Probability and Probability Distributions Problems

Q.1. Among male birds of a species, 20% have a particular gene. Among females of the species, 10% have the gene. The males comprise 40% of all the birds of the species (thus, females comprise 60%).

p.1.a. What is the probability a randomly selected bird of this species has the gene?

p.1.b. What is the probability the bird is male, given it has the gene?

Q.2. You are going to a foreign nation to conduct your research. On a weather website you see that the average high temperature during the period you will be there has been historically 20 degrees Celsius. What is the average in degrees Fahrenheit? Hint: F=32+(9/5)C

Q.3. In a population of 100 watt light bulbs manufactured by a company, 80% (0.80 as a proportion) have lifetimes exceeding 800 hours. An inspector samples 10 bulbs at random. What is the probability that all 10 bulbs' lifetimes exceed 800 hours?

Q.4. Among students taking a standardized exam, scores are normally distributed with a mean of 550 and standard deviation 100. What proportion of the students score above 700?

Q.5. A sample of 5 animals of a particular species is selected at random from the population being managed in a wildlife refuge. If 5% (0.05) of the population have a particular trait, what is the probability that none of the 5 tested have the trait?

Q.6. Based on the following contingency table, complete the following parts:

	Concussion	No Concussion	Total
Male	40	25596	25636
Female	60	27107	27167
Total	100	52703	52803

p.6.a. Among Males, what is Probability of concussion?
p.6.b. Among Females, what is Probability of concussion?

Q.7. A quality engineer in a factory is interested in the proportion of all computer chips that her assembly line produces that meet a particular quality requirement. She selects a sample of 100 chips and finds that 85 pass the test. This means that π=0.85 is the proportion of all chips the assembly line produces that meet the requirement. **TRUE or FALSE**

Q.8. A sample of 6 animals of a particular species is selected at random from the population being managed in a wildlife refuge. If 15% (0.15) of the population have a particular trait, what is the probability that none of the 6 tested have the trait?

Q.9. A new simpler (LA) test for bird flu is compared with the existing gold standard (HI) test, with the following results. Assume the gold standard (HI) test is completely accurate:

	HI (+)	HI (-)	Total
LA (+)	664	2	666
LA (-)	84	80	164
Total	748	82	830

p.9.a. What is the probability a person with the bird flu tests positive on the LA test?
p.9.b. What is the probability a person without the bird flu tests negative on the LA test?

Q.10. A magazine publisher includes winning coupons for an advertised product in 1% ($\pi = 0.01$) of the October issues of the magazine. A national firm buys a random sample of n=500 of the issues (assume the total number of issues is many times > 500). Let Y be the number of winning coupons the firm receives. Give the expected value of Y, and the probability that they get 1 or fewer winning coupons.

p.10.a. $E(Y) =$ p.10.b. $P(Y \leq 1) =$

Q.11. Two reviewers (Rev1 and Rev2) are compared by their positive and negative reviews of 1000 movies:

	Rev2 (+)	Rev2(-)	Total
Rev1 (+)	400	200	600
Rev1 (-)	100	300	400
Total	500	500	1000

p.11.a. What is the probability both reviewers give the same review? ______________________

p.11.b. What is the probability Reviewer 2 was positive, given Reviewer 1 was negative? ___________________

Q.12. Heights of adult males (cm) are approximately normally distributed with $\mu_M = 167$ and $\sigma_M = 6$. Heights of adult females (cm) are approximately normally distributed with $\mu_F = 160$ and $\sigma_F = 5$.

p.12.a. What proportion of males are taller than 175 cm?

p.12.b. What is the 95th %-ile among female heights?

Q.13. In a population of people on a "Singles Cruise", 60% are females and 40% are males. Among the females, 20% are actually married (and cheating on their spouse), among males, 40% are married.

p.13.a. What is the probability a randomly selected "single" is actually married?

p.13.b. Given the randomly selected "single" is married, what is the probability that it is male?

Q.14 The probability of randomly selecting the correct response on a multiple choice question with five choices is 0.20 (assuming zero knowledge). Suppose an exam consists of 6 multiple choice questions, each with five choices.

p.14.a. How many correct responses would you expect a student to pick by randomly selecting answers?

p.14.b. What is the probability a student gets none of the questions correct?

Q.15 A bridge holds up to 25 cars. It is known the weight of individual cars is normally distributed with $\mu = 2100$ lbs. and $\sigma = 500$ lbs.

p.15.a. What is the sampling distribution of the sample mean weight for n=25 cars?

p.15.b. If the maximum load for which the bridge is designed is 80,000 lbs., what is the probability a full load of cars (n=25) will exceed the design limits?

Q.16. A diagnostic test has 95% sensitivity (the probability a person with the condition tests positive = 0.95) and 95% specificity (the probability a person without the condition tests negative = 0.95). In a population of people given the test, 1% of the people have the condition (probability a person has the condition = 0.01).

p.16.a. What proportion of the people will test positive?

p.16.b. Given a person has tested positive, what is the probability he/she has the condition?

Q.17. A study of the effect of parents' smoking habits on the smoking habits of students in a high school produced the following table of proportions.

Suppose a student is randomly selected from this population.

Parents	Student smokes	Student does not smoke	Total
Both parents smoke	0.07	0.26	0.33
One smokes	0.08	0.34	0.42
Neither smokes	0.03	0.22	0.25
Total	0.18	0.82	1.00

p.17.a. What is the probability the student smokes and at least one parent of the student smokes?

p.17.b. Given the student smokes, what is the probability neither parent smokes?

Q.18. The weight of tomato juice in mechanically filled cans varies from can to can according to a normal distribution with μ = 454 grams and σ = 8 grams. A sample of 25 cans is selected.

What is the distribution of the sample mean, $\overline{Y}$ **?**

Q.19. A school system employs teachers at salaries between \$18,000 and \$40,000. The teachers' union and the school board are negotiating the form of next year's increase. Circle the correct response to each of the following.

p.19.a. If every teacher is given a flat \$1,000 raise, what is the effect on each of the following measures of salaries?

Mean: (1) decreases by \$1,000
(2) increases by \$1,000
(3) remains the same

Standard Deviation: (1) decreases by \$1,000
(2) increases by \$1,000
(3) remains the same

p.19.b. If every teacher is given a 5% raise, the amount of the raise will vary from \$900 to \$2,000, depending on the teacher's current salary. What is the effect on each of the following descriptive measures of the salaries?

Mean: (1) decreases by 5%
(2) increases by 5%
(3) remains the same

Standard deviation; (1) decreases by 5%
(2) increases by 5%
(3) remains the same

Q.20. In a population of birds on a desert island, 40% are Red, 30% are Yellow, 20% are Black, and 10% are Green. Among the Red birds, 5% have a genetic trait, among the Yellows, 10% have the trait, among the Blacks, 20% have the trait, and among the Greens, 25% have the trait. Let T be the event that a bird has the trait. Complete the following table.

Color	P(Color)	P(T|Color)	P(T&Color)	P(Color|T)
Red				
Yellow				
Black				
Green				
Total		#N/A		

Q.21. A soccer player on the UF soccer team has a probability of scoring on a penalty kick against a particular goalie of $\pi = 0.80$. Suppose in the course of a game, she has 3 penalty kicks against the goalie.

p.21.a. What is the probability she scores on all 3 kicks (assuming independence)?

p.21.b. What is the probability she fails to score on all 3 kicks?

p.21.c. Suppose each day she attempts 64 penalty kicks, and observes $\hat{\pi}$, the sample proportion of successful attempts. The sampling distribution of $\hat{\pi}$ is approximately:

Shape ___________ Mean ____________________ Standard Error ___________________________

Q.22. In a large scale study of SAR levels in cellphone models for 3 brands (LG, Motorola, Nokia), one characteristic reported was whether the model had low SAR levels. There were 457 total models studied, with P(LG) = 0.25, P(Motorola) = 0.40, and P(Nokia) = .35. Among LG phones, the proportion with low SAR levels is 0.24, among Motorola, the proportion is 0.20, and among Nokia, the proportion is 0.28.

p.22.a. Compute the probability a randomly selected phone has a low SAR level.

p.22.b. Compute the probability of each brand, given the phone has a low SAR level.

Q.23. A movie preview is shown independently to n = 10 people. Each person is then offered the chance to purchase a ticket. The probability any individual person, after seeing the preview, will buy the ticket is 0.4.

p.23.a. What is the probability that exactly 4 of the 10 buy the ticket?

p.23.b. What is the mean and variance of the number of the 10 people to buy the ticket?

Q.24. Body Mass Indices (BMI) for National Hockey League (NHL) players are approximately normally distributed with a mean of 26.50 and standard deviation of 1.45.

p.24.a. What is the probability a randomly selected NHL player has a BMI below 25.0?

p.24.b. Between what 2 BMI levels do the middle 95% of all NHL players fall?

p.24.c. What is the sampling distribution of sample means of sample size = 25 from this population? Give the distribution symbolically and draw a graph of it.

Q.25. During the years 1960-2015 the mean and standard deviation for July high temperatures at Orlando International airport were $\mu_F = 91.4°F$ and $\sigma_F = 8.3°F$, respectively. You have friends arriving from a country that uses the Celsius system, where °C = 0.56F – 17.78. Report to your friends the mean and standard deviation in terms of degrees Celsius.

Q.26. Chicago food establishments are classified by 3 levels of Risk (High, Medium, and Low). The proportions (probabilities) are: P(High) = .65, P(Medium) = .22, P(Low) = .13. The probabilities of Failing inspection are .21 among High risk, .23 among Medium Risk, and .33 among Low Risk.

p.26.a. Compute the probability a randomly selected establishment Fails inspection.

p.26.b. Compute the probability of each risk type, given the establishment fails inspection..

P(High Risk | Fail) = __________ P(Medium Risk | Fail) = _________ P(Low Risk | Fail) = __________

Q.27. An examination is given with n = 5 multiple-choice questions, each with 4 choices, and 1 correct answer. A student arrives for the exam completely unprepared, and will randomly guess on each question.

p.27.a. What is the probability the student will get at least 1 correct.

p.27.b. What are the mean and standard deviation of the number correct answers if this exam was given many times to people randomly guessing answers?

Q.28. Body Mass Indices (BMI) for English Premier League (EPL) football players are approximately normally distributed with a mean of 23.00 and standard deviation of 1.70.

p.28.a. What is the probability a randomly selected EPL player has a BMI above 24.5?

p.28.b. Between what 2 BMI levels do the middle 50% of all EPL players fall?

p.28.c. What is the sampling distribution of sample means of sample size = 16 from this population? Give the distribution symbolically and draw a graph of it.

Q.29. Chicago food establishments are classified by 3 levels of Risk (High, Medium, and Low). The proportions (probabilities) are: P(High) = .65, P(Medium) = .22, P(Low) = .13. The probabilities of Failing inspection are .21 among High risk, .23 among Medium Risk, and .33 among Low Risk.

p.29.a. Compute the probability a randomly selected establishment Fails inspection.

p.29.b. Compute the probability of each risk type, given the establishment fails inspection..

P(High Risk | Fail) = __________ P(Medium Risk | Fail) = _________ P(Low Risk | Fail) = __________

Q.30. The June monthly rainfall totals (in inches) for a sample of 5 Orlando years were: 10, 5, 5, 6, 9.

Give the sample mean, median, and standard deviation of the monthly rainfall totals (show all work).

Mean = ______________ Median = ________________ Std. Deviation = _________________

Q.31. An examination is given with n = 5 multiple-choice questions, each with 4 choices, and 1 correct answer. A student arrives for the exam completely unprepared, and will randomly guess on each question.

p.31.a. What is the probability the student will get at least 1 correct.

p.31.b. What are the mean and standard deviation of the number correct answers if this exam was given many times to people randomly guessing answers?

Q.32. Elite female hammer thrower Anita Włodarczyk has a competitive mean distance thrown of 73.36 meters and standard deviation of 2.74 meters. Translate her mean and standard deviation to feet. (1 foot = 0.3048 meters).