474$$

$$\bar{y}_1-\bar{y}_2=1.564-2.391=-0.827$$ (3)

$$se\{\bar{y}_1-\bar{y}_2\}=1.126(0.474)=0.534 \qquad t_{obs}=\frac{-0.827}{0.534}=-1.549$$ (2)

(6)

$$t_{.025,18-2}=2.120$$ (3)

(3)

Test Statistic $t_{obs}=-1.549$ Rejection Region $|t_{obs}|\geq 2.120$ P-value >0.05 or <0.05

(2)

32

Q.6. An experiment was conducted as a paired experiment to compare the effects of using a rinse cycle softener and not using it. The researchers created n = 24 matched pairs, where the conditions with respect to other factors were identical, and one fabric was washed with softener, and the other was not. The response was length shrinkage of the fabric after drying.

p.6.a. The sample mean difference was -0.188 and the sample standard deviation was 0.292. Compute the 95% Confidence Interval for the population mean difference μ_D.

5 3

$\bar{d} = -0.188$ 3 $\quad \frac{s_d}{\sqrt{n}} = \frac{0.292}{\sqrt{24}} = 0.060 \quad t_{.025,24-1} = 2.069$

$-0.188 \pm 2.069(0.060) \equiv -0.188 \pm 0.123 \equiv (-.311, -.065)$ 3

95% CI for μ_D $(-.311, -.065)$ 3

p.6.b. Based on your confidence interval, would the P-value for H_0: $\mu_D = 0$ vs H_A: $\mu_D \neq 0$ be: **> 0.05** or **< 0.05** ?

p.6.c. Conduct the large-sample Wilcoxon Signed-Rank Test of H_0: $M_1 = M_2$ vs H_A: $M_1 \neq M_2$. $T^+ = 35$, $n^* = 21$ pairs with non-zero differences.

$E\{T\} = \frac{n^*(n^*+1)}{4} = \frac{21(22)}{4} = 115.5$ 4

$\sigma_T = \sqrt{\frac{21(21+1)(2(21)+1)}{24}} = \sqrt{\frac{19866}{24}} \quad \sqrt{827.75} = 28.77$ 4

$z_{obs} = \frac{35-115.5}{28.77} = -2.798$ 3

3 3

Test Statistic $z_{obs} = -2.798$ Rejection Region $|z_{obs}| \geq 1.96$ P-value is **> 0.05** or **< 0.05**

p.6.d. What is T^- for this experiment? $\cancel{T^+}$

$T^+ + T^- = \frac{21(22)}{2} = 231 \Rightarrow T^- = 231-35 = 196$

4

38

Q.7. An engineer wishes to compare 2 brands of propane gas tank firms in terms of mean hours of cooking per tank at a fixed temperature. She believes the standard deviation for each brand is 25 hours based on a pilot study. She would like to estimate the population mean difference $\mu_1 - \mu_2$ within 4 hours with 95% confidence. How many tanks should she sample from each firm?

$\sigma = 25 \quad E = 4 \quad z_{\alpha/2} = 1.96$ 3

$$n = \frac{2 z_{\alpha/2}^2 \sigma^2}{E^2} = \frac{2(1.96^2)(625)}{16} = 300.125 \approx 301$$ 8 2

$n_1 = n_2 =$ 301

Q.8. Anita Wlodarczyk is considered the all-time greatest women's hammer thrower. A random sample of n = 15 of her competitive throws yields a sample standard deviation of s = 3.055 meters. Assuming her distribution of throws is normally distributed, compute a 95% Confidence Interval for her "true" standard deviation of throws. Hint: Compute a CI for the variance, then compute the CI for the standard deviation from that.

$$\frac{(n-1)s^2}{\chi^2_u}, \frac{(n-1)s^2}{\chi^2_l}$$

$\chi^2_u = \chi^2_{.025, 15-1} = 26.119$ 3

$\chi^2_l = \chi^2_{.975, 15-1} = 5.629$ 3

$s^2 = 3.055^2 = 9.333$ 3

$(n-1)s^2 = 14(9.333) = 130.662$

5

95% CI for σ^2: $\left[\frac{130.662}{26.119} = 5.003, \frac{130.662}{5.629} = 23.212\right)$

95% Confidence Interval for σ: $\left[\sqrt{5.003} = 2.237, \sqrt{23.212} = 4.818\right]$

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