Chapter 9 Solutions

9.1 $\rho = \begin{bmatrix} 1.0 & 0.63 & 0.45 \\ 0.63 & 1.0 & 0.35 \\ 0.45 & 0.35 & 1.0 \end{bmatrix}$ $p=3$, $m=1$

$L = \begin{bmatrix} .9 \\ .7 \\ .5 \end{bmatrix}$ $F = [F_1]$ $\Psi = \begin{bmatrix} .19 & 0 & 0 \\ 0 & .51 & 0 \\ 0 & 0 & .75 \end{bmatrix}$

$LL' = \begin{bmatrix} .81 & .63 & .45 \\ .63 & .49 & .35 \\ .45 & .35 & .25 \end{bmatrix}$ $LL' + \Psi = \rho$

9.2

a) $h_i^2 = \sum_{j=1}^{m} l_{ij}^2 \Rightarrow h_1^2 = .9^2 = .81$ $h_2^2 = .49$ $h_3^2 = .25$

81% of variance of Z_1 is contributed by the Factor

49% " " " Z_2 " " " " "

25% " " " Z_3 " " " " "

b) $Cov\{Z_i, F_1\} = l_{i1} = Corr\{Z_i, F_1\} = \begin{bmatrix} .9 \\ .7 \\ .5 \end{bmatrix}$

Z_1 (highest correlation w/ factor

9.3 $L = \sqrt{1.96} \begin{bmatrix} .625 \\ .593 \\ .507 \end{bmatrix} = \begin{bmatrix} .875 \\ .830 \\ .710 \end{bmatrix}$

a)

$\Psi_1 = 1 - .875^2 = .234$ $\Psi_2 = 1 - .830^2 = .311$

$\Psi_3 = 1 - .710^2 = .496$

9.3 b) $\frac{\lambda_1}{3} = \frac{1.96}{3}$

9.6 a) ① $(I+L'\Psi^{-1}L)(I+L'\Psi^{-1}L)^{-1}L'\Psi^{-1}L = L'\Psi^{-1}L$

② $(I+L'\Psi^{-1}L)\left[I-(I+L'\Psi^{-1}L)^{-1}\right] = I+L'\Psi^{-1}L - I = L'\Psi^{-1}L$

b) ① $(LL'+\Psi)^{-1}(LL'+\Psi) = I$

② $\left[\Psi^{-1} - \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}L'\Psi^{-1}\right](LL'+\Psi)$

$= \Psi^{-1}(LL'+\Psi) - \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}L'\Psi^{-1}(LL'+\Psi)$

$= \Psi^{-1}LL' + I - \Psi^{-1}L\left[I-(I+L'\Psi^{-1}L)^{-1}\right]L'$

$\quad - \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}L'$

$= \Psi^{-1}LL' + I - \Psi^{-1}LL' + \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}L'$

$\quad - \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}L' = I$

c) $(LL'+\Psi)^{-1}L = \left[\Psi^{-1} - \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}L'\Psi^{-1}\right]L$

$= \Psi^{-1}L - \Psi^{-1}L\left[I-(I+L'\Psi^{-1}L)^{-1}\right]$

$= \Psi^{-1}L(I+L'\Psi^{-1}L)^{-1}$

$\Rightarrow L'(LL'+\Psi)^{-1} = (I+L'\Psi^{-1}L)^{-1}L'\Psi^{-1}$

9.10

$$\hat{L}_z \hat{L}_z' = \begin{bmatrix} .602 & .200 \\ .467 & .154 \\ .926 & .143 \\ 1.000 & .000 \\ .874 & .476 \\ .894 & .327 \end{bmatrix} \begin{bmatrix} .602 & .467 & .926 & 1.000 & .874 & .894 \\ .200 & .154 & .143 & .000 & .476 & .327 \end{bmatrix}$$

diagonal elements

$$= \begin{bmatrix} .4024 \\ .2418 \\ .8779 \\ 1.0000 \\ .9905 \\ .9062 \end{bmatrix}$$

a) $\hat{\Psi}_z$ diagonal elements $= \begin{bmatrix} .5976 \\ .7582 \\ .1221 \\ 0 \\ .0095 \\ .0938 \end{bmatrix}$ $\begin{matrix} .598 \\ .758 \\ .122 \\ .000 \\ .010 \\ .094 \end{matrix}$

b) Communalities .4024, .2418, .8779, 1.0000, .9905, .9062

c) Proportions ; F_1 : $\dfrac{.602^2 + .467^2 + .926^2 + 1^2 + .874^2 + .894^2}{6} = \dfrac{4.00}{6} = .667$

F_2 : $\dfrac{.200^2 + .154^2 + .143^2 + 0^2 + .476^2 + .327^2}{6} = \dfrac{.418}{6} = .070$

d)

$$\hat{L}_z \hat{L}_z' = \begin{bmatrix} .4024 & .312 & .5860 & .6020 & .6210 & .604 \\ .312 & .242 & .454 & .467 & .481 & .468 \\ .586 & .454 & .878 & .926 & .877 & .875 \\ .602 & .467 & .926 & 1.000 & .874 & .894 \\ .621 & .481 & .877 & .874 & .991 & .937 \\ .604 & .468 & .875 & .894 & .937 & .906 \end{bmatrix}$$

9.10 d. Continued

$$R = \hat{L}_z L_z' - \hat{\Psi}_z = \begin{bmatrix} .000 & .193 & -.017 & .000 & .000 & -.001 \\ .193 & .000 & -.032 & .000 & .001 & -.018 \\ -.017 & -.032 & .000 & .000 & .000 & .003 \\ .000 & .000 & .000 & .000 & .000 & .000 \\ .000 & .001 & .000 & .000 & .000 & .000 \\ -.001 & -.018 & .003 & .000 & .000 & .000 \end{bmatrix}$$

9.12 $\hat{L} = \begin{bmatrix} .1022 \\ .0752 \\ .0765 \end{bmatrix} \Rightarrow \hat{L}\hat{L}' = \begin{bmatrix} .01044 & .00769 & .00782 \\ .00769 & .00566 & .00575 \\ .00782 & .00575 & .00585 \end{bmatrix}$

a) Specific Variances $\hat{\Psi}_1 = .01107 - .01044 = .00163$

$\hat{\Psi}_2 = .00642 - .00566 = .00076 \quad \hat{\Psi}_3 = .00677 - .00585 = .00092$

b) Communalities $h_1^2 = .01044 \quad h_2^2 = .00566 \quad h_3^2 = .00585$

c) Proportion variance explained by Factor 1: $\dfrac{.01044 + .00566 + .00585}{.01107 + .00642 + .00677}$

$= \dfrac{.02195}{.02426} = .905$

d) $S_1 = \dfrac{23}{24} S = \begin{bmatrix} .01061 & .00768 & .00782 \\ .00768 & .00615 & .00575 \\ .00782 & .00575 & .00649 \end{bmatrix}$

$S_1 - \hat{L}\hat{L}' - \hat{\Psi} = \begin{bmatrix} -.00146 & -.00001 & .00000 \\ -.00001 & -.00027 & .00000 \\ .00000 & .00000 & -.00028 \end{bmatrix}$

9.13 TS: $\left(n-1-(2p+4m+5)/6\right) \ln \dfrac{|\hat{L}\hat{L}'+\hat{\Psi}|}{|S_n|}$

$n=24 \quad p=3 \quad m=1 \Rightarrow$ multiplier $= 24-1-\frac{6+4+5}{6} = 20.5$

$$|\hat{L}\hat{L}'+\hat{\Psi}| = \begin{vmatrix} .01107 & .00769 & .00782 \\ .00769 & .00642 & .00575 \\ .00782 & .00575 & .00677 \end{vmatrix}$$

$$|S_n| = \begin{vmatrix} .01061 & .00768 & .00782 \\ .00768 & .00615 & .00575 \\ .00782 & .00575 & .00649 \end{vmatrix}$$

From excel: $\dfrac{|\hat{L}\hat{L}'+\hat{\Psi}|}{|S_n|} = 3.0771$

$TS = 20.5 \ln(3.0771) = 20.5(1.124) = 23.04$

$df = (p-m)^2 - p - m = (3-1)^2 - 3 - 1 = 4-3-1 = 0$

Cannot conduct test.