Inference Concerning Variances

Q.1. A researcher samples 9 computer monitors from each of two manufacturers. The lifetime of each monitor is observed and recorded (in 1000s of hours). The following table gives the summary results:

Brand	Mean	SD
A	4.20	1
B	4.60	0.8

- o Test $H_0: \sigma_A^2 = \sigma_B^2$ versus $H_A: \sigma_A^2 \neq \sigma_B^2$ at the $\alpha = 0.05$ significance level.
  - ▪ Test Statistic: $F_{obs} = 1^2 / (0.8)^2 = 1.5625$
  - ▪ Rejection Region: $F_{obs} \leq F_{.975,8,8} = 1 / F_{.025,8,8} = 1/4.433 = 0.226$, $F_{obs} \geq F_{.025,8,8} = 4.433$
  - ▪ Conclusion (circle best answer):
    - • Conclude $\sigma_A^2 > \sigma_B^2$
    - • Conclude $\sigma_A^2 < \sigma_B^2$
   - • **Do not reject $H_0: \sigma_A^2 = \sigma_B^2$**

Q.2. A confidence interval for a population variance can include both positive and negative values. T/**F**

Q.3. A chemist is interested in comparing the variation of chemical purity for the 2 suppliers her lab purchases the chemical from. She takes random samples of 9 vials from each supplier, and observes sample standard deviations of 10.0 for supplier 1 and 7.0 for supplier 2.

p.3.a. Give the test statistic for testing $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_A: \sigma_1^2 \neq \sigma_2^2$ $F_{obs} = (10)^2 / 7^2 = 2.041$

p.3.b. Clearly Give the Rejection Region for her Test Statistic ($\alpha = 0.05$ significance level).

$F_{obs} \leq F_{.975,8,8} = 1 / F_{.025,8,8} = 4.433 = 0.226$, $F_{obs} \geq F_{.025,8,8} = 4.433$
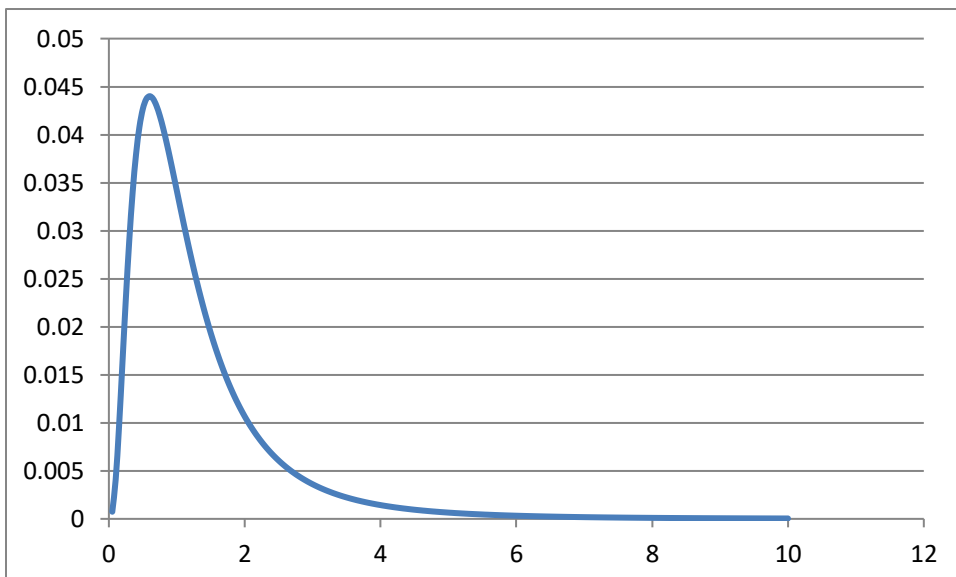
Q.4. A researcher samples 7 adult males from each of two species of squirrels and measures their body mass index . The following table gives the summary results:

Species	Mean	SD
A	7.20	2.4
B	5.70	3.2

- o Test H_0: $\sigma_A^2 = \sigma_B^2$ versus H_A: $\sigma_A^2 \neq \sigma_B^2$ at the $\alpha = 0.05$ significance level.
  - Test Statistic: **F_{obs} = (2.4)^2 / (3.2)^2 = 0.5625**
  - Rejection Region: **$F_{obs} \leq F_{.975,6,6}$ = 1 / $F_{.025,6,6}$ = 1/5.820 = 0.172, $F_{obs} \geq F_{.025,6,6}$ = 5.820**
  - Conclusion (circle best answer):
    - Conclude $\sigma_A^2 > \sigma_B^2$
    - Conclude $\sigma_A^2 < \sigma_B^2$
    - **Do not reject H_0: $\sigma_A^2 = \sigma_B^2$**

Q.5. A chemist is interested in the mean and variance of batches of a compound received from a supplier. A random sample of n=18 batches are obtained and the amount of the compound is measured from each batch. The sample mean and standard deviation are 23.8 ounces and 0.8 ounces, respectively. Give a 95% confidence interval for the standard deviation among all batches from this supplier.

$$n = 18 \quad s = 0.8 \quad df = 18-1=17 \quad \chi^2_{.975,17} = 7.564 \quad \chi^2_{.025,17} = 30.191$$

$$\text{95\% CI for } \sigma^2: \quad \left(\frac{(18-1)(0.8)^2}{30.191}, \frac{(18-1)(0.8)^2}{7.564} \right) \equiv (0.360, 1.438) \qquad \text{95\% CI for } \sigma: \quad (0.600, 1.199)$$

Q.6. Ultrasound measurements were made on mice of 2 Age Groups (Adult Mice, n_1=10 and Old Mice, n_2=9). The standard deviations of the pulse rates were S_1 = 70 and S_2 = 49, respectively. Test:

H_0: $\sigma_1^2 = \sigma_2^2$ vs H_A: $\sigma_1^2 \neq \sigma_2^2$

Test Statistic = **F_{obs} = 2.041**   Rejection Region: **$F_{obs} \leq 0.244$, $F_{obs} \geq 4.357$**   P-value >   < 0.05

Q.7. A sample of n = 93 lobsters at Devil's Hole was obtained with sample mean and standard deviation carapace lengths measured as 37.6 mm and 7.1 mm, respectively. Obtain a 95% Confidence Interval for the population standard deviation of lengths at this site. Hint: Obtain a 95% CI for the variance, then obtain square roots of the upper and lower bounds.

$$\text{95\% Confidence Interval for } \sigma: \quad \left(\sqrt{\frac{92(7.1)^2}{120.427}}, \sqrt{\frac{92(7.1)^2}{67.356}} \right) \equiv (6.206, 8.298)$$

Q.8. A machine that fills cans with coffee is working properly if the standard deviation of net weights is small (less than 0.5 ounces). The production manager will take random samples of filled cans and measure their net weights. If his test shows significant evidence that the variation is too large, he will stop the filling process for corrections.

i) $H_0 : \sigma \geq 0.5 \; H_A : \sigma < 0.5$ ii) $H_0 : \sigma \leq 0.5 \; H_A : \sigma > 0.5$ iii) $H_0 : \sigma = 0.5 \; H_A : \sigma \neq 0.5$

Answer: *ii*)

Q.9. Two types of measuring instruments may be used to determine a measurement that is critical in a laboratory analysis. Each instrument is used to make measurements of a known substance. Do the sample results indicate there is a difference in the precision (variability) of the measurements for the two instruments?

	Intrument1	Intrument2
n	8	8
Mean	8.2	8.1
SD	1.24	0.64

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad H_A : \sigma_1^2 \neq \sigma_2^2 \quad TS : F_{obs} = \frac{(1.24)^2}{(0.64)^2} = 3.754 \quad RR : F_{obs} \leq F_{.975,7,7} = \frac{1}{4.995} = 0.200, F_{obs} \geq F_{.025,7,7} = 4.995$$

Q.10. An textile engineer is interested whether the variation in breaking strength of Yarn Type A is larger than the variation in Type B. Random samples of $n_A = n_B = 7$ measurements were obtained, and the sample variances were $s_A^2 = 5.0$ and $s_B^2 = 1.25$. Do the data provide sufficient evidence to conclude $\sigma_A^2 > \sigma_B^2$? Test by using $\alpha = 0.05$.

   a) Test Statistic: **F_{obs} = 5.0 / 1.25 = 4.00**

   b) Rejection Region: **$F_{obs} \geq F_{.05,6,6}$ = 4.284**

   c) The P-value is **Larger** or Smaller than 0.05.

Q.11. An experiment is conducted to compare breaking strengths of 2 types of fibers. The means, standard deviations, and sample sizes of random samples from each fiber type are: $\bar{y}_{1\bullet} = 50 \quad s_1 = 12 \quad n_1 = 10 \quad \bar{y}_{2\bullet} = 45 \quad s_2 = 8 \quad n_2 = 10$

p.11.a. Test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_A: \sigma_1^2 \neq \sigma_2^2$ at α = 0.10 significance level

Test Statistic: **2.25**   Reject H_0 if Test Statistic **< 0.248** or **> 4.026**