Chapter 8 Solutions

8.1 $\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \qquad |\Sigma - \lambda I| = 0$

$$\Rightarrow \left| \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = \begin{bmatrix} 5-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix} = (5-\lambda)(2-\lambda) - 4$$

$$= 10 - 5\lambda - 2\lambda + \lambda^2 - 4 = \lambda^2 - 7\lambda + 6 = 0$$

$$\Rightarrow \lambda = \frac{-(-7) \pm \sqrt{49 - 4(1)(6)}}{2(1)} = \frac{7 \pm 5}{2} = [6, 1]$$

$$\Sigma x_1 = \lambda_1 x_1 \Rightarrow \begin{bmatrix} 5x_{11} + 2x_{12} \\ 2x_{11} + 2x_{12} \end{bmatrix} = \begin{bmatrix} 6x_{11} \\ 6x_{12} \end{bmatrix}$$

$$\Rightarrow \begin{matrix} x_{11} = 2x_{12} \\ 4x_{11} = 2x_{12} \end{matrix} \Rightarrow \underset{\sim}{x}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \underset{\sim}{e}_1 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

$$\Sigma \underset{\sim}{x}_2 = \lambda_2 \underset{\sim}{x}_2 \Rightarrow \begin{bmatrix} 5x_{21} + 2x_{22} \\ 2x_{21} + 2x_{22} \end{bmatrix} = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}$$

$$\Rightarrow \begin{matrix} -4x_{21} = 2x_{22} \\ -x_{22} = 2x_{21} \end{matrix} \Rightarrow \underset{\sim}{x}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow e_2 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$$\underset{\sim}{e}_1' \underset{\sim}{e}_2 = -\frac{2}{5} + \frac{2}{5} = 0$$

$$\frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{6}{6+1} = \frac{6}{7} = .857$$

8.2 $\rho = \frac{2}{\sqrt{5(2)}} = .632 \qquad \rho = \begin{bmatrix} 1 & .632 \\ .632 & 1 \end{bmatrix}$

8.2a)

$$|\rho - \lambda I| = 0 \Rightarrow \left| \begin{bmatrix} 1 & .632 \\ .632 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 1-\lambda & .632 \\ .632 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - .632^2$$

$$= \lambda^2 - 2\lambda + 1 - 0.40 = \lambda^2 - 2\lambda + 0.6 = 0$$

$$\Rightarrow \lambda = \frac{-(-2) \pm \sqrt{4 - 4(1)(.6)}}{2(1)} = \frac{2 \pm \sqrt{1.6}}{2}$$

$$= \frac{2 \pm 1.265}{2} = 1.632, \ .368$$

$$\begin{bmatrix} 1 & .632 \\ .632 & 1 \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{12} \end{bmatrix} = 1.632 \begin{bmatrix} X_{11} \\ X_{12} \end{bmatrix} \Rightarrow \begin{matrix} .632X_{12} = .632X_{11} \\ .632X_{11} = .632X_{12} \end{matrix}$$

$$\Rightarrow X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow e_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\begin{pmatrix} 1 & .632 \\ .632 & 1 \end{pmatrix} \begin{pmatrix} X_{21} \\ X_{22} \end{pmatrix} = 0.368 \begin{bmatrix} X_{21} \\ X_{22} \end{bmatrix} \Rightarrow \begin{matrix} .632X_{22} = -.632X_{21} \\ .632X_{21} = -.632X_{22} \end{matrix}$$

$$\Rightarrow X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow e_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

Proportion for Y_1: $\frac{1.632}{2} = .816$

8.2b. Components are diffrent

8.2.c $\rho_{Y_1,Z_1} = e_{11}\sqrt{\lambda_1} = \frac{1}{\sqrt{2}}\sqrt{1.632} = .903$

$\rho_{Y_1,Z_2} = e_{12}\sqrt{\lambda_1} = \frac{1}{\sqrt{2}}\sqrt{1.632} = .903$

$\rho_{Y_2,Z_1} = e_{21}\sqrt{\lambda_2} = \frac{1}{\sqrt{2}}\sqrt{.368} = .429$

8.3 $\Sigma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$|\Sigma - \lambda I| = \begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 4-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{vmatrix} = (2-\lambda)(4-\lambda)(4-\lambda) = 0$

$\Rightarrow \lambda_1 = \lambda_2 = 4 \quad \lambda_3 = 2$

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = 4\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \Rightarrow \begin{bmatrix} 2x_{11} = 4x_{11} \\ 4x_{12} = 4x_{12} \\ 4x_{13} = 4x_{13} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_{11} = 0 \\ x_{12} = 1 \\ x_{13} = 0 \end{bmatrix}$ and $\begin{bmatrix} x_{21} = 0 \\ x_{22} = 0 \\ x_{23} = 1 \end{bmatrix}$ work for $\underline{e}_1, \underline{e}_2$

$\Rightarrow \begin{pmatrix} x_{31} = 1 \\ x_{32} = 0 \\ x_{33} = 0 \end{pmatrix}$ $\underline{e}_1, \underline{e}_2$ not unique

8.4 $$|\Sigma - \lambda I| = \left| \begin{bmatrix} \sigma^2 & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2 & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} \sigma^2-\lambda & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2-\lambda & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2-\lambda \end{vmatrix} = \left[(\sigma^2-\lambda)^3 + 0 + 0\right] - \left[0 + \sigma^4\rho^2(\sigma^2-\lambda)(2)\right]$$

$$= (\sigma^2-\lambda)^3 - 2(\sigma^2-\lambda)\sigma^4\rho^2$$

$$= (\sigma^2-\lambda)\left[(\sigma^2-\lambda)^2 - 2\sigma^4\rho^2\right]$$

$$-\tfrac{1}{\sqrt{2}} < \rho < \tfrac{1}{\sqrt{2}}$$
$$\Rightarrow 0 < \rho^2 < \tfrac{1}{2}$$
$$\Rightarrow 1-2\rho^2 > 0$$

$$= (\sigma^2-\lambda)\left[\sigma^4 - 2\sigma^2\lambda + \lambda^2 - 2\sigma^4\rho^2\right]$$

$$= (\sigma^2-\lambda)\left[\lambda^2 - 2\sigma^2\lambda - \sigma^4(1-2\rho^2)\right]$$

(1) $\lambda = \sigma^2$

(2) $$\frac{2\sigma^2 \pm \sqrt{4\sigma^4 - 4(1)\sigma^4(1-2\rho^2)}}{2(1)}$$

$$= \frac{2\sigma^2 \pm 2\sigma^2\sqrt{1-2\rho^2}}{2(1)} \equiv \sigma^2\left[1 \pm \sqrt{1-2\rho^2}\right]$$

Roots $\equiv \sigma^2\left[1+\sqrt{1-2\rho^2}\right],\ \sigma^2,\ \sigma^2\left[1-\sqrt{1-2\rho^2}\right]$

8.4 Continued

(1) $$\begin{bmatrix} \sigma^2 & \sigma^2\rho & 0 \\ \sigma^2\rho & \sigma^2 & \sigma^2\rho \\ 0 & \sigma^2\rho & \sigma^2 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \sigma^2\left[1+\sqrt{1-2\rho^2}\right]\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix}$$

$$\Rightarrow \quad \sigma^2 x_{11} + \sigma^2\rho x_{12} = \sigma^2\left[1+\sqrt{1-2\rho^2}\right]x_{11}$$

$$\Rightarrow \quad \rho x_{12} = \sqrt{1-2\rho^2}\, x_{11} \Rightarrow x_{11} = \frac{\rho}{\sqrt{1-2\rho^2}}\, x_{12}$$

$$\sigma^2\rho x_{12} + \sigma^2 x_{13} = \sigma^2\left[1+\sqrt{1-2\rho^2}\right]x_{13}$$

$$\Rightarrow \quad \rho x_{12} = \sqrt{1-2\rho^2}\, x_{13} \Rightarrow x_{13} = \frac{\rho}{\sqrt{1-2\rho^2}}\, x_{12}$$

$$\underset{\sim}{x}_1 = \begin{bmatrix} \frac{\rho}{\sqrt{1-2\rho^2}} \\ 1 \\ \frac{\rho}{\sqrt{1-2\rho^2}} \end{bmatrix} \qquad x_1'x_1 = \frac{\rho^2}{1-2\rho^2} + 1 + \frac{\rho^2}{1-2\rho^2}$$

$$= \frac{2\rho^2 + (1-2\rho^2)}{1-2\rho^2} = \frac{1}{1-2\rho^2}$$

$$\Rightarrow \underset{\sim}{e}_1 = \sqrt{1-2\rho^2}\, \underset{\sim}{x}_1 = \begin{bmatrix} \rho \\ \sqrt{1-2\rho^2} \\ \rho \end{bmatrix}$$

(2) $$\sigma^2 x_{21} + \sigma^2\rho x_{22} = \sigma^2 x_{21} \quad \Rightarrow x_{22} = 0$$

$$\sigma^2\rho x_{22} + \sigma^2 x_{23} = \sigma^2 x_{23} \quad \cancel{\Rightarrow x_{23} = 0}$$

$$\underset{\sim}{e}_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$$

$$\sigma^2\rho x_{21} + \sigma^2 x_{22} + \sigma^2\rho x_{23} = \sigma^2 x_{22}$$

$$\Rightarrow \sigma^2\rho x_{21} + \sigma^2\rho x_{23} = 0 \Rightarrow x_{21} = -x_{23}$$

8.4 Continued

$$\sigma^2 x_{31} + \sigma^2\rho x_{32} = \sigma^2\left[1-\sqrt{1-2\rho^2}\right] x_{31}$$

$$\sigma^2\rho x_{32} + \sigma^2 x_{33} = \sigma^2\left[1-\sqrt{1-2\rho^2}\right] x_{33}$$

$$\Rightarrow \quad x_{31} = \frac{-\rho}{\sqrt{1-2\rho^2}} x_{32} \qquad x_{33} = \frac{-\rho}{\sqrt{1-2\rho^2}} x_{32}$$

$$\underset{\sim}{e}_3 = \begin{bmatrix} -\rho \\ \sqrt{1-2\rho^2} \\ -\rho \end{bmatrix}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = \sigma^2\left[1 + \sqrt{1-2\rho^2} + 1 + 1 - \sqrt{1-2\rho^2}\right] = 3\sigma^2$$

Proportion by 1^{st} Princomp: $\dfrac{\sigma^2\left[1+\sqrt{1-2\rho^2}\right]}{3\sigma^2} = \dfrac{1+\sqrt{1-2\rho^2}}{3}$

2^{nd} ~~P~~Princomp: $\dfrac{\sigma^2}{3\sigma^2} = \dfrac{1}{3}$

3^{rd} Princomp: $\dfrac{1-\sqrt{1-2\rho^2}}{3}$

8.6 $|S - \lambda I| = 0 \Rightarrow$

a) $(7476.45 - \lambda)(26.19 - \lambda) - 303.62^2$

$= 195808.2255 - 7502.64\lambda + \lambda^2 - 92185.1044$

$= \lambda^2 - 7502.64\lambda + 103623.1211 = 0$

$$\Rightarrow \lambda = \frac{7502.64 \pm \sqrt{7502.64^2 - 4(1)(103623.1211)}}{2(1)}$$

$$= \frac{7502.64 \pm 7474.97}{2} = \underbrace{7488.81}_{=\lambda_1}, \underbrace{13.84}_{=\lambda_2}$$

$$\begin{bmatrix} 7476.45 & 303.62 \\ 303.62 & 26.19 \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{12} \end{bmatrix} = \begin{matrix} 7488.81 X_{11} \\ 7488.81 X_{12} \end{matrix}$$

$\Rightarrow 303.62 X_{12} = 12.36 X_{11} \Rightarrow X_{11} = 24.56 X_{12}$

$\cancel{303.62 X_{11} \quad 26.19 \Rightarrow}$

$\underset{\sim}{X}_1 = \begin{bmatrix} 24.56 \\ 1 \end{bmatrix} \qquad \underset{\sim}{X}_1' \underset{\sim}{X}_1 = 604.19$

$\sqrt{\underset{\sim}{X}_1' \underset{\sim}{X}_1} = 24.58$

$\Rightarrow \underset{\sim}{e}_1 = \begin{bmatrix} .9992 \\ .0407 \end{bmatrix} \qquad \underset{\sim}{e}_2 = \begin{bmatrix} -.0407 \\ .9992 \end{bmatrix}$

b) $\frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{7488.81}{7502.65} = .9982$

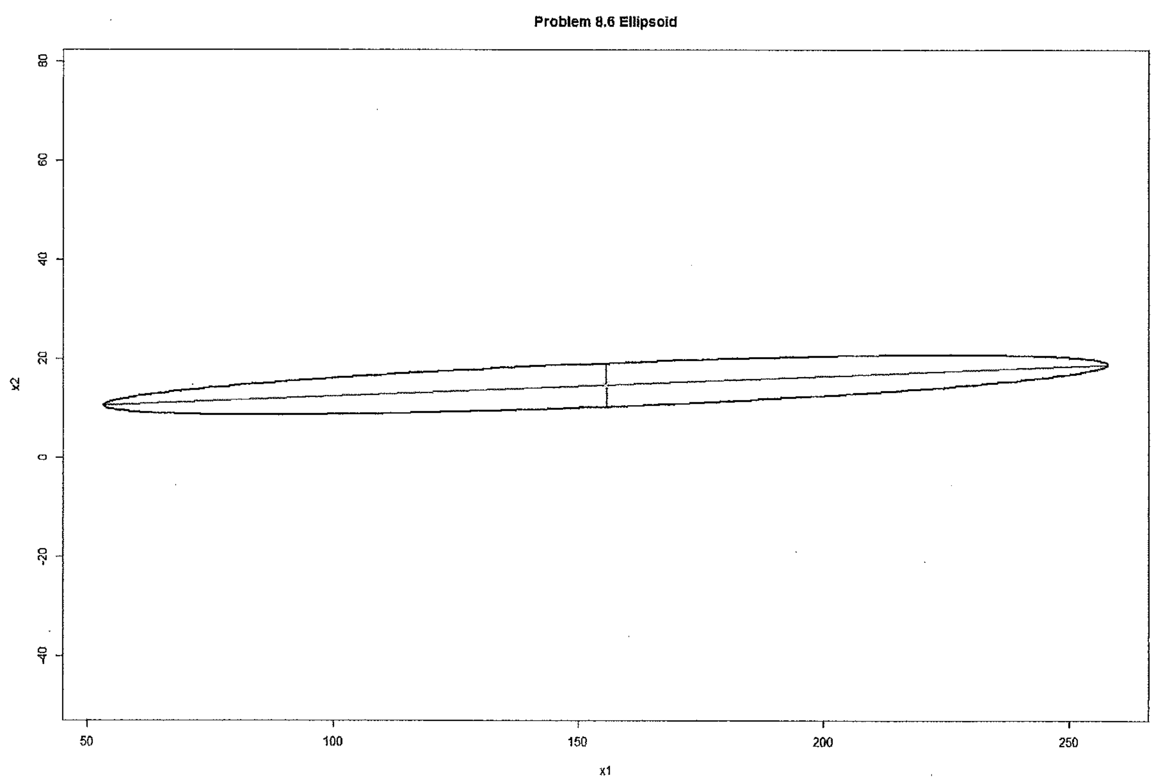
8.8

Problem 8.6.

R Program for Ellipsoid

```
crit.dist <- sqrt(1.4)
A <- matrix(c(7476.45, 303.62, 303.62, 26.19),ncol=2)
# mu.test <- c(155.60,14.70)

ctr <- c(155.60,14.70)
angles <- seq(0, 2*pi, length.out=200)


eigVal  <- eigen(A)$values
eigVec  <- eigen(A)$vectors
eigScl  <- eigVec %*% diag(sqrt(eigVal))  # scale eigenvectors to length = square-root
xMat    <- rbind(ctr[1] + eigScl[1, ]*crit.dist, ctr[1] - eigScl[1, ]*crit.dist)
yMat    <- rbind(ctr[2] + eigScl[2, ]*crit.dist, ctr[2] - eigScl[2, ]*crit.dist)
ellBase <- cbind(sqrt(eigVal[1])*crit.dist*cos(angles),
sqrt(eigVal[2])*crit.dist*sin(angles)) # normal ellipse
ellRot  <- eigVec %*% t(ellBase)                                            # rotated
ellipse

plot((ellRot+ctr)[1, ], (ellRot+ctr)[2, ], asp=1, type="l", lwd=2,
main="Problem 8.6 Ellipsoid",
xlab="x1", ylab="x2")
matlines(xMat, yMat, lty=1, lwd=2, col="blue")
points(ctr[1], ctr[2], pch=4, col="orange", lwd=3)

eigVal
eigVec
```

Graph



8.7

a) $R = \begin{bmatrix} 1 & 0.686 \\ 0.686 & 1 \end{bmatrix}$ $\quad r_{12} = \dfrac{303.62}{\sqrt{7476.45(26.19)}}$

$$(1-\lambda)^2 - .686^2 = \lambda^2 - 2\lambda + 1 - .471 = \lambda^2 - 2\lambda + .529 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4(1)(.529)}}{2(1)} = \frac{2 \pm 1.373}{2} = (1.686, 0.314)$$

$$\begin{bmatrix} 1 & .686 \\ .686 & 1 \end{bmatrix}\begin{bmatrix} \cancel{x}_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} 1.686\,\cancel{x}_{11} \\ 1.686\,\cancel{x}_{12} \end{bmatrix} \Rightarrow .686\,\cancel{x}_{12} = .686\,\cancel{x}_{11}$$

$$\Rightarrow \cancel{x}_{12} = x_{11} \Rightarrow \underset{\sim}{e}_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & .686 \\ .686 & 1 \end{bmatrix}\begin{bmatrix} \cancel{x}_{21} \\ \cancel{x}_{22} \end{bmatrix} = \begin{bmatrix} .314\,\cancel{x}_{21} \\ .314\,\cancel{x}_{22} \end{bmatrix} \Rightarrow .686\,\cancel{x}_{22} = -.686\,\cancel{x}_{21}$$

$$\Rightarrow \cancel{x}_{21} = -x_{22} \Rightarrow \underline{e}_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\hat{y}_1 = \frac{1}{\sqrt{2}}\cancel{z}_1 + \frac{1}{\sqrt{2}}\cancel{z}_2 \qquad V\{\hat{y}_1\} = \lambda_1 = 1.686$$

$$\hat{y}_2 = -\frac{1}{\sqrt{2}}\cancel{z}_1 + \frac{1}{\sqrt{2}}\cancel{z}_2 \qquad V\{\hat{y}_2\} = \lambda_2 = .314$$

b) $\dfrac{\lambda_1}{\lambda_1 + \lambda_2} = \dfrac{1.686}{2} = .843$

c) $r_{\hat{y}_1, z_1} = \hat{e}_{11}\sqrt{\hat{\lambda}_1} = \dfrac{1}{\sqrt{2}}\sqrt{1.686} = .918$ $\qquad r_{\hat{y}_i, z_k} = \hat{e}_{ik}\sqrt{\hat{\lambda}_k}$

$r_{\hat{y}_1, z_2} = \hat{e}_{12}\sqrt{\hat{\lambda}_1} = \dfrac{1}{\sqrt{2}}\sqrt{1.686} = .918$

$\hat{y}_1 \equiv$ simple average of z_1, z_2

8.7 d) Due to diffent scales (sales much more variable than profits), should probably use correlation matrix. (See top of p. 439).

$V\{X_1\}$ is 285 times larger than $V\{X_2\}$

8.8 Example 8.5

a) $\hat{e}_1' = [.469, .532, .465, .387, .361]$ $\hat{\lambda}_1 = 2.437$
$\hat{e}_2' = [-.368, -.236, -.315, .585, .606]$ $\hat{\lambda}_2 = 1.407$

$$r_{\hat{y}_i, z_k} = e_{ik}\sqrt{\hat{\lambda}_i}$$

k

i	1	2	3	4	5
1	.732	.830	.726	.604	.564
2	-.437	-.280	-.374	.694	.719

Monotonically related. Virtually no difference.

b) $$R = \begin{bmatrix} 1.000 & .632 & .511 & .115 & .155 \\ .632 & 1.000 & .574 & .322 & .213 \\ .511 & .574 & 1.000 & .183 & .146 \\ .115 & .322 & .183 & 1.000 & .683 \\ .155 & .213 & .146 & .683 & 1.000 \end{bmatrix}$$

$\bar{r}_1 = \frac{1}{4}[.632 + .511 + .115 + .155] = .353$
$\bar{r}_2 = \frac{1}{4}[.632 + .574 + .322 + .213] = .435$
$\bar{r}_3 = \frac{1}{4}[.511 + .574 + .183 + .146] = .354$

8.8b Continued

$$\bar{r}_4 = \frac{1}{4}[.115 + .322 + .183 + .683] = .326$$

$$\bar{r}_5 = \frac{1}{4}[.155 + .213 + .146 + .683] = .299$$

$$\bar{r} = \frac{2}{5(4)}[.632 + .511 + .115 + .155 + .574 + .322 + .213 + .183 + .146 + .683]$$

$$= \frac{1}{10}\{3.534\} = .353$$

$$\hat{\gamma} = \frac{(5-1)^2[1-(1-.353)^2]}{5-(5-2)(1-.353)^2} = \frac{9.302}{3.744} = 2.485$$

$$\sum\sum_{i<k}(r_{ik}-\bar{r})^2 = (.632-.353)^2 + \cdots + (.683-.353)^2$$

$$= \sum\sum_{i<k} r_{ik}^2 - 10\bar{r}^2 = 1.698 - 10(.353)^2 = 1.698 - 1.246$$

$$= 0.452$$

$$\sum_{k=1}^{p}(\bar{r}_k-\bar{r})^2 = (.353-.353)^2 + (.435-.353)^2 + (.354-.353)^2$$

$$+ (.326-.353)^2 + (.299-.353)^2 = 0 + .0067 + .0000$$

$$+ .0007 + .0029 = .0103$$

$$\hat{\gamma}\sum_k(\bar{r}_k-\bar{r})^2 = 2.485(.0103) = .0256$$

$n = 103$

$$T = \frac{103-1}{(1-.353)^2}[0.452 - .0256] = 243.664[.4264] = 103.9$$

$(p+1)(p-2)/2 = 6(4)/2 = 12$ $\quad \chi^2_{12}(.05) = 21.0$ Reject Ho.

Problem 8.10

R Program

```
prob8.10 <- read.table("http://www.stat.ufl.edu/~winner/sta4702/data/wichern/T8-4.DAT",
     header=F,col.names=c("JPM","CB","WF","RDS","EM"))
attach(prob8.10)

X <- cbind(JPM,CB,WF,RDS,EM)
n <- nrow(X); p <- ncol(X)

I_n <- diag(n)
J_n <- matrix(rep(1,n^2),ncol=n)

(xbar <- (1/n) * (t(X) %*% rep(1,n)))
(S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n)*J_n) %*% X))

cov(X)

(lambda <- eigen(S)$val)
(E <- eigen(S)$vec)
lambda <- as.matrix(lambda,ncol=1)
(prop1 <- lambda[1]/sum(lambda))
(prop2 <- lambda[2]/sum(lambda))
(prop3 <- lambda[3]/sum(lambda))
prop1+prop2+prop3

lambda.lo <- lambda[1:3,]/(1+qnorm(1-.10/(2*3))*sqrt(2/n))
lambda.hi <- lambda[1:3,]/(1-qnorm(1-.10/(2*3))*sqrt(2/n))

lambda.out <- cbind(lambda.lo, lambda[1:3,], lambda.hi)
colnames(lambda.out) <- c("Lower Bound", "Estimate", "Upper Bound")

round(lambda.out, 6)
```

R Output

```
> (xbar <- (1/n) * (t(X) %*% rep(1,n)))
            [,1]
JPM 0.0010627806
CB  0.0006554204
WF  0.0016260816
RDS 0.0040491252
EM  0.0040386417
> (S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n)*J_n) %*% X))
             JPM           CB           WF          RDS           EM
JPM 4.332695e-04 0.0002756679 1.590265e-04 6.411929e-05 8.896616e-05
CB  2.756679e-04 0.0004387172 1.799737e-04 1.814512e-04 1.232623e-04
WF  1.590265e-04 0.0001799737 2.239722e-04 7.341348e-05 6.054612e-05
RDS 6.411929e-05 0.0001814512 7.341348e-05 7.224964e-04 5.082772e-04
EM  8.896616e-05 0.0001232623 6.054612e-05 5.082772e-04 7.656742e-04
>
```

Continued

8.13

```
> cov(X)
             JPM           CB           WF          RDS           EM
JPM 4.332695e-04 0.0002756679 1.590265e-04 6.411929e-05 8.896616e-05
CB  2.756679e-04 0.0004387172 1.799737e-04 1.814512e-04 1.232623e-04
WF  1.590265e-04 0.0001799737 2.239722e-04 7.341348e-05 6.054612e-05
RDS 6.411929e-05 0.0001814512 7.341348e-05 7.224964e-04 5.082772e-04
EM  8.896616e-05 0.0001232623 6.054612e-05 5.082772e-04 7.656742e-04
>
> (lambda <- eigen(S)$val)
[1] 0.0013676780 0.0007011596 0.0002538024 0.0001426026 0.0001188868
> (E <- eigen(S)$vec)
          [,1]       [,2]        [,3]       [,4]        [,5]
[1,] 0.2228228 -0.6252260  0.32611218  0.6627590  0.11765952
[2,] 0.3072900 -0.5703900 -0.24959014 -0.4140935 -0.58860803
[3,] 0.1548103 -0.3445049 -0.03763929 -0.4970499  0.78030428
[4,] 0.6389680  0.2479475 -0.64249741  0.3088689  0.14845546
[5,] 0.6509044  0.3218478  0.64586064 -0.2163758 -0.09371777
> lambda <- as.matrix(lambda,ncol=1)
> (prop1 <- lambda[1]/sum(lambda))
[1] 0.5292607
> (prop2 <- lambda[2]/sum(lambda))
[1] 0.271333
> (prop3 <- lambda[3]/sum(lambda))
[1] 0.09821584
> prop1+prop2+prop3
[1] 0.8988095
>
> lambda.lo <- lambda[1:3,]/(1+qnorm(1-.10/(2*3))*sqrt(2/n))
> lambda.hi <- lambda[1:3,]/(1-qnorm(1-.10/(2*3))*sqrt(2/n))
>
> lambda.out <- cbind(lambda.lo, lambda[1:3,], lambda.hi)
> colnames(lambda.out) <- c("Lower Bound", "Estimate", "Upper Bound")
>
> round(lambda.out, 6)
     Lower Bound Estimate Upper Bound
[1,]    0.001055 0.001368    0.001944
[2,]    0.000541 0.000701    0.000997
[3,]    0.000196 0.000254    0.000361
```

Problem 8.11

R Program

```
prob8.11 <- read.table("http://www.stat.ufl.edu/~winner/sta4702/data/wichern/T8-5.DAT",
  header=F,col.names=c("totpop","prodeg","empo16","govemp","medhv"))
attach(prob8.11)

medhv <- medhv*10

X <- cbind(totpop,prodeg,empo16,govemp,medhv)
n <- nrow(X); p <- ncol(X)

I_n <- diag(n)
J_n <- matrix(rep(1,n^2),ncol=n)

(S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n)*J_n) %*% X))

(lambda <- eigen(S)$val)
(E <- eigen(S)$vec)

(prop1 <- lambda[1]/sum(lambda))
(prop2 <- lambda[2]/sum(lambda))
prop1+prop2

r.Y.X <- matrix(rep(0,p^2),ncol=p)


for (i in 1:p) {
  for (k in 1:p) {
     r.Y.X[k,i] <- -E[k,i]*sqrt(lambda[i])/sqrt(S[k,k])
  }
}
r.Y.X
```

R Output

```
> (S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n)*J_n) %*% X))
           totpop    prodeg      empo16     govemp      medhv
totpop  3.3968990 -1.102139   4.3055548  -2.078285  0.2720391
prodeg -1.1021394  9.672775  -1.5132363  10.953232 12.0306366
empo16  4.3055548 -1.513236  55.6259116 -28.937464 -0.4355907
govemp -2.0782852 10.953232 -28.9374642  89.066612  9.5729973
medhv   0.2720391 12.030637  -0.4355907   9.572997 31.8625082
>
> (lambda <- eigen(S)$val)
[1] 108.271939  43.139674  31.267127   4.598098   2.347868
> (E <- eigen(S)$vec)
            [,1]        [,2]        [,3]        [,4]        [,5]
[1,]  0.03762881 -0.06230915 -0.03997936 -0.55553173  0.82733777
[2,] -0.11892964 -0.24930105  0.26052476  0.76839232  0.51517455
[3,]  0.47967291 -0.75967654 -0.43064872  0.02807896 -0.08098582
[4,] -0.85891177 -0.31639989 -0.39364417 -0.06867379 -0.04989847
[5,] -0.12893518 -0.50670427  0.76818907 -0.30895506 -0.20262977
>
```

Continued

8.15

```
> (prop1 <- lambda[1]/sum(lambda))
[1] 0.5709801
> (prop2 <- lambda[2]/sum(lambda))
[1] 0.2275003
> prop1+prop2
[1] 0.7984804
>
> r.Y.X
           [,1]      [,2]       [,3]         [,4]         [,5]
[1,] -0.2124404 0.2220491  0.1212938  0.646333476 -0.687825655
[2,]  0.3978987 0.5264862 -0.4684006 -0.529781248 -0.253814076
[3,] -0.6692133 0.6690038  0.3228707 -0.008072938  0.016638235
[4,]  0.9469980 0.2202000  0.2332332  0.015603522  0.008101522
[5,]  0.2376782 0.5895939 -0.7609781  0.117366575  0.055004739
```

Problem 8.15

R Program/Output

```
R <- matrix(c(1,.7501,.6329,.6363, .7501,1,.6925,.7386,
  .6329,.6925,1,.6625, .6363,.7386,.6625,1), ncol=4)

sd.vec <- c(32.9909,33.5918,36.5534,37.3517)
V.half <- diag(sd.vec)

(S <- V.half %*% R %*% V.half)

(lambda <- eigen(S)$val)
(E <- eigen(S)$vec)



> (S <- V.half %*% R %*% V.half)
          [,1]      [,2]      [,3]      [,4]
[1,] 1088.3995  831.2786  763.2328  784.0910
[2,]  831.2786 1128.4090  850.3169  926.7294
[3,]  763.2328  850.3169 1336.1511  904.5322
[4,]  784.0910  926.7294  904.5322 1395.1495
>
> (lambda <- eigen(S)$val)
[1] 3779.0086  468.2480  452.1289  248.7236
> (E <- eigen(S)$vec)
           [,1]       [,2]        [,3]        [,4]
[1,] -0.4541429  0.2842829  0.67918134 -0.50164743
[2,] -0.4941262  0.2240730  0.18158053  0.82015797
[3,] -0.5123013 -0.8554764 -0.03422706 -0.06734959
[4,] -0.5358553  0.3703168 -0.71033072 -0.26674854
```

Problem 8.26

R Program

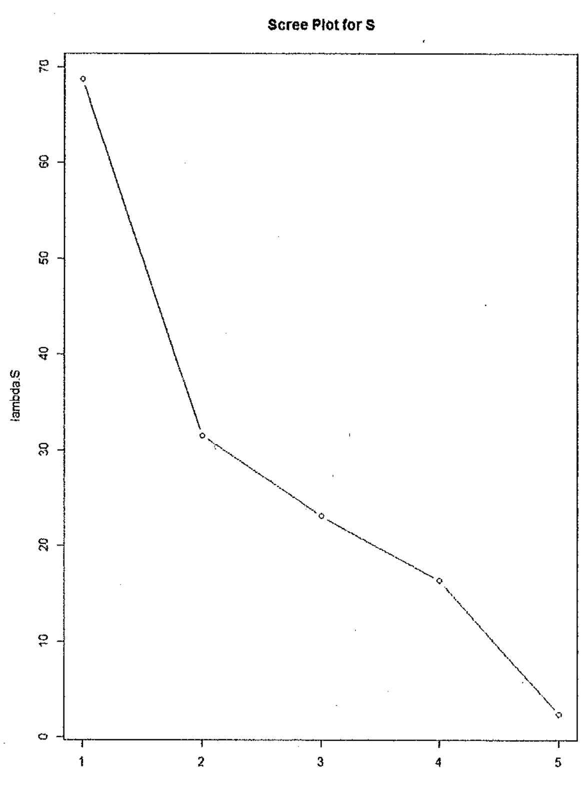
```
prob8.26 <- read.table("http://www.stat.ufl.edu/~winner/sta4702/data/wichern/T4-6.DAT",
     header=F,col.names=c("X1","X2","X3","X4","X5","X6","X7"))
attach(prob8.26)

X <- cbind(X1,X2,X3,X4,X5)
n <- nrow(X); p <- ncol(X)

I_n <- diag(n)
J_n <- matrix(rep(1,n^2),ncol=n)

(S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n)*J_n) %*% X))
V.12 <- diag(sqrt(diag(S)))
(R <- solve(V.12) %*% S %*% solve(V.12))

(lambda.S <- eigen(S)$val)
(E.S <- eigen(S)$vec)
(lambda.R <- eigen(R)$val)
(E.R <- eigen(R)$vec)

par(mfrow=c(1,2))
plot(lambda.S,type="b",main="Scree Plot for S")
plot(lambda.R,type="b",main="Scree Plot for R")

Y1.S <- X %*% E.S[,1];  Y2.S <- X %*% E.S[,2]
GS.group <- 3*X6+X7

plot(Y2.S,Y1.S,pch=GS.group,main="PC1 vs PC2 - Based on S")

lambda1.S.lo <- lambda.S[1]/(1+qnorm(.975)*sqrt(2/n))
lambda1.S.hi <- lambda.S[1]/(1-qnorm(.975)*sqrt(2/n))

cbind(lambda1.S.lo, lambda.S[1], lambda1.S.hi)
```

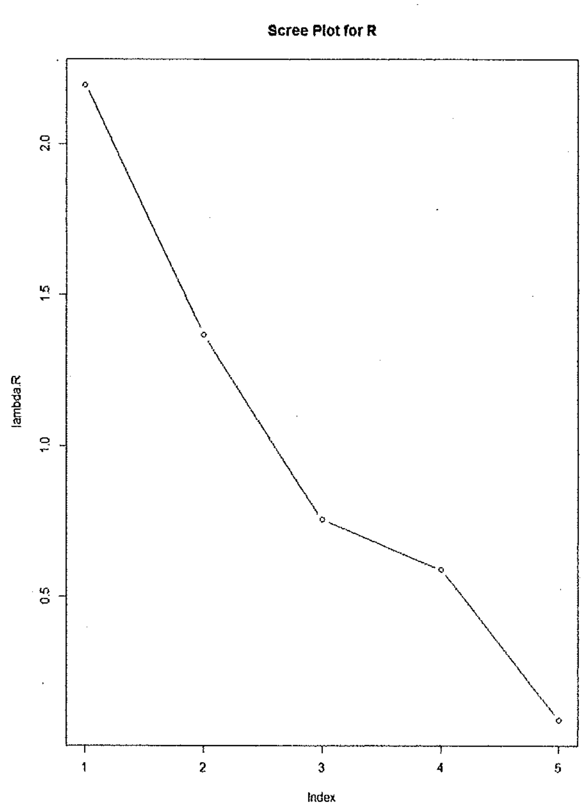
R Text Output

```
> (S <- (1/(n-1)) * (t(X) %*% (I_n - (1/n)*J_n) %*% X))
           X1         X2          X3         X4         X5
X1  34.750209 -4.2766846 -18.0717949 -15.972868   5.716458
X2  -4.276685 17.5134168   0.4197973  -7.868217  -8.723315
X3 -18.071795  0.4197973  29.8447227   9.348837 -13.942159
X4 -15.972868 -7.8682171   9.3488372  33.042636  -9.941860
X5   5.716458 -8.7233154 -13.9421586  -9.941860  26.957961
> (R <- solve(V.12) %*% S %*% solve(V.12))
           [,1]        [,2]        [,3]       [,4]       [,5]
[1,]  1.0000000 -0.17335767 -0.56116271 -0.4713753  0.1867690
[2,] -0.1733577  1.00000000  0.01836202 -0.3270797 -0.4014696
[3,] -0.5611627  0.01836202  1.00000000  0.2977052 -0.4915331
[4,] -0.4713753 -0.32707967  0.29770524  1.0000000 -0.3331093
[5,]  0.1867690 -0.40146956 -0.49153305 -0.3331093  1.0000000
```

Continued

```
> (lambda.S <- eigen(S)$val)
[1] 68.752385 31.508994 23.100973 16.354182  2.392411
> (E.S <- eigen(S)$vec)
            [,1]        [,2]       [,3]        [,4]      [,5]
[1,]  0.57943538  0.07917988  0.6428795 -0.30939267 0.3859629
[2,] -0.04165689  0.61192825 -0.1399143  0.51462195 0.5825777
[3,] -0.52428496  0.21883511 -0.1192554 -0.73403767 0.3524249
[4,] -0.49309245 -0.57215650  0.4221873  0.30427403 0.3983365
[5,]  0.38013742 -0.49398633 -0.6120997 -0.08970196 0.4782893

> (lambda.R <- eigen(R)$val)
[1] 2.19662443 1.36824960 0.75586304 0.58878599 0.09047694
> (E.R <- eigen(R)$vec)
           [,1]         [,2]       [,3]         [,4]      [,5]
[1,]  0.5209626  0.086521361  0.6674512  0.253099293 0.4599582
[2,] -0.1213677  0.788216689 -0.1870605 -0.350892684 0.4537257
[3,] -0.5482732 -0.007941356 -0.1150943  0.732694760 0.3863226
[4,] -0.4391410 -0.490952547  0.2949415 -0.525281896 0.4507873
[5,]  0.4694885 -0.360736798 -0.6475184 -0.007238184 0.4797052

> lambda1.S.lo <- lambda.S[1]/(1+qnorm(.975)*sqrt(2/n))
> lambda1.S.hi <- lambda.S[1]/(1-qnorm(.975)*sqrt(2/n))
>
> cbind(lambda1.S.lo, lambda.S[1], lambda1.S.hi)
     lambda1.S.lo          lambda1.S.hi
[1,]     55.30704 68.75238     90.83461
```





8.19