<u>Randomized Block Design Problems</u>

Q.1. A study is conducted to compare 4 formulations of a new drug in terms of the availability of the drug in the bloodstream over time. Ten healthy subjects are selected and each subject receives each drug in random order in a randomized block design. The researcher conducts the appropriate F-test for testing for formulation differences. If the test is conducted at the α=0.05 significance level, he will conclude formulation differences exist if the F-statistic falls in what range?

$$df_{\text{Trt}} = 4-1=3 \quad df_{\text{Err}} = (10-1)(4-1)=27 \quad F_{.05,3,27} = 2.960$$

Q.2. A randomized block design is conducted to compare the output of three weaving looms (treatments) for a sample of 10 operators (blocks), where each operator's output is measured on each loom. The Mean Square Error from the ANOVA is MSE = 500. Compute Bonferroni's B, the minimum significant difference for concluding that two looms' population means differ if their sample means differ by at least B.

$$t=3 \quad b=10 \quad MSE=500 \quad df_{\text{Err}} = (3-1)(10-1)=18 \quad c=\frac{3(3-1)}{2}=3 \text{ comparisons}$$

$$t_{.025,c=3,df=18} = 2.639 \qquad B = 2.639\sqrt{\frac{2(500)}{10}} = 26.39$$

Q.3. An experiment was conducted to compare 3 mixtures of components in making rocket propellant. Eight samples of each mixture were obtained for testing. Eight investigators were sampled, and each investigator measured the propellant thrust (Note each investigator measured each mixture once, in random order).

- For this experiment the treatments are mixtures and the blocks are investigators. **True / False**
- Complete the following ANOVA table:

Source	df	SS	MS	F_obs	F(.05)
Mixture	2	235,000	117500	1175	3.739
Investigator	7	3,600	514.28571		
Error	14	1,400	100		
Total	23	240,000			

- Do you conclude that the mixture effects differ at the 0.05 significance level? **Yes** / No
- Compute the Minimum significant Difference for comparing mixture means based on Bonferroni's method

  $t_{.025,3,14}$ = 2.718 B = 13.59

- Compute the Relative Efficiency of having used this Randomized Complete Block (RCB) design as opposed to a Completely Randomized (CR) Design. That is, compute RE(RCB,CR)

$$RE(RCB,CR)=\frac{(8-1)514.3+8(3-1)100}{(8(3)-1)100}=\frac{3600+1600}{2300}=2.26$$

- How many replicates would you have needed per mixture to have the same standard error for the difference between two mixture means for a Completely Randomized (CR) Design?

2.26(8) $\approx$ 18 reps/treatment

Q.4. A college's volleyball coach is interested in whether her team's players prefer one brand of shoe among 3 brands. She has each player practice wearing each brand (in random order), and has each player rate the comfort on a 0-100 visual analogue scale. She analyzes the experiment as a Randomized Block Design with *t* = 3 treatments (shoe brands) and *b* = 18 (team players). She obtains the following results from the Analysis of Variance:

$$\overline{y}_{1\bullet}=65 \quad \overline{y}_{2\bullet}=80 \quad \overline{y}_{3\bullet}=55 \quad MSE=225$$

p.4.a. Use Bonferroni's method to obtain simultaneous 95% confidence intervals for the population mean ratings differences among all pairs of brands.

$$df_{\text{Err}}=2(17)=34 \quad c=3 \quad t_{.025,3,34}=2.518 \quad B=2.518\sqrt{\frac{2(225)}{18}}=12.59$$

$$\mu_1-\mu_2:(65-80)\pm 12.6\equiv(-27.6,-2.4) \quad \mu_1-\mu_3:(-2.6,22.6) \quad \mu_2-\mu_3:(12.4,37.6)$$

p.4.b. Show your results by drawing lines that join means that are not significantly different. **3 1 2**

Q.5. An experiment is conducted as a Randomized Block Design, and you obtain the following partial Analysis of Variance Table.

Source	DF	SS	MS	F	F(.05)	Conclude Effects???
Treatments	5	1500	300	5	2.534	Yes
Blocks	6	3600	600	#N/A	#N/A	#N/A
Error	30	1800	60	#N/A	#N/A	#N/A
Total	41	6900	#N/A	#N/A	#N/A	#N/A

p.5.a. Complete the table.

p.5.b. Number of treatments = **6** p.5.c. Number of blocks = **7** p.5.d. Number of observations = **42**

Q.6. A researcher reports the Relative Efficiency of a Randomized Block Design, relative to a Completely Randomized Design of 5. The Randomized Block Design had 5 treatments and 8 blocks. How many observations would be needed to have as precise of estimates of treatment means, if the experiment was conducted as to a Completely Randomized Design?

p.6.a. observations per treatment = **5(8) = 40** p.6.b. total number of observations = **5(40) = 200**

Q.7. Compute Bonferroni's Minimum Significant Difference (B_{ij}) for following experiment (with experimentwise error rate of 0.05): Randomized Block Design: t = 3 treatments, b = 11 blocks, MSE = 80

B = 9.97

Q.8. An experiment is conducted as a Randomized Block Design, with 10 blocks and 5 treatments, reports a relative efficiency (relative to a Completely Randomized Design) of RE = 5. How many TOTAL subjects would be needed if we ran a CRD, to obtain the same standard error of a difference between two treatment means?

i) 50 ii) 10 iii) 25 iv) **250** v) 500

Q.9. A study was conducted to determine how consistent forensic examiners ("treatments") are when determining the number of minutiae that appear on latent fingerprints ("blocks"). The study involved t = 10 examiners and b = 10 latent fingerprints. Each fingerprint was observed by each examiner. The means are given below, as well as some summary statistics.

Means	1	2	3	4	5	6	7	8	9	10
Fingerprint	20.6	13.4	20.1	9.8	10.7	8.4	12.1	15.6	7.1	9.1
Examiner	10.2	11.3	11.8	13.5	11.3	12.9	13.6	11.3	17.4	13.6

$$\overline{y}_{\bullet\bullet} = 12.7 \quad \sum_{i=1}^{10}\sum_{j=1}^{10}\left(y_{ij} - \overline{y}_{\bullet\bullet}\right)^2 = 3359$$

$$10\left[(20.6-12.7)^2 + \cdots + (9.1-12.7)^2\right] = 2016 \quad 10\left[(10.2-12.7)^2 + \cdots + (13.6-12.7)^2\right] = 373$$

p.9.a. Complete the following ANOVA table.

Source	df	SS	MS	F
Examiner (T)	9	373	41.44444	3.460825
Fingerprint(B)	9	2016	224	
Error	81	970	11.97531	
Total	99	3359		

p.9.b. Compute Bonferroni's Minimum significant difference for comparing all pairs of examiners.

$$c = \frac{10(10-1)}{2} = 45 \quad t_{.025,45,81} = 3.382 \quad B = 3.382\sqrt{\frac{2(11.98)}{10}} = 5.24$$

Q.10. A Randomized Block Design was conducted where 7 food items (treatments) by 103 evaluators (blocks) to determine whether there are any differences in true mean "liking scores" among the 7 food items. The model fit and null hypothesis are:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \quad i = 1,...,7;\ j = 1,...,103 \qquad H_0 : \alpha_1 = ... = \alpha_7 = 0$$

P10.a. Complete the following ANOVA table:

Source	df	SS	MS	F_obs	F(0.05)
Treatments	6	459	76.51429	19.918	2.113
Blocks	102	23136	226.8233	#N/A	#N/A
Error	612	2351	3.841464	#N/A	#N/A
Total	720	25946	#N/A	#N/A	#N/A

p.10.b. Compute and interpret the relative efficiency of the Randomized Block to the Completely Randomized Design:

Relative Efficiency : **9.23** # Raters per treatment needed for CRD: **9.23(103) ≈ 950**

p.10.c. Compute Bonferroni's minimum significant difference for comparing all pairs to food items.

$$c = \frac{7(7-1)}{2} = 21 \quad t_{.025,21,612} = 3.051 \quad B = 3.051\sqrt{\frac{2(3.84)}{103}} = 0.833$$

Q.11. A consumer is interested in comparing prices among 3 grocery store chains in his city. He randomly samples 8 branded products and obtains the regular price of each product at each store. He finds that between store variation accounts for 20% of the total variation and between product variation accounts for 60% of the total variation.

(a) Complete the following ANOVA table.

Source	DF	SS	MS	F_{obs}
Stores	**2**	**120**	**60**	**7.00**
Products	**7**	**360**	**51.43**	
Error	**14**	**120**	**8.57**	
Total	24-1=23	600		

(b) Test whether we can conclude that the population mean prices differ among the three stores by completing the following sentence (use α=0.05 significance level and circle any correct bold type options and fill appropriate numeric values in the blanks).

Conclude / Do Not conclude means differ since **7.00** is **above /** below **$F_{.05,2,14}$ = 3.739**

(c) This is an example of dependent samples because (circle the best answer):

i. The sample sizes for each store are the same

ii. The same products are observed at each store

iii. There are 3 stores

Q.12. A Randomized Block Design is used to compare 3 types of sounds on sleep in 9 people suffering from sleeping disorders. Each subject is assigned to each sound type (in random order) and the time until the person falls asleep is measured The following calculations are obtained from a statistical software program.

$$\overline{y}_{1\bullet} = 40 \quad \overline{y}_{2\bullet} = 60 \quad \overline{y}_{3\bullet} = 50 \quad MSE = 112.5$$

Use Bonferroni's method to obtain simultaneous 95% confidence intervals for all pairs of population means. Give the correct conclusion based on the intervals (NSD="Not significantly different).

$$t_{.025,3,16} = 2.673 \quad B = 2.673\sqrt{\frac{2(112.5)}{9}} = 13.4$$

Sounds	Point Estimate	Confidence Interval	Conclude
1 vs 2	**-20**	**(-33.4 , -6.6)**	$\mu_1 < \mu_2$
1 vs 3	**-10**	**(-23.4 , 3.4)**	**NSD**
2 vs 3	**10**	**(-3.4 , 23.4)**	**NSD**

Q.13. A Randomized Block Design (when it is feasible to be conducted) is generally preferred to a Completely Randomized Design because it typically increases the experimental error variance for a given sample size.

TRUE / **FALSE**

Q.14. An ergonomic study was conducted to compare comfort ratings among 3 chair models (treatments). The study involved 12 subjects (blocks) who each rated each chair (in random order) in a Randomized Block Design (RBD).

p.14.a. Complete the Analysis of Variance table.

Source	df	SS	MS	F	F(0.05)
ChairType	2	210.91	105.455	24.32896	3.443
Subject	11	368.10	33.46364	#N/A	#N/A
Error	22	95.36	4.334545	#N/A	#N/A
Total	35	674.37	#N/A	#N/A	#N/A

p.14.b. Compute the Relative Efficiency of the RBD versus a Completely Randomized Design (CRD). How many subjects per treatment would be needed to have equivalent standard errors of chair type means?

Relative Efficiency = **3.11** # of Subjects per treatment in CRD **= 3.11(12)** ≈ **38**

p.14.c. Compute Tukey's Honest Significant difference (W) for comparing treatment means.

$$q_{.05,3,22} = 3.553 \quad HSD = 3.553\sqrt{\frac{4.335}{12}} = 2.135$$

Q.15. A Randomized Complete Block Design was conducted to compare 3 energy drinks in terms of endurance in 6 subjects (each subject drinks each energy drink). The response is time to exhaustion on a treadmill. Due to some outlying observations, use Friedman's test to determine whether the 3 energy drinks differ in their effects on endurance.

Subject	Drink1	Drink2	Drink3
1	42 (1)	48 (2)	62 (3)
2	36 (2)	34 (1)	48 (3)
3	54 (1)	56 (2)	75 (3)
4	44 (1)	46 (2)	52 (3)
5	28 (1)	32 (2)	44 (3)
6	45 (1)	50 (2)	65 (3)

$$T_1 = 1+2+1+1+1+1 = 7 \quad T_2 = 2+1+2+2+2+2 = 11 \quad T_3 = 6(3) = 18$$

p.15.a. Test Stat: $F_r = \frac{12}{6(3)(3+1)}\left[(7)^2 + (11)^2 + (18)^2\right] - 3(6)(3+1) = \frac{12(494)}{72} - 72 = 82.333 - 72 = 10.333$

p.15.b. Rejection Region: $F_r \geq \chi^2_{.05,3-1} = 5.991$